SOME MATLAB FILES

JUHA-MATTI HUUSKO

Abstract. Here are some of my Matlab files, and some calculations.

Contents

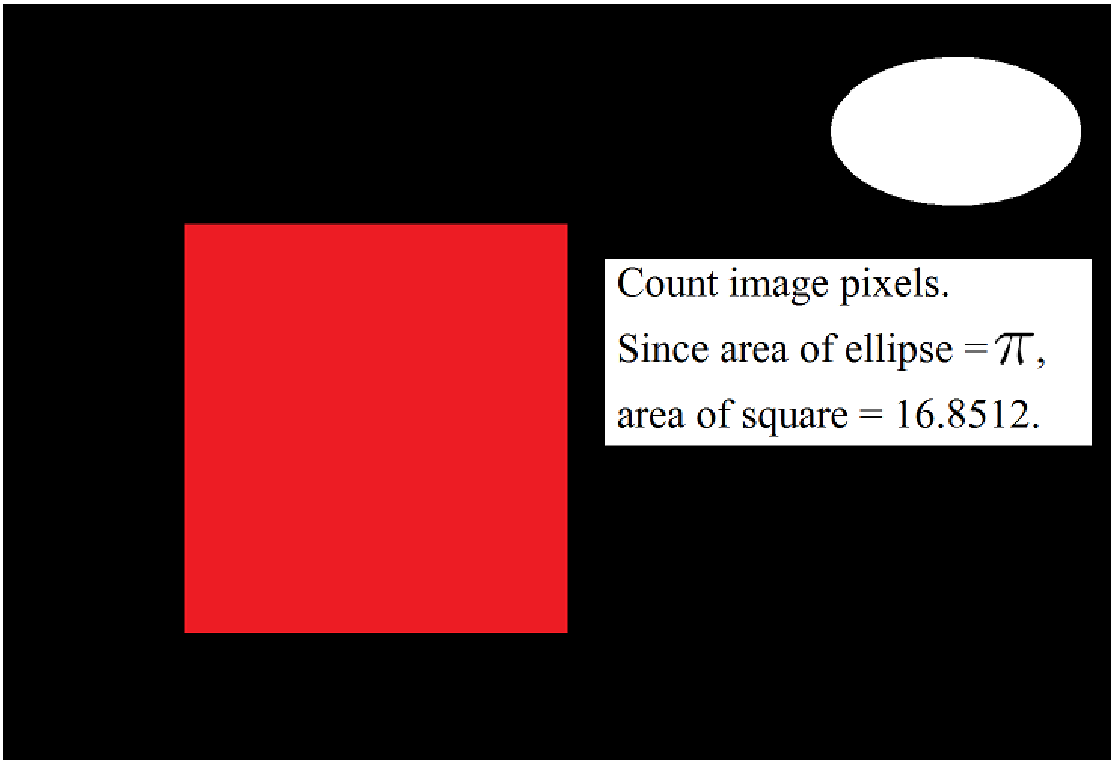
Most files are here:
`http://integraali.com/stuff/`

Date: December 1, 2017.

1. Image editing

1.1. **Area calculator.** Counts pixels in a picture. Given a picture with two domains E, D in different colors and a black background, the function counts the pixels in each domain and calculates the quotient $\operatorname{area}(E)/\operatorname{area}(D)$.

