Contents

1 Limits and Continuity **6**
- 1.1 Examples of Velocity, Growth Rate, and Area 6
  - 1.1.1 Average Velocity and Instantaneous Velocity 6
  - 1.1.2 The Growth of an Algal Culture 6
  - 1.1.3 The Area of a Circle 6
- 1.2 Limits of Functions 6
  - 1.2.1 One-Sided Limits 6
  - 1.2.2 Rules for Calculating Limits 6
  - 1.2.3 The Squeeze Theorem 6
- 1.3 Limits at Infinity and Infinite Limits 6
  - 1.3.1 Limits at Infinity 6
  - 1.3.2 Limits at Infinity for Rational Functions 6
  - 1.3.3 Infinite Limits 6
  - 1.3.4 Using Maple to Calculate Limits 6
- 1.4 Continuity 6
  - 1.4.1 Continuity at a Point 6
  - 1.4.2 Continuity on an Interval 6
  - 1.4.3 There Are Lots of Continuous Functions 6
  - 1.4.4 Continuous Extensions and Removable Discontinuities . . 6
  - 1.4.5 Continuous Functions on Closed, Finite Intervals 6
  - 1.4.6 Finding Roots of Equations 6
- 1.5 The Formal Definition of Limit 6
  - 1.5.1 Using the Definition of Limit to Prove Theorems 6
  - 1.5.2 Other Kinds of Limits 6
  - 1.5.3 Chapter Review 6

2 Differentiation **6**
- 2.1 Tangent Lines and Their Slopes 6
  - 2.1.1 Normals 6
- 2.2 The Derivative 6
  - 2.2.1 Some Important Derivatives 6
  - 2.2.2 Leibniz Notation 6
  - 2.2.3 Differentials 6
  - 2.2.4 Derivatives Have the Intermediate-Value Property 6
- 2.3 Differentiation Rules 6
  - 2.3.1 Sums and Constant Multiples 6
  - 2.3.2 The Product Rule 6
  - 2.3.3 The Reciprocal Rule 6
  - 2.3.4 The Quotient Rule 6
- 2.4 The Chain Rule 6
  - 2.4.1 Finding Derivatives with Maple 6
  - 2.4.2 Building the Chain Rule into Differentiation Formulas . . 6
  - 2.4.3 Proof of the Chain Rule (Theorem 6) 6
- 2.5 Derivatives of Trigonometric Functions 6

2.5.1 Some Special Limits . . . 6
2.5.2 The Derivatives of Sine and Cosine . . . 6
2.5.3 The Derivatives of the Other Trigonometric Functions . . 6
2.6 Higher-Order Derivatives . . . 6
2.7 Using Differentials and Derivatives . . . 6
2.7.1 Approximating Small Changes . . . 6
2.7.2 Average and Instantaneous Rates of Change . . . 6
2.7.3 Sensitivity to Change . . . 6
2.7.4 Derivatives in Economics . . . 6
2.8 The Mean-Value Theorem . . . 6
2.8.1 Increasing and Decreasing Functions . . . 6
2.8.2 Proof of the Mean-Value Theorem . . . 6
2.9 Implicit Differentiation . . . 6
2.9.1 Higher-Order Derivatives . . . 6
2.9.2 The General Power Rule . . . 6
2.10 Antiderivatives and Initial-Value Problems . . . 6
2.10.1 Antiderivatives . . . 6
2.10.2 The Indefinite Integral . . . 6
2.10.3 Differential Equations and Initial-Value Problems . . . 6
2.11 Velocity and Acceleration . . . 6
2.11.1 Velocity and Speed . . . 6
2.11.2 Acceleration . . . 6
2.11.3 Falling Under Gravity . . . 6
2.11.4 Chapter Review . . . 6

3 Transcendental Functions 6
3.1 Inverse Functions . . . 6
3.1.1 Inverting Non-One-to-One Functions . . . 6
3.1.2 Derivatives of Inverse Functions . . . 6
3.2 Exponential and Logarithmic Functions . . . 6
3.2.1 Exponentials . . . 6
3.2.2 Logarithms . . . 6
3.3 The Natural Logarithm and Exponential . . . 6
3.3.1 The Natural Logarithm . . . 6
3.3.2 The Exponential Function . . . 6
3.3.3 General Exponentials and Logarithms . . . 6
3.3.4 Logarithmic Differentiation . . . 6
3.4 Growth and Decay . . . 6
3.4.1 The Growth of Exponentials and Logarithms . . . 6
3.4.2 Exponential Growth and Decay Models . . . 6
3.4.3 Interest on Investments . . . 6
3.4.4 Logistic Growth . . . 6
3.5 The Inverse Trigonometric Functions . . . 6
3.5.1 The Inverse Sine (or Arcsine) Function . . . 6
3.5.2 The Inverse Tangent (or Arctangent) Function . . . 6
3.5.3 Other Inverse Trigonometric Functions . . . 6

3.6 Hyperbolic Functions 6
3.6.1 Inverse Hyperbolic Functions 6
3.7 Second-Order Linear DEs with Constant Coefficients 6
3.7.1 Recipe for Solving ay" + by' + cy = 0 6
3.7.2 Simple Harmonic Motion 6
3.7.3 Damped Harmonic Motion 6
3.7.4 Chapter Review 6

4 More Applications of Differentiation 6
4.1 Related Rates 6
4.1.1 Procedures for Related-Rates Problems 6
4.2 Finding Roots of Equations 6
4.2.1 Discrete Maps and Fixed-Point Iteration 6
4.2.2 Newtons Method 6
4.2.3 Solve" Routines 6
4.3 Indeterminate Forms 6
4.3.1 l'Hospital's Rules 6
4.4 Extreme Values 6
4.4.1 Maximum and Minimum Values 6
4.4.2 Critical Poi., Singular Points, and Endpoints 6
4.4.3 Finding Absolute Extreme Values 6
4.4.4 The First Derivative Test 6
4.4.5 Functions Not Defined on Closed, Finite Intervals 6
4.5 Concavity and Inflections 6
4.5.1 The Second Derivative Test 6
4.6 Sketching the Graph of a Function 6
4.6.1 Asymptotes 6
4.6.2 Examples of Formal Curve Sketching 6
4.7 Graphing with Computers 6
4.7.1 Numerical Monsters and Computer Graphing 6
4.7.2 Floating-Point Representation of Numbers in Computers 6
4.7.3 Machine Epsilon and Its Effect on Figure 4.45 6
4.7.4 Determining Machine Epsilon 6
4.8 Extreme-Value Problems 6
4.8.1 Procedure for Solving Extreme-Value Problems 6
4.9 Linear Approximations 6
4.9.1 Approximating Values of Functions 6
4.9.2 Error Analysis 6
4.10 Taylor Polynomials 6
4.10.1 Taylor's Formula 6
4.10.2 Big-O Notation 6
4.10.3 Evaluating Limits of Indeterminate Forms 6
4.11 Roundoff Error, Truncation Error, and Computers 6
4.11.1 Taylor Polynomials in Maple 6
4.11.2 Persistent Roundoff Error 6
4.11.3 Truncation, Roundoff, and Computer Algebra 6

4.11.4 Chapter Review 6

5 Integration **6**
5.1 Sums and Sigma Notation 6
5.1.1 Evaluating Sums 6
5.2 Areas as Limits of Sums 6
5.2.1 The Basic Area Problem 6
5.2.2 Some Area Calculations 6
5.3 The Definite Integral 6
5.3.1 Partitions and Rieman Sums 6
5.3.2 The Definite Integral 6
5.3.3 General Rieman Sums 6
5.4 Properties of the Definite Integral 6
5.4.1 A Mean-Value Theorem for Integrals 6
5.4.2 Definite Integrals of Piecewise Continuous Functions . . . 6
5.5 The Fundamental Theorem of Calculus 6
5.6 The Method of Substitution 6
5.6.1 Trigonometric Integrals 6
5.7 Areas of Plane Regions 6
5.7.1 Areas Between Two Curves 6
5.7.2 Chapter Review 6

6 Techniques of Integration **6**
6.1 Integration by Parts 6
6.1.1 Reduction Formulas 6
6.2 Integrals of Rational Functions 6
6.2.1 Linear and Quadratic Denominators 6
6.2.2 Partial Fractions 6
6.2.3 Completing the Square 6
6.2.4 Denominators with Repeated Factors 6
6.3 Inverse Substitutions 6
6.3.1 The Inverse Trigonometric Substitutions 6
6.3.2 Inverse Hyperbolic Substitutions 6
6.3.3 Other Inverse Substitutions 6
6.3.4 The tan(x/2) Substitution 6
6.4 Other Methods for Evaluating Integrals 6
6.4.1 The Method of Undetermined Coefficients 6
6.4.2 Using Maple for Integration 6
6.4.3 Using Integral Tables 6
6.4.4 Special Functions Arising from Integrals 6
6.5 Improper Integrals 6
6.5.1 Improper Integrals of Type I 6
6.5.2 Improper Integrals of Type II 6
6.5.3 Estimating Convergence and Divergence 6
6.6 The Trapezoid and Midpoint Rules 6
6.6.1 The Trapezoid Rule 6

