(FORM 2)

FY 2018 JSPS International Fellowships for Research in Japan

APPLICATION FORM FOR JSPS POSTDOCTORAL FELLOWSHIP FOR RESEARCH IN JAPAN (SHORT-TERM)

<This form should be sent to your proposed host researcher in Japan. Applications should be typed or printed. >

1. Full Name (Your name must be written in the alphabet used to write the English language and match the name in your passport.)									
FAMILY		First			Middle				
HUUSKO		Juha-Matti			Aleksanteri				
2. Nationality	Permanent Residency	3. Date of Birth			4. Sex (Check the box below.)				
Finnish	Finland	Year	Month	Day	x	Male	□	Female	
		1987	04	20					

5. Current Appointment (If you do not have current appointment, type N/A)	
Institution	University of Eastern Finland
Department	Department of Physics and Mathematics
Current Position or Status	Postdoctoral Researcher
Country/Region	Finland

6. Academic Degree (Check the box below and fill in the blanks.)											
Type (PhD, or an equivalent.)					PhD						
Date Obtained	x	Obtained	□	Expected	Year	2017	Month	06	Day	15	
Field	Complex Analysis / Theoretical Mathematics										
Institution	University of Eastern Finland										
Country/Region	Finland										

7. JSPS Fellowship(s) you were granted in the past (Check the box below and fill in the blanks.)				
x	N/A			
□	Postdoctoral Fellowship(Strategic)	Fiscal Year		ID Number that starts with GR
□	Postdoctoral Fellowship (Summer Program)	Fiscal Year		ID Number that starts with SP

8. Names of other fellowship(s) for which you are applying (Check the box below and fill in the blanks.)	
x	N/A
□	JSPS Postdoctoral Fellowship for Research in Japan (Standard)
□	JSPS Postdoctoral Fellowship for Research in Japan (Standard) application through overseas nominating authorities
□	JSPS Postdoctoral Fellowship for Research in Japan (Short-term) application through overseas nominating authorities
□	Other Fellowship(s) ( )

9. Contact Information (Fill in the blanks and indicate your preference by checking the box below to receive the award package from JSPS in case you were selected.)			
Office/Institute	x	Home	□
Postal Code	80100	Postal Code	
Country/Region	Finland	Country/Region	
Mailing Address		Mailing Address	
Department of Physics and Mathematics University of Eastern Finland Juha-Matti Huusko Yliopistokatu 7 80100 Joensuu Finland			

Phone 1(Main and Personal) Give the phone number that begins with country code.	+	358405282815		
Phone 2(Sub) Give the phone number that begins with country code.	+			
E-mail Address 1 (Main and Personal)	juha-matti.huusko@uef.fi			
E-mail Address 2 (Sub)	juhisteri@gmail.com			

10. Letter of Reference/Recommendation				
Name of the Recommender	FAMILY	RÄTTYÄ	First	Jouni
Institution of the Recommender	University of Eastern Finland			

Full Name of the Candidate: Juha-Matti Aleksanteri HUUSKO

11. Higher Education (Start from the latest one. Include your current status if you are a doctoral student.)

Name of University	Location	Degree	Field	Completion Date (Month, Year)
University of Eastern Finland	Finland	PhD	Mathematics	6, 2017
University of Eastern Finland	Finland	Master of Science	Teacher of Mathematics and Physics	9, 2013
University of Eastern Finland	Finland	Bachelor of Science	Mathematics	2, 2013

12. Previous Employment (Start from the latest one. Include your current appointment.)

Name of Institution	Location	Position	From – To (Month, Year)
University of Eastern Finland	Joensuu, Finland	Postdoctoral Researcher	7/2017-9/2017
University of Eastern Finland	Joensuu, Finland	Early Stage Researcher	10/2013-6/2017

