PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
FACULTAD DE MATEMÁTICAS

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Faculty Council
Faculty of Science and Forestry
University of Eastern Finland

Dear Members of the Council:

It has been a pleasure for me to serve as a preliminary examiner of the dissertation *Methods for complex Odes based on localization, integration and operator theory,* presented by Juha-Matti Huusko as part of the requirements to obtain the doctoral degree. Hereby I am submitting the requested report.

The present dissertation represents an interesting study of certain aspects of complex differential equations in the plane or subdomains. In my opinion, the topic is very adequate for a doctoral thesis, with specific questions to address and sufficient freedom for candidates to explore on their own. The results obtained are numerous and of different flavors, and constitute in quality a very good doctoral thesis in mathematics.

This dissertation continues with investigations on complex linear differential equations, an important topic with a long history in mathematics. During the past decades, there has been a renewed interest in the subject with a fair amount of new results. The supervisors are experts in the field. Besides still intriguing questions regarding oscillation, a terminology borrowed from the real case that in the complex setting cannot be interpreted literally, an important theme has emerged in trying to understand the relationship between the spaces to which the coefficients versus the solutions of linear equations can belong. By now, a fair amount of information has been gathered, and the thesis is a contribution is this aspect.

The author has investigated on three ideas in the thesis, namely, on results based on localization, on repeated integration, and on connections with operator theory and function spaces. New techniques have been developed and there are valuable contributions in each topic. In order to be successful, the candidate must have had to amass a fair amount of information both theoretical as well as quite technical. In this sense, the candidate should be in a good stand to continue with independent research as well as collaborating with other mathematicians. The thesis has produced papers in the three areas, which are already published or are accepted for publication in very good journals. The dissertation is very well written and organized, and the ideas and goals are presented very clearly. It makes a good reading. References are complete, and all in all, as I mentioned above, the thesis represents a well balanced and thorough

study. I could not find corrections to indicate. Some comments or suggestions have been added in the following paragraphs.

In terms of the specific results, I must say that I find all of them interesting.

In the first paper, the author uses a technique of localization to explore on the exponential iterated maximal growth of solutions f of linear equations in the disc for which the coefficient accompanying f itself has a specific exponential singularity at a given boundary point where the remaining coefficients are regular. This is very interesting, in the sense that the author has isolated the conditions for the strong conclusion that all non-trivial solutions will inherit a certain exponential growth from the singular coefficient. The localization technique depends in a crucial manner on identifying a specific localizing conformal map that will allow for explicit computations. It would probably be interesting to pursue similar results with multiple coefficients in the equation exhibiting simultaneously a singular behavior.

The second paper deals with the use of the fundamental theorem of calculus and repeated integration, to obtain sufficient conditions on the coefficients of the linear equations to ensure that all solutions will belong to a weighted $H^{\infty}(\omega)$ space that depends on a parameter function ω. This is very good and offers interesting instances for certain choices of ω. Despite the nature of the technique involved, it does render sharp results in certain important cases such as the second order linear equation. Corollaries 5.3 and 5.4 as special cases are very good. The iterated integration should give the correct answer when all coefficients are constant.

The results of the third paper are probably the most interesting ones to this reviewer. It uses fine descriptions of certain function spaces that deal with holomorphic mappings in the disc with controlled behavior of different kinds at the boundary. The author provides sufficient conditions on the coefficient of the second order equation for all solutions to belong to either the H^{∞}, the *BMOA* or the *B* space. This is a delicate result. I also found the oscillations results quite interesting, in particular, the analysis presented on the rare third order equation.

In summary, the thesis is a very good contribution to the subject. An open question that the author may have already considered is whether a natural symmetry like differential equations on the real line or an interval, with real analytic coefficients, would render results with new information.

Sincerely,

Martin Chuaqui
Professor of Mathematics