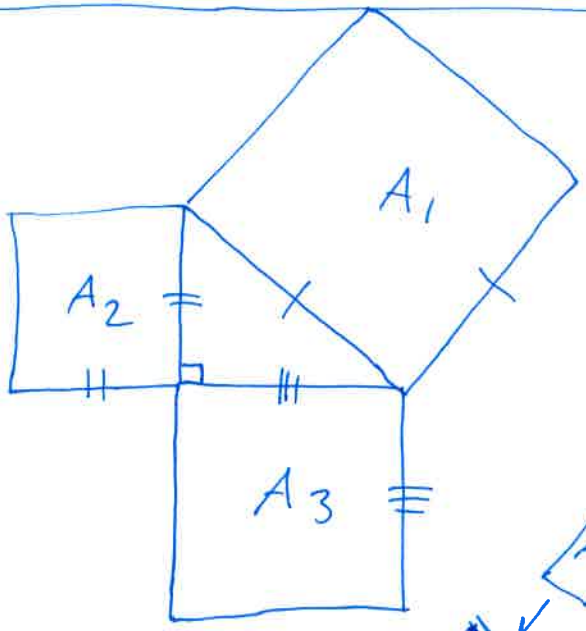
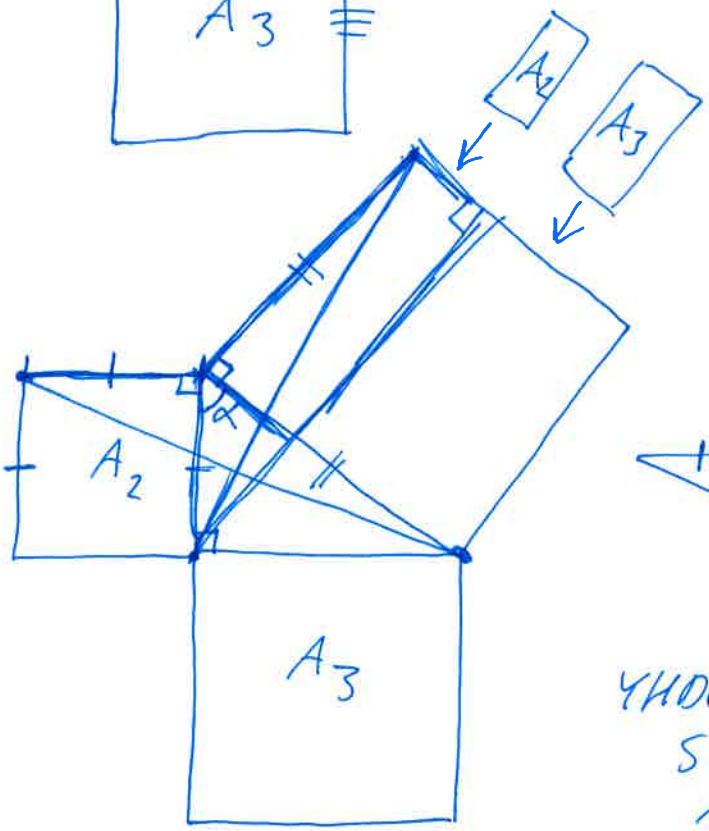
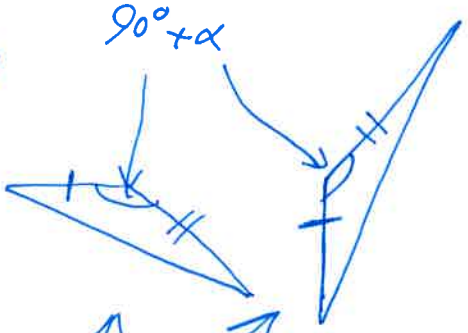
LAUSE 1.47 (PYTHAGORAAN LAUSE)



$\Rightarrow \quad A_1 = A_2 + A_3$

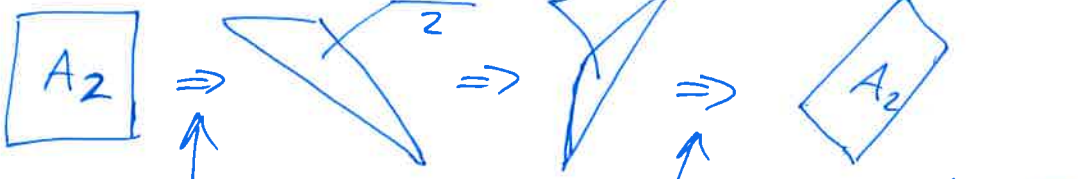
Tod.