13. Awards

Title	Organization	Year
N/A		

14. Language Ability (5: excellent ……1: poor)

	Reading	Writing	Understanding	Speaking
Japanese	5 4 3 2 (1)	5 4 3 2 (1)	5 4 3 2 (1)	5 4 3 2 (1)
English	(5) 4 3 2 1	(5) 4 3 2 1	(5) 4 3 2 1	5 (4) 3 2 1
Finnish	(5) 4 3 2 1	(5) 4 3 2 1	(5) 4 3 2 1	(5) 4 3 2 1
Swedish	5 4 (3) 2 1	5 4 (3) 2 1	5 4 (3) 2 1	5 4 (3) 2 1
	5 4 3 2 1	5 4 3 2 1	5 4 3 2 1	5 4 3 2 1

15. Past/Present Stay(s) in Japan over 3 months

City	From – To (Month, Year)	Purpose of the stay
N/A		

Full Name of the Candidate: Juha-Matti Aleksanteri HUUSKO

16. Research Achievements and Results

During his Ph. D. studies, the problems considered by the candidate where mostly related to different results concerning linear differential equations

$$f^{(k)} + A_{k-1} f^{(k-1)} + \ldots + A_1 f' + A_0 f = 0, \qquad (1)$$

where $k \geq 2$ and $A_0, \ldots, A_{k-1}$ are analytic functions in a simply connected domain D of the complex plane (typically, the unit disk). It is well known that in this case each solution f turns out to be analytic in D.

Localization is a general method which allows to implement known results to new domains. Nevanlinna theory combined with the standard order reduction method yields if-and-only-if relations between the so-called iterated M-order of growth of the coefficients and solution (see J. Heittokangas, R. Korhonen and J. Rättyä, Growth estimates for solutions of linear complex differential equations, *Ann. Acad. Sci. Fenn. Math.* **29** (2004), 233 – 246, for instance).

Integration methods have proved to be an efficient tool when all solutions of (1) or their derivatives are forced to be in the weighted space $H_omega^infty(D)$ by giving a sufficient condition on the coefficients A_j. Such conditions have earlier been consider by other authors (Gröhn, Heittokangas, Korhonen and Rättyä) in a series of different papers, using Picard's successive approximations and integral estimates based on Gronwall's lemma or Herold's comparison theorem.

An operator theoretic approach, originated by Pommerenke (On the mean growth of the solutions of complex linear differential equations in the disc, *Complex Var. Theory Appl.* **1** (1982), 23–38), is based on the fact that if X is an admissible normed space of analytic functions in D, f is a solution of (1), and certain integral operator is a contraction in X, then f can be described completely in terms of f(0), f'(0) and the image of f under the given operator.

Huusko's thesis consisted of the publications [H1], [H2] and [H4] listed in the following section. The most relevant research achievements and results obtained in these papers are:

- Description of a general localization method which can be applied to the analysis of solutions of differential equations in the unit disc of the complex plane. In particular, Huusko introduces a particular mapping which can detect exponential growth of solutions of the equation near a given boundary point (see [H1] in Section 17).
- In [H2], we propose sufficient conditions for the coefficients A_j in (1) that guarantee that the growth of the corresponding solutions of the equation is controlled by a given weight ω.
- A sharp condition on the coefficient A, analytic in the unit disc, such that all solutions of the equation

  $$f'' + Af = 0 \qquad (2)$$

  belong to the classical Bloch space of analytic functions in the unit disk which are Lipschitz as applications between the following metric spaces: the unit disk endowed with the hyperbolic metric and the complex plane endowed with the Euclidean metric.
- First result in the literature concerning the BMOA solutions (that is, analytic functions in the unit disk whose boundary values have bounded mean oscillation) of equation (2) is introduced in [H4]
- Also, in [H4], and among other results, a counterpart of the Hardy-Stein-Spencer formula for higher order derivatives is presented.

Two more papers have been produced by the candidate that parallel but do not take part of his Ph. D. Studies.

- In the paper [H3] (see Section 17 below), the authors introduce a criterion for the finite valence of a given complex-valued harmonic function in a simply connected domain of the complex plane in terms of the Pre-Schwarzian and Schwarzian derivatives, thus generalizing different theorems by Gehring and Pommerenke (On the Nehari univalence criterion and quasicircles, *Comment. Math. Helvetici* **59**, 226– 242) and Becker and Pommerenke (Locally univalent functions and the Bloch and Dirichlet norm, *Comput. Methods Funct. Theory* **16** (2016), 43–52) in the analytic setting to the more general case when the functions considered are merely harmonic.
- Finally, in [H5], the authors consider some "Becker criterion of univalence type conditions". They obtain sufficient conditions which imply that the analytic function f is bounded, belongs to the Bloch space or belongs to the class of normal functions, are discussed. Moreover, generalizations to locally univalent harmonic functions are obtained.

These results open a new line of research that is related but different from the ones considered in his Ph. D. Thesis.

17. List of Major Publications
Authors (all), title, Journal, Vol. , No , pp. - , Month, Year
(The list includes the peer-reviewed studies that have been accepted for publication and in pre-publication stage.)

In order to distinguish the published papers due to the applicant from those due to other authors that will be listed in Section 18 below, we have decided to list the first ones using the nomenclature [H*] for the first ones and a number for the seconds.

Here, we list the first group of paper by according to the date of publication (or acceptance).

- [H1] J.-M. Huusko, Localisation of linear differential equations in the unit disc by a conformal map, *Bull. Aust. Math. Soc.* **93**, no. 2, 260 – 271, October, 2015.
- [H2] J.-M. Huusko, T. Korhonen, A. Reijonen, Linear differential equations with solutions in the growth space H_omega^∞, *Ann. Acad. Sci. Fenn. Math.* **41**, no. 1, 399 – 416, January, 2016.
- [H3] J.-M. Huusko, M. Martín, Criteria for bounded valence of harmonic mappings, *Comput. Methods Funct. Theory*, DOI: 10.1007/s40315-017-0197-z. Published electronically on March 28th. 2017.
- [H4] J. Gröhn, J-M. Huusko, J. Rättyä, Linear differential equations with slowly growing solutions, to appear in *Trans. Amer. Math. Soc.*
- [H5] J-M. Huusko, T, Vesikko, On Becker's univalence criterion, to appear in *J. Math. Anal. Appl.*

18. Research Plan in Japan
a. Background of proposed research plan
b. Purpose of proposed research
c. Proposed plan
d. Expected results and impacts

a. As was mentioned before, in his PhD Huusko studied linear differential equations in the unit disc of the complex plane (see publications [H1], [H2] and [H4] in the previous section.). In [H1], Huusko studied how to use localization maps in the study of differential equations. In particular, a particularly useful univalent map from the unit disc into itself was used.

   In [H2], the authors studied how solutions (or the derivatives of solutions) of differential equations can be placed in certain growth spaces. As an important special case, the authors obtain a sharp condition on the coefficient A, analytic in the unit disc, such that all solutions of the equation f´´+Af=0 belong to the Bloch space (or to a more general Q_K space). This topic was studied in more detail in [H4], where the authors obtain an essentially equivalent result by a different method. As the reference [7] below suggests, the complete solution of the question: when do solutions of (1) belong to the Bloch space? requires some different approach. Also solutions in some other function spaces, such as BMOA, and bounded solutions where studied in [H4].

   After his PhD, the applicant has done research on univalence properties of harmonic mappings. In fact, recently, the classical theory of univalent functions has been under active research recently. Classical results concerning analytic functions are being generalized for harmonic functions, for example, by Hernandez, Liu, Ponnusamy, Martín, and other authors (see [12] and the references therein). Also, the sharpness of classical univalence results has resently been studied in detail, see [6], for example. There are several extremal problems mentioned here, where the sharp constants have been conjectured but nowhere near proved.