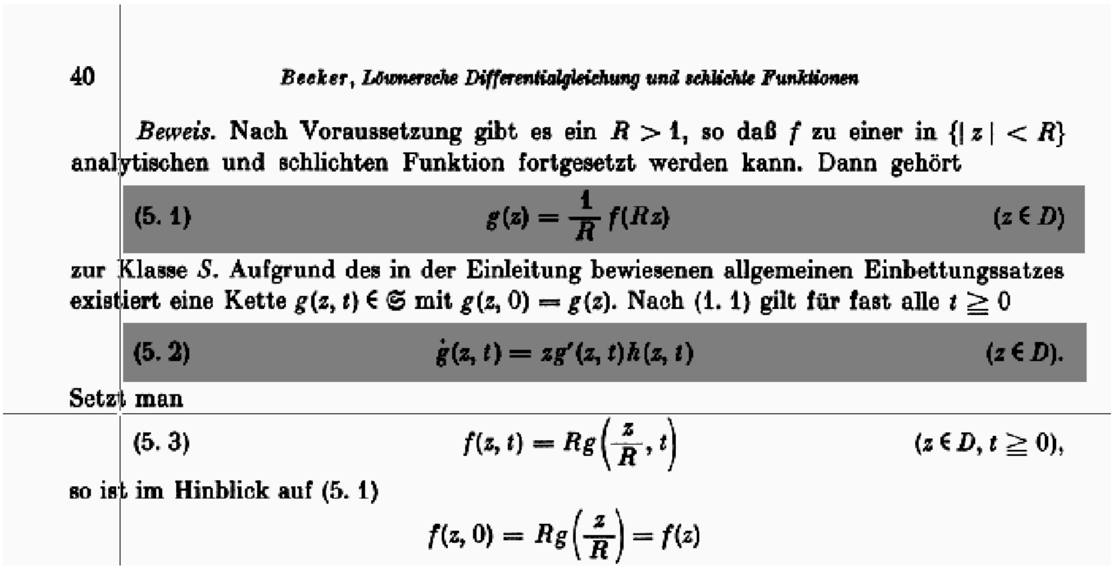
Second part of the function assumes that $\operatorname{area}(D) = \pi$ and calculates $\operatorname{area}(E)$ by the formula $\operatorname{area}(E) = \pi * \operatorname{area}(E)/\operatorname{area}(D)$.



Download in:
https://se.mathworks.com/matlabcentral/fileexchange/62210-area-pix-picture-filename-

1.2. **Automatic picture snipping tool.** An automatic snipping tool for pictures and formulas. Saves the formula images automatically to .PNG and .EPS format.

If a pdf document is translated through OCR-software and Google Translate into a Word/LaTeXfile, the formulas can be added in a simple way.



40 *Becker, Löwnersche Differentialgleichung und schlichte Funktionen*

Beweis. Nach Voraussetzung gibt es ein $R > 1$, so daß f zu einer in $\{|z| < R\}$ analytischen und schlichten Funktion fortgesetzt werden kann. Dann gehört

$$g(z) = \frac{1}{R} f(Rz) \qquad (z \in D) \tag{5. 1}$$

zur Klasse S. Aufgrund des in der Einleitung bewiesenen allgemeinen Einbettungssatzes existiert eine Kette $g(z, t) \in \mathfrak{S}$ mit $g(z, 0) = g(z)$. Nach (1. 1) gilt für fast alle $t \geqq 0$

$$\dot{g}(z, t) = zg'(z, t)h(z, t) \qquad (z \in D). \tag{5. 2}$$

Setzt man

$$f(z, t) = Rg\left(\frac{z}{R}, t\right) \qquad (z \in D, t \geqq 0), \tag{5. 3}$$

so ist im Hinblick auf (5. 1)

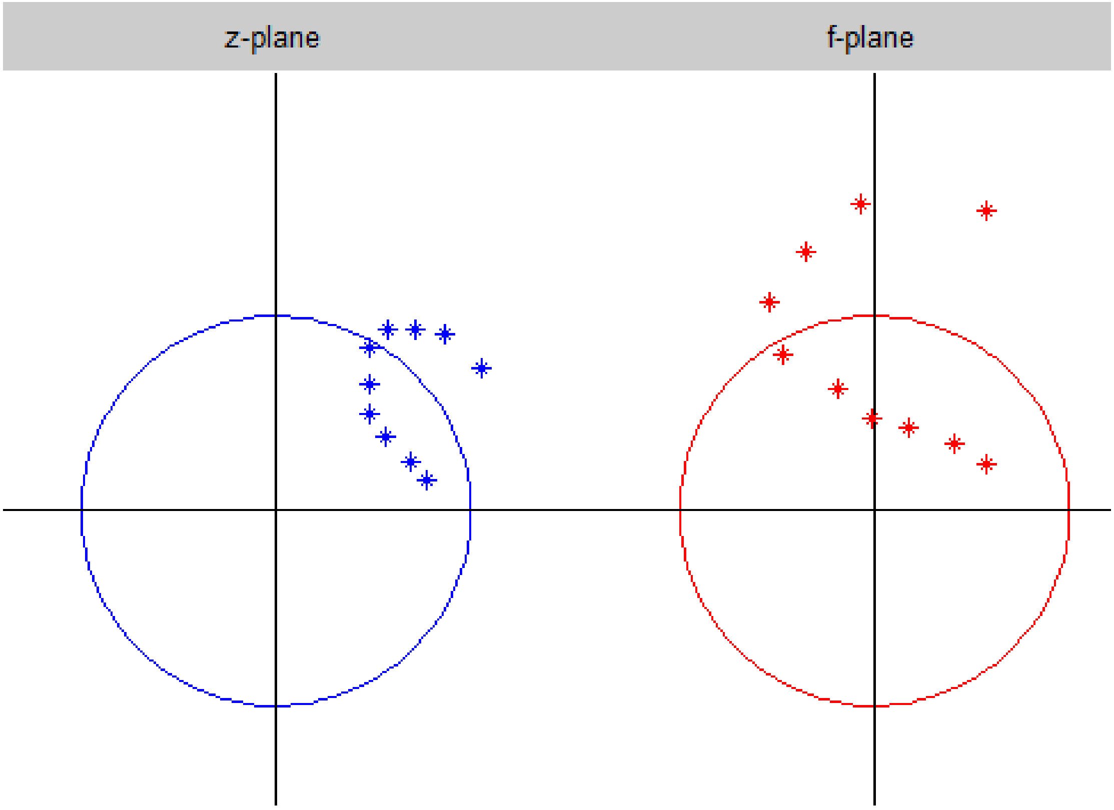
$$f(z, 0) = Rg\left(\frac{z}{R}\right) = f(z)$$

Download in:

https://se.mathworks.com/matlabcentral/fileexchange/62211-formula-snip-filename-filetype-opt-

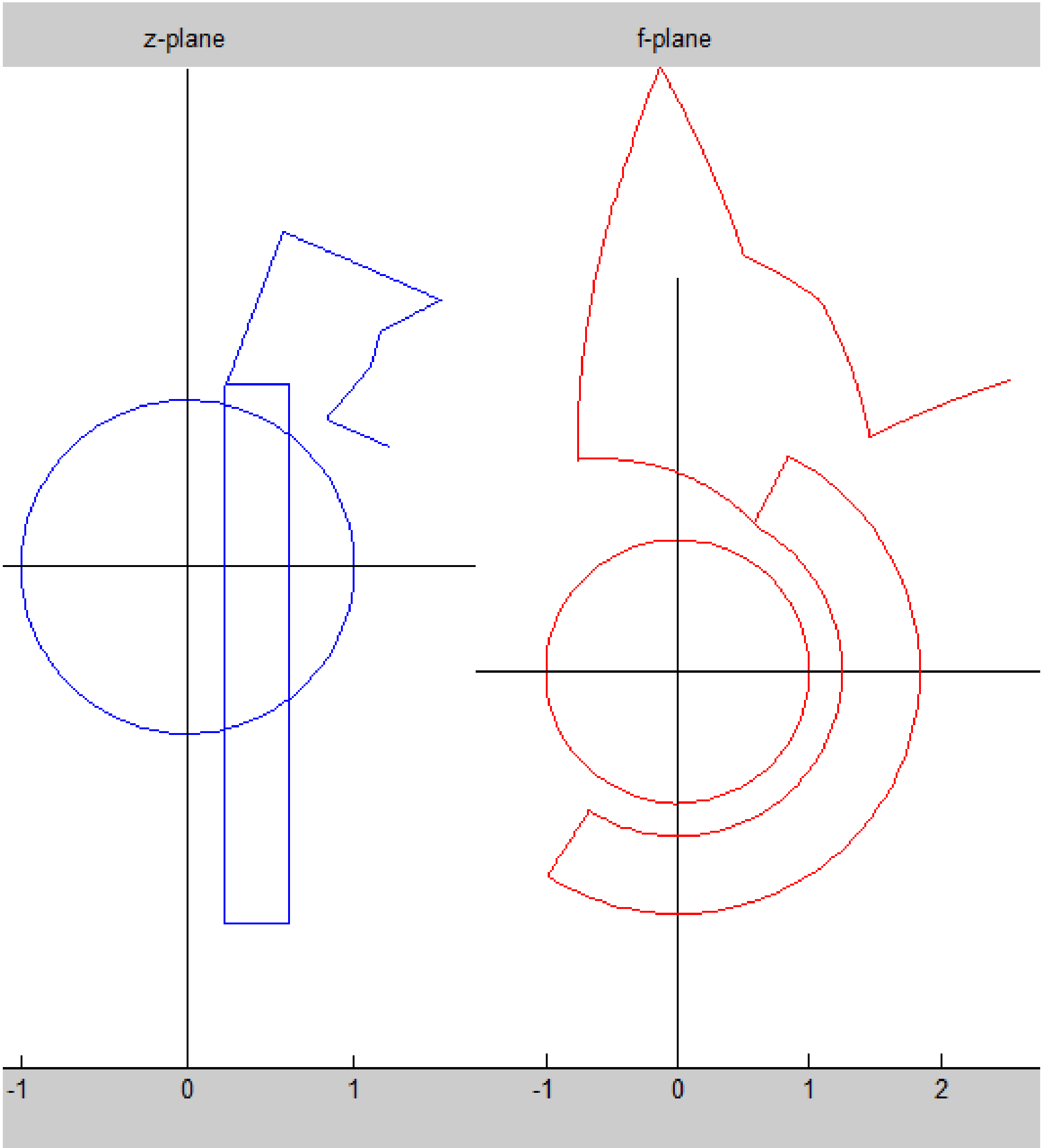
2. Complex analysis

2.1. **Complex map tool 1.** Maps dots by a complex map $z \mapsto f(z)$. Dots are given by graphical input. In the example picture, the map $z \mapsto z^2$ is applied.



Download in:
`https://se.mathworks.com/matlabcentral/fileexchange/62212-complex1-f-`

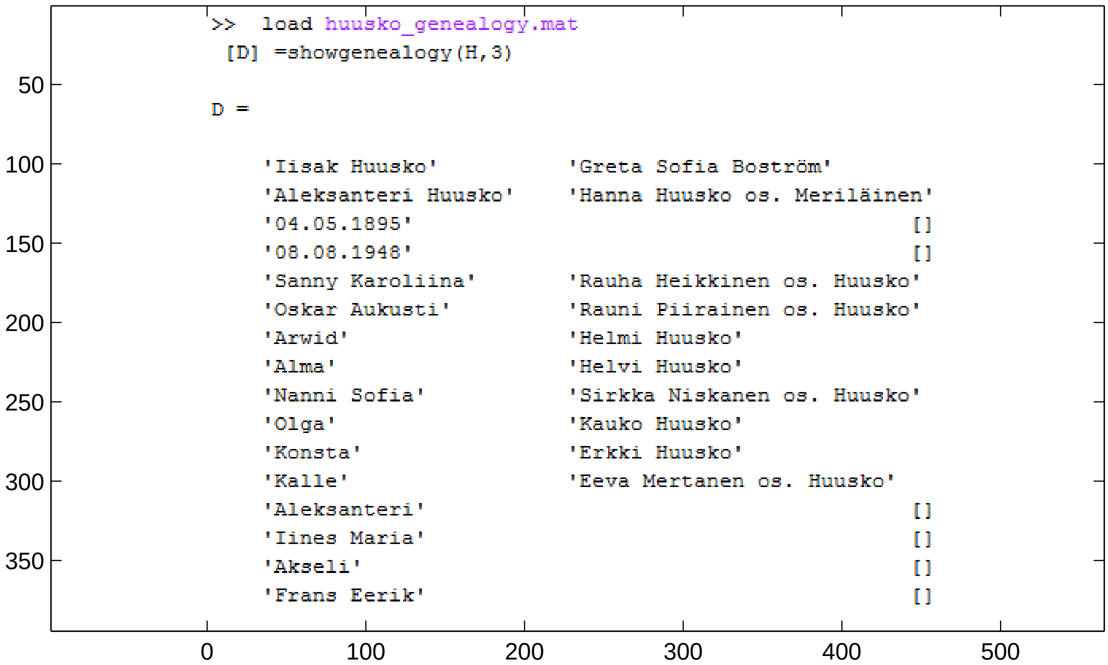
2.2. **Complex map tool 2.** Maps an polygonal chain by a complex map $z \mapsto f(z)$. Vertices are given by graphical input. In the example picture, the map $z \mapsto e^z$ is applied.