6.6.2 The Midpoint Rule . . . 6
6.6.3 Error Estimates . . . 6
6.7 Simpson's Rule . . . 6
6.8 Other Aspects of Approximate Integration . . . 6
6.8.1 Approximating Improper Integrals . . . 6
6.8.2 Using Taylor's Formula . . . 6
6.8.3 Romberg Integration . . . 6
6.8.4 The Importance of Higher-Order Methods . . . 6
6.8.5 Other Methods . . . 6
6.8.6 Chapter Review . . . 6

7 Applications of Integration 6
7.1 Volumes by Slicing, Solids of Revolution . . . 6
7.1.1 Volumes by Slicing . . . 6
7.1.2 Solids of Revolution . . . 6
7.1.3 Cylindrical Shells . . . 6
7.2 More Volumes by Slicing . . . 6
7.3 Arc Length and Surface Area . . . 6
7.3.1 Arc Length . . . 6
7.3.2 The Arc Length of the Graph of a Function . . . 6
7.3.3 Areas of Surfaces of Revolution . . . 6
7.4 Mass, Moments, and Centre of Mass . . . 6
7.4.1 Mass and Density . . . 6
7.4.2 Moments and Centres of Mass . . . 6
7.4.3 Two- and Three-Dimensional Examples . . . 6
7.5 Centroids . . . 6
7.5.1 Pappus's Theorem . . . 6
7.6 Other Physical Applications . . . 6
7.6.1 Hydrostatic Pressure . . . 6
7.6.2 Work . . . 6
7.6.3 Potential Energy and Kinetic Energy . . . 6
7.7 Applications in Business, Finance, and Ecology . . . 6
7.7.1 The Present Value of a Stream of Payments . . . 6
7.7.2 The Economics of Exploiting Renewable Resources . . . 6
7.8 Probability . . . 6
7.8.1 Discrete Random Variables . . . 6
7.8.2 Expectation, Mean, Variance, and Standard Deviation . . . 6
7.8.3 Continuous Random Variables . . . 6
7.8.4 The Normal Distribution . . . 6
7.8.5 Heavy Tails . . . 6
7.9 First-Order Differential Equations . . . 6
7.9.1 Separable Equations . . . 6
7.9.2 First-Order Linear Equations . . . 6
7.9.3 Chapter Review . . . 6

1 Limits and Continuity

1.1 Examples of Velocity, Growth Rate, and Area

1.1.1 Average Velocity and Instantaneous Velocity

1.1.2 The Growth of an Algal Culture

1.1.3 The Area of a Circle

1.2 Limits of Functions

1.2.1 One-Sided Limits

1.2.2 Rules for Calculating Limits

1.2.3 The Squeeze Theorem

1.3 Limits at Infinity and Infinite Limits

1.3.1 Limits at Infinity

1.3.2 Limits at Infinity for Rational Functions

1.3.3 Infinite Limits

1.3.4 Using Maple to Calculate Limits

1.4 Continuity

1.4.1 Continuity at a Point

1.4.2 Continuity on an Interval

1.4.3 There Are Lots of Continuous Functions

1.4.4 Continuous Extensions and Removable Discontinuities

1.4.5 Continuous Functions on Closed, Finite Intervals

1.4.6 Finding Roots of Equations

1.5 The Formal Definition of Limit

1.5.1 Using the Definition of Limit to Prove Theorems

1.5.2 Other Kinds of Limits

1.5.3 Chapter Review

2 Differentiation

2.1 Tangent Lines and Their Slopes

2.1.1 Normals

2.2 The Derivative

2.2.1 Some Important Derivatives

2.2.2 Leibniz Notation

2.2.3 Differentials

2.2.4 Derivatives Have the Intermediate-Value Property

2.3 Differentiation Rules

2.3.1 Sums and Constant Multiples

2.3.2 The Product Rule

2.3.3 The Reciprocal Rule

2.3.4 The Quotient Rule