   The study of univalence properties of harmonic mappings is progressing very slowly. Just recently, a definition of the pre-Schwarzian and Schwarzian derivatives of such mappings has been presented, see [9]. This new operators have allowed to extend classical theorems for analytic mappings to those cases when the functions considered are just harmonic. Now these operators are widely accepted and under extensive study, see for example [12]. Due to non-sufficient insight, powerful tools (such as Loewner chains) available in the theory of analytic functions, have not yet been established for harmonic functions. It would be beneficial for the theory, due, for instance, to the relation between Teichmüller spaces and quasiconformal mappings if this connections would be investigated.

   One important topic is the univalence criteria based on the classical well-known univalence criteria by Nehari and Becker (see [13] and [1], respectively). The assumptions in these criteria many times imply geometrical properties for the image domain. The image domain can be, for example, bounded [2], a quasi-disc [15] or a John disc [8].

   In 2016, a finite valence version of Nehari's criterion was proved in [2]. In [H3], Huusko and Martín generalized this result for harmonic functions. In the preprint [H5], the authors studied the effect of Becker criterion with a linear error on the univalence properties of the function.

b. The proposed host, Professor Hiroshige Shiga, is an expert on many sophisticated areas in Mathematics. For example, Riemann surfaces, Teichmüller theory, Kleinian and Fuchsian groups and quasiconformal mappings. Teichmüller theory has intimate connections to univalent functions, see for example [11].

c. During the visit, Huusko will learn about following things:

   1) Knowledge about Riemann surfaces allows Huusko to pursue the study of differential equations in Riemann surfaces.
   2) Teichmüller theory has connections to classical function spaces (Bloch space, BMOA, Q_K spaces), see [16], and offers more insight to the study of differential equations.
   3) Poincaré introduced Kleinian groups in the 1880s as the monodromy groups of certain second order differential equations on the complex plane [4]. During a minicourse in Joensuu, Autumn 2016, Huusko learned about solving second order differential equations by means of Gaussian hypergeometric function (see [5]). Learning Kleinian groups will give Huusko more insight to differential equations.
   4) Teichmüller is intimately related the theory of univalent functions. The use of Teichmüller theory in the study of univalence properties of harmonic functions will be investigated.

d. The applicant and the prospective supervisor will do research and write a survey paper on the interrelationship of Teichmüller spaces and univalence properties of harmonic functions. Huusko will keep in touch with María Martín, a colleague from Joensuu that will move to Universidad Autónoma de Madrid, Spain, next year.
   It is expected, that Teichmüller theory will lead to new inventions in the study of univalence properties of harmonic functions. Research papers about this will be done.