Download in:
https://se.mathworks.com/matlabcentral/fileexchange/62213-complex2-f-

3. Text editing

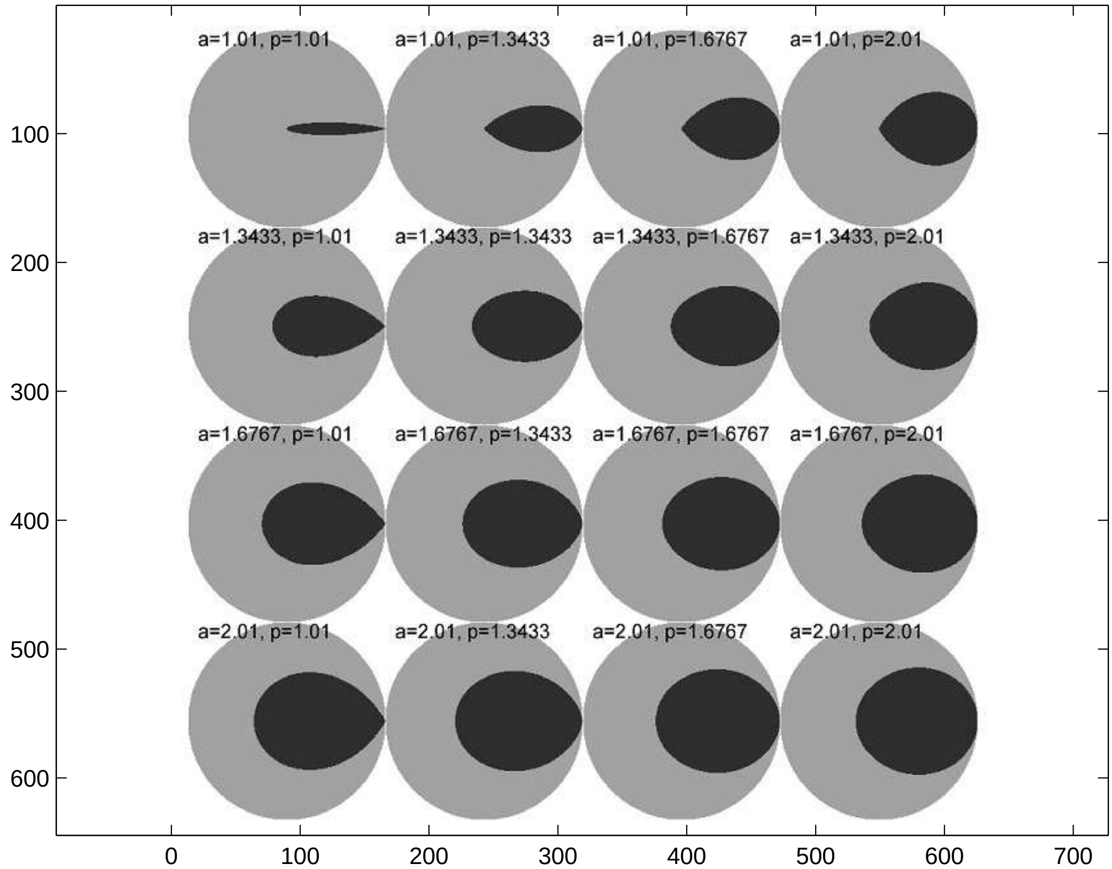
3.1. **Genealogy.** I have written some genealogy details to a file. The function showgenealogy.mat shows the information nicely. The graphics need to be improved.

