REPORT ON "METHODS FOR COMPLEX ODES BASED ON LOCALIZATION, INTEGRATION AND OPERATOR THEORY" BY JUHA-MATTI HUUSKO

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The author considers the following linear differential equations

$$(*) \qquad f^{(n)}(z) + A_{n-1}(z)f^{(n-1)}(z) + \cdots + A_1(z)f'(z) + A_0 f(z) = A_n(z),$$

where $n \geq 2$ and coefficients $A_0, \ldots, A_n$ are analytic functions in a simply connected domain D of the complex plane. Typically D is the unit disc. The thesis deals with conditions of coefficients such that all analytic solutions of (*) belong to certain classical function spaces.

The thesis contains two published papers and a submitted manuscript.

In the paper entitled "Localisation of linear differential equations in the unit disc by a conformal map", Juha-Matti Huusko obtains lower bounds for the growth of solutions of linear differential equations (*) by localising the equations via conformal maps. The author also explains how a localisation method can be used to study the growth of solutions of (*) when information on the coefficients is available near some boundary point only.

In the paper entitled "Linear differential equations with solutions in the growth space H_ω^∞", the authors give sufficient conditions for solutions of linear differential equations (*) or their derivatives to be in H_ω^∞ by limiting the growth of coefficients $A_0, \ldots, A_n$. Here H_ω^∞ consists of those analytic functions f in a domain D for which $|f(z)|\omega(z)$ is uniformly bounded and $\omega : D \to (0, \infty)$ is bounded and measurable. The results in this paper generalize and improve certain results in the literature. For example, the authors show that under some conditions the solutions of linear differential equations (*) belong to $(\frac{1}{2} - \epsilon)$-Bloch space, whereas in the literature it is obtained that the solutions lie in a strictly larger Q_K space.

In the paper entitled "Linear differential equations with slowly growing solutions", the authors concern linear differential equations (*) and the separation of zeros (of maximal multiplicity) of solutions; they establish sufficient conditions for the coefficient of (*) for the second order case such that all solutions of linear differential equations (*) belong to H^{∞}, or BMOA, or the Bloch space.

The methods used in the thesis come from function spaces theory and operator theory. The thesis is essentially correct and is very carefully written in all aspects. Some results of the thesis are very interesting and the proofs are often technically complicated. I strongly recommend this thesis manuscript for the public defence of the doctoral degree.

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