18. Research Plan in Japan (Continued)

References

- [H1] – [H5]; see section 17.
- [1] J. Becker, Löwnersche Differentialgleichung und quasikonform fortsetzbare schlichte Funktionen, (German) *J. Reine Angew. Math.* 255 (1972), 23–43.
- [2] J. Becker, Ch. Pommerenke, Locally univalent functions and the Bloch and Dirichlet norm. *Comput. Methods Funct. Theory* **16** (2016), no. 1, 43–52.
- [3] L. Bers, Finitely generated Kleinian groups. An introduction. *Ann. Acad. Sci. Fenn. Ser. A I Math.* **13** (1988), no. 3, 313–327.
- [4] A. Cano, J.-P. Navarrete, J. Seade, *Complex Kleinian groups*, http://www.matcuer.unam.mx/~jseade/Kleinian-Groups.pdf
- [5] E. Y.-M. Chiang, Complex analysis and special functions. minicourse in 2016 in Joensuu, Finland
- [6] J. Gevirtz, The set of infinite valence values of an analytic function, preprint. arXiv:1508.05416v1 [math.CV].
- [7] J. Gröhn, Slowly growing solutions of ODEs revisited, preprint. arXiv:1707.09760v1 [Math.CA].
- [8] K. Hag, P. Hag, John disks and the pre-Schwarzian derivative. *Ann. Acad. Sci. Fenn. Math.* **26** (2001), no. 1, 205–224.
- [9] R. Hernández, M. J. Martín, Pre-Schwarzian and Schwarzian derivatives of harmonic mappings. *J. Geom. Anal.* **25** (2015), no. 1, 64–91.
- [10] I. Laine, *Nevanlinna Theory and Complex Differential Equations*, Walter de Gruyter, Berlin, 1993.
- [11] O. Lehto, *Univalent Functions and Teichmüller Spaces*. Graduate Texts in Mathematics, 109. Springer-Verlag, New York, 1987.
- [12] G. Liu, S. Ponnusamy, Uniformly locally univalent harmonic mappings associated with the pre-Schwarzian norm, preprint.
- [13] Z. Nehari, The Schwarzian derivative and schlicht functions, *Bull. Amer. Math. Soc.* **55** (1949), 545–551.
- [14] A. Papadopoulos, *Introduction to Teichmüller Theory, Old and New, VI*. Handbook of Teichmüller theory. Vol. VI, 1–29, IRMA Lect. Math. Theor. Phys., 27, Eur. Math. Soc., Zürich, 2016
- [15] H. Shiga, *Modulus of Continuity, a Hardy-Littlewood Theorem and its Application*. Infinite dimensional Teichmüller spaces and moduli spaces, 127–133, RIMS Kôkyûroku Bessatsu, B17, Res. Inst. Math. Sci. (RIMS), Kyoto, 2010.
- [16] H. Wulan, F. Ye, Universal Teichmüller space and QK spaces. *Ann. Acad. Sci. Fenn. Math.* **39** (2014), no. 2, 691–709.

19. Your Academic Goals and Career Prospects after the Fellowship

After the fellowship, I will seek for

1) another fellowship in Japan, to continue work with new contacts
2) a grant to stay 1 year in Madrid, Spain, together with María Martín and Dragan Vukotić
3) a grant to stay 1 year in Chile, together with Martin Chuaqui, Iason Efraimidis and Rodrigo Hernández

Then I will search for a tenure position from Finland, Japan, Spain or Chile.

20. Required Conditions for the Applicants of this Fellowship
Please read the following conditions carefully. **You are not eligible to apply if you do not meet all the conditions.**

☑ I do not have Japanese nationality/permanent residency. (If you are applying for or planning to obtain Japanese nationality/permanent residency, please note that the Fellowship will terminate on the day you acquire either of them.)

☑ I have not been received the Fellowship(s) of the following program(s) in the past. If you were awarded one or more of the following Fellowship(s) but did not accept it/them, you are eligible to apply.
a) JSPS Postdoctoral Fellowship for Research in Japan (Standard)
b) JSPS Postdoctoral Fellowship for Research in Japan (Pathway)
c) JSPS Postdoctoral Fellowship for Research in Japan(Short-term)

☑ I do not have residence card with mailing address in Japan in the duration of the call for application issued by JSPS.

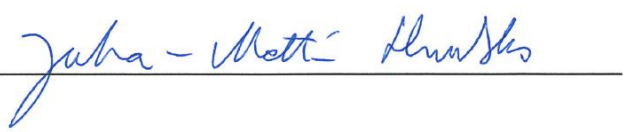
☑My doctorate was/will be conferred by the institution outside of Japan.

☑ My research is not related to military affairs.

I have read the above conditions and I meet all of them. I certify that the information provided on this application is true and correct.

Date: 20.09.2017

Full Name (Print) : Juha-Matti Aleksanteri Huusko

Signature: 

(Notes)

1. Please sign this form and forward it first to your proposed host researcher in Japan, instead of sending it to JSPS.
2. The following documents must be attached: A letter of reference/recommendation from your current or previous supervisor (not from your prospective Japanese host researcher). **The letter should be no longer than one page (single-side).**

Full Name of the Candidate: Juha-Matti Aleksanteri HUUSKO