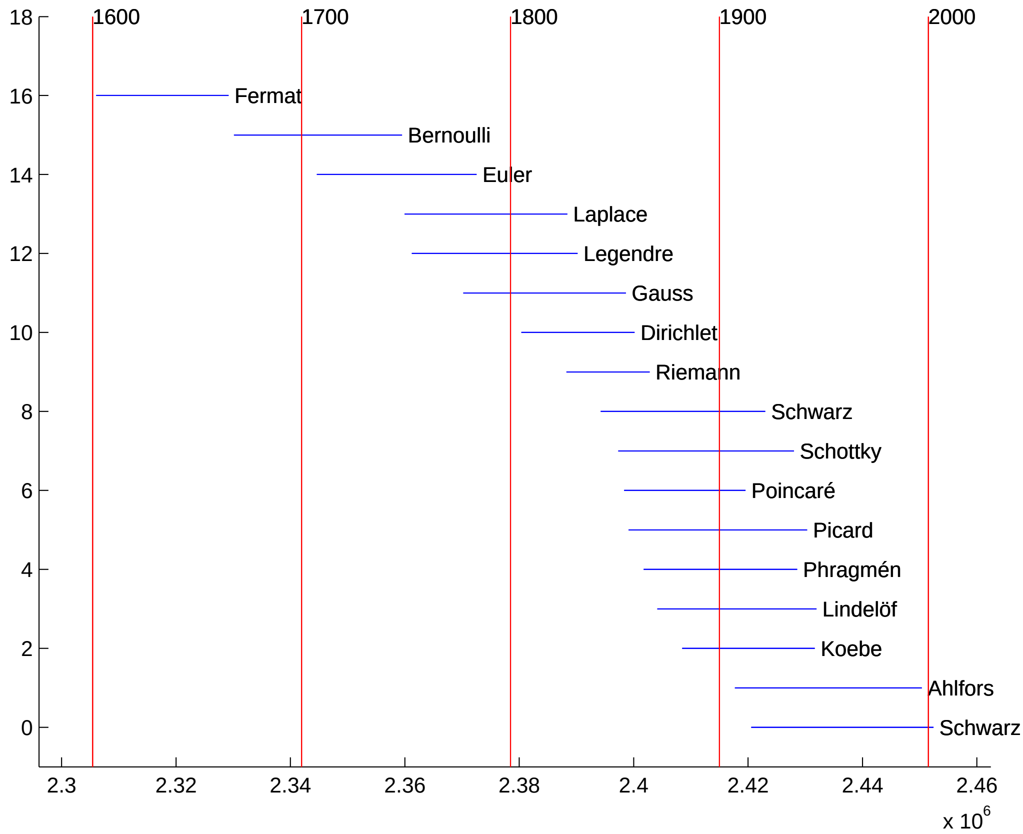


Download in:
`http://integraali.com/stuff/showgenealogy.m`

4. Pictures

4.1. **Various pictures.** During my academics, I have made the following pictures:


a=1.01, p=1.01
a=1.01, p=1.3433
a=1.01, p=1.6767
a=1.01, p=2.01
a=1.3433, p=1.01
a=1.3433, p=1.3433
a=1.3433, p=1.6767
a=1.3433, p=2.01
a=1.6767, p=1.01
a=1.6767, p=1.3433
a=1.6767, p=1.6767
a=1.6767, p=2.01
a=2.01, p=1.01
a=2.01, p=1.3433
a=2.01, p=1.6767
a=2.01, p=2.01
100
200
300
400
500
600
0
100
200
300
400
500
600
700



Download in:

http://integraali.com/stuff/mathematicians/

5. Some calculations

5.1. **Hyperbolic geometry.** Let

$$\varphi_a(z) = \frac{a-z}{1-\overline{a}z}$$

for $a, z \in \mathbb{D}$, be the disc automorphism. Then $\varphi_a^{-1} = \varphi_a$. A hyperbolic segment between two points $a, b \in \mathbb{D}$, can be parametrized by

$$\langle a, b\rangle = \{\varphi_a\left(\varphi_a(b)t\right) \,:\, 0 \le t \le 1\}\,.$$

The hyperbolic midpoint of $\langle a, b\rangle$, denoted by $\zeta = \varphi_a(\varphi_a(b)t)$, satisfies

$$|\varphi_a(\zeta)| = |\varphi_b(\zeta)|\,.$$

We wish to calculate a formula for ζ. By choosing $a = 0$, we obtain $\varphi_a(z) = -z$, $\zeta = bt$ and

$$|b|t = \left|\frac{b-bt}{1-|b|^2t}\right| = |b|\frac{1-t}{1-|b|^2t}.$$

This implies that

$$t = \frac{1-\sqrt{1-|b|^2}}{|b|^2} = \frac{1}{1+\sqrt{1-|b|^2}} \tag{5.1}$$

In the general case, map the segment $\langle a, b\rangle$ by φ_a, so that points a, ζ, b map to points $0, \varphi_a(\zeta), \varphi_a(b)$. Since the automorphism preserves hyperbolic distances, $\varphi_a(\zeta)$ is the midpoint of $[0, \varphi_a(b)]$ and we obtain by (5.1) that

$$\varphi_a(\zeta) = \varphi_a(b)t,$$

that is,

$$\zeta = \varphi_a\left(\varphi_a(b)t\right), \quad t = \frac{1}{1+\sqrt{1-|\varphi_a(b)|^2}},$$

that is,

$$\zeta = \varphi_a\left(\frac{\varphi_a(b)}{1+\sqrt{1-|\varphi_a(b)|^2}}\right). \tag{5.2}$$