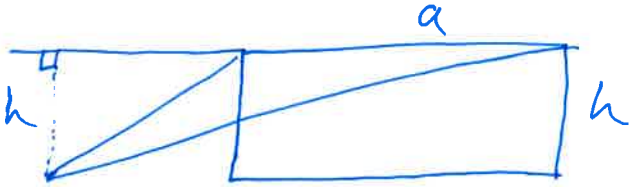
YHDENMUOTOISET SKS-LAUSEEN NOJALLA

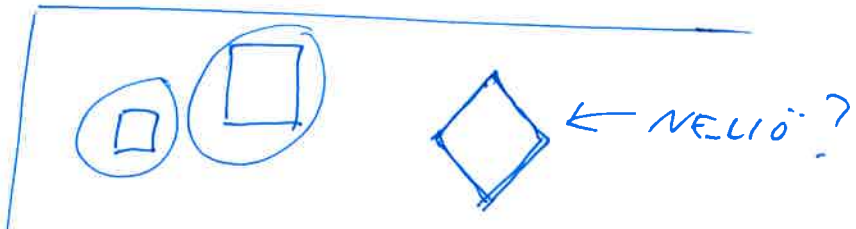
SIIS



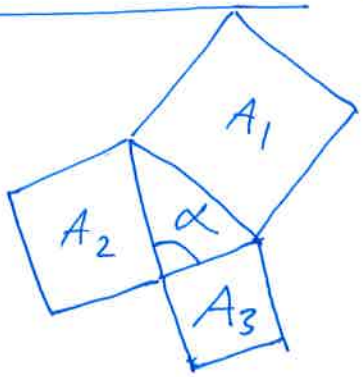
L1.41 L1.41

$A(\triangle) = \frac{1}{2}ah$ $A(\square) = ah$





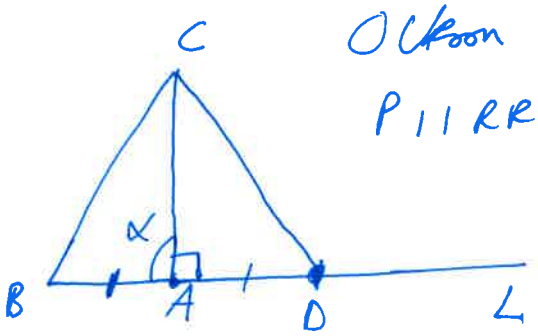
LAUSE 1.48 (KÄÄNTEINEN PYTHAGORAAN LAUSE)



$\Rightarrow \quad \alpha = 90°$

$A_1 = A_2 + A_3$

Tod.



Olkoon $BC^2 = AC^2 + BA^2$.

Piirretään $L \perp CA$, jolle $A \in L$.

Ota $D \in L$, jolle $\underline{BA = AD}$.

Piirrä CD.

Pythagoras $\Rightarrow AC^2 + AD^2 = CD^2$

$(BA = AD) \Rightarrow AC^2 + BA^2 = CD^2$ $\Rightarrow CD^2 = BC^2$

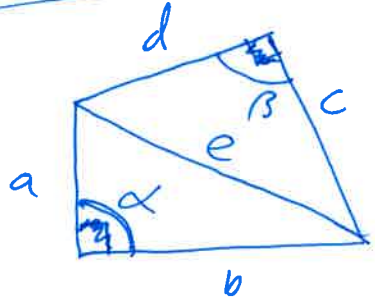
oletus $\Rightarrow AC^2 + BA^2 = BC^2$ $\Rightarrow \underline{CD = BC}$

SSS

$\Rightarrow \quad \triangle BAC \cong \triangle DAC$ $\quad \underline{AC = AC}$

$\Rightarrow \quad \alpha = \sphericalangle CAB = \sphericalangle DAC = 90°$.

□



$a^2 + b^2 = e^2$
$c^2 + d^2 = e^2$ $\Rightarrow \alpha = \beta = 90°$

$$\varphi = \frac{1+\sqrt{5}}{2}$$

KUINKA LASKEA ARVO $\sqrt[3]{2}$ KÄSIN?

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1) HAARUKOIDAAN. ETSITÄÄN x, JOLLE $x^3 = 2$.

$x = 1$? $\quad 1^3 = 1 < 2$

$x \mapsto x^3$ KASVAVA $\Rightarrow x > 1$

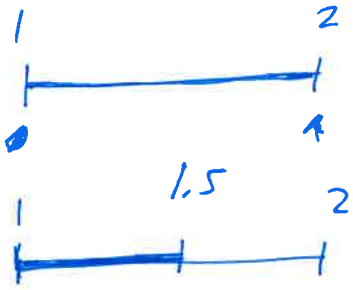
$x = 2$? $\quad 2^3 = 8 > 2 \Rightarrow 1 < x < 2$

$x = 1.5$? $\quad 1.5^3 = 1.5\,(1.5 + 0.75)$

$= 1.5\,(2.25)$

$= 2.25 + 1.125$

$= 2.375 > 2 \Rightarrow 1 < x < 1.5$



JATKAMALLA TÄTÄ TAPAAN, ARVIO TARKENTUU ASKEL ASKELEELTA.

ARVION "MAKSIMIVIRHE" PUOLITTUU JOKA ASKELEELLA (KOSKA VÄLI PUOLITETAAN).

SIIS ε_n = "VIRHE ASKELEELLA n"

$= \tilde{x} - \sqrt[3]{2} \Rightarrow \boxed{\varepsilon_{n+1} = \frac{\varepsilon_n}{2}}$

MENETELMÄ SUPPENEE LINEAARISESTI $\boxed{y = ax}$

2) SEKANTTI MENETELMÄ:

ALKUARVAUKSET $\begin{cases} x_0 = 1 \\ x_1 = 1.5 = \frac{3}{2} \end{cases}$

REKURSIOKAAVA:

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$f(x) = x^3 - 2$

NOLLAKOHTAA ETSITÄÄN

SOVELTAMALLA KAAVAA SAA OHJEN LