Statement of the official opponent
on the Doctoral Dissertation of Juha-Matti Huusko
"METHODS FOR COMPLEX ODES BASED ON LOCALISATION, INTEGRATION AND OPERATOR THEORY"

Research of Juha-Matti Huusko concerns study of linear differential equations with analytic coefficients in simply connected domains of the complex plane $\mathbb{C}$ in general and in the unit disc D. The author studies conditions when solutions of linear differential equations belong to some classical classes of functions. Interest to these problems is based on many different approaches and techniques of investigation.

The doctoral thesis of Juha-Matti Huusko is based on three papers. Two of them are already published and one is accepted for publication. The candidate demonstrates both a good individual and team work. He wrote two papers in collaboration with two more co-authors and one paper individually.

The Academic Dissertation is written in accordance to the candidate's publications. JM Huusko gives clear introduction to the problems which are discussed in the research papers. Some results are compared with results from past papers of other mathematicians. Special attention is paid on methods of investigations. The candidate apply different methods for study solutions of differential equations.

In the paper "Localisation of linear differential equations in the unit disc by a conformal map" J-M Huusko studies growth of solutions of the linear differential equation

$$f^{(k)} + a_{k-1}(z)f^{(k-1)} + \ldots + a_0(z)f = 0$$

with analytic coefficients $a_j(z)$ in the unit disc D. Special attention is paid for exponential growth near a boundary point. He considers iterated M-order of solution. Main tool of investigation is a composition of unknown function with some locally conformal mapping. Method is illustrated by nice figure and several examples.

The paper "Linear differential equations with solutions in the growth space H^∞_ω" is a joint

work with T. Korhonen and A. Reijonen. The authors study sufficient conditions for solutions of

$$f^{(n)} + A_{n-1}(z)f^{(n-1)} + \ldots + A_0(z)f = A_n(z)$$

to be in H^∞_ω, Q_K, $\mathcal{B}^\alpha$ and other classes. The authors use straightforward integral estimates instead of standard methods. Obtained results are illustrated by good examples.

In the paper "Linear differential equations with slowly growing solutions" (joint work with J. Gröhn and J. Rättyä) the authors study solutions of the linear differential equation

$$f^{(k)} + A_{k-1}(z)f^{(k-1)} + \ldots + A_0(z)f = 0$$

with analytic coefficients $A_j(z)$ in the unit disc D. This type of equations was studied by different methods and by many mathematicians. The authors propose a new technique of study conditions when solution of the equation have given growth. Special cases are classes H^∞, $BMOA$ and $\mathcal{B}$ classes. It is important to mention that main technique is based methods of operator theory (integral operators, Volterra-type operators). This paper contains a lot of interesting ideas for future research.

Conclusion

The candidate illustrates strong understanding the topic and ability for independent research. All results in his publications are new and demonstrate good technique of study of linear differential equations with analytic coefficients, understanding of methods of composition and integral operators. Obtained results are explained clear and illustrated by examples.

In accordance to results obtained by Juha-Matti Huusko in the papers and his oral presentation I can recommend the candidate for the doctoral degree with the scale **approved**.

Professor Shamil Makhmutov

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Joensuu, Finland