Since ζ is the midpoint of a and b, formula (5.3) should remain the same, when points a and b are exchanged. By considering the mapping of $\langle a, b\rangle$ to both $[0, \varphi_a(b)]$ and $[0, \varphi_a(b)]$ and noting that $|\varphi_a(b)| = |\varphi_b(a)|$, this is really the case. Therefore

$$\zeta = \varphi_a\left(\frac{\varphi_a(b)}{1+\sqrt{1-|\varphi_a(b)|^2}}\right) = \varphi_b\left(\frac{\varphi_b(a)}{1+\sqrt{1-|\varphi_b(a)|^2}}\right). \tag{5.3}$$

We have

$$\zeta = \frac{a(1-\overline{a}b)-t(a-b)}{1-\overline{a}b-\overline{a}t(a-b)}, \quad t = \frac{1}{1+\sqrt{1-|\varphi_a(b)|^2}}.$$

6. Math ideas

6.1. **A coefficient $A = -f''/f \notin H_2^\infty$ such that f grows exponentially.** Let A be analytic in $\mathbb{D}$ and let $\{f, g\}$ be a solution base for the equation

$$f'' + Af = 0$$

such that $W(f, g) = fg' - f'g \equiv 1$. Let

$$A \in H_{2+2\varepsilon}^\infty \setminus \bigcup_{0<p<2+2\varepsilon} H_p^\infty,$$

where $h \in H_p^\infty$ if and only if

$$\sup_{z\in\mathbb{D}} |h(z)|(1 - |z|^2)^p < \infty.$$

Assume that there exists $\{z_n\} \subset \mathbb{D}$ such that $|z_n| \to 1^{-1}$ as $n \to \infty$ and

$$|A(z_n)| \geq \frac{1}{(1 - |z_n|)^{2+2\varepsilon}}, \quad n \in \mathbb{N}.$$

Since

$$A = -\frac{f''}{f} = -\frac{f''}{f'}\frac{f'}{f},$$

we have

$$\max\left\{\left|\frac{f''(z_n)}{f'(z_n)}\right|, \left|\frac{f'(z_n)}{f(z_n)}\right|\right\} \geq \frac{1}{(1 - |z_n|)^{1+\varepsilon}}.$$

The condition

$$\frac{f''(x)}{f'(x)} = \frac{1}{(1 - x)^{1+\varepsilon}}$$

heuristically implies

$$(\log f(x))' = \frac{1}{(1 - x)^{\varepsilon}} + C,$$

that is,

$$f(x) = e^{\frac{1}{(1-x)^{\varepsilon}}}.$$

6.2. **A question.**

7. Math sketches

Define the spherical distance

$$\sigma(z,w) = \frac{2}{\pi} \inf_{\gamma} \int_{\gamma} \frac{|d\zeta|}{1+|\zeta|^2}.$$

where γ is a curve joining z and w. Now $\sigma(0,1) = 1/2$ and $\sigma(0,\infty) = 1$. Moreover,

$$\sigma(0,z) = \frac{2}{\pi} \tan^{-1}(|z|).$$

Let $D \subset \widehat{\mathbb{C}}$ be a domain and $f : D \to \widehat{\mathbb{C}}$. Let $A \subset \widehat{\mathbb{C}}$ be a non-empty set. Define

$$M(r,f,A) = \max\left\{|f(z)| \,:\, \sigma(z,A) = r\right\}.$$

Hence, in case of an entire function

$$M_\infty(r,f) = \max_{|z|=r} |f(z)| = M(2(1-\tan^{-1}(r))/\pi, f, A)$$

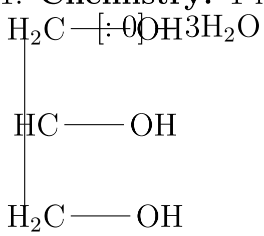
and

$$M_\infty(r,f) = M(2(1/2 - tan^{-1}(r))/\pi, f, \partial\mathbb{D})$$

Since for $z \approx w$, we have $\sigma(z,w) \approx 2|z-w|/\pi$, we have

$$M(r,f,z_0) \sim \max\left\{|f(z)| \,:\, |z-z_0| = \pi r/2\right\}.$$

7.1. Chemistry. $1\ \mathrm{kg\,m\,s^{-1}}$



Department of Physics and Mathematics, University of Eastern Finland, P.O. Box 111, FI-80101 Joensuu, Finland
E-mail address: `juha-matti.huusko@uef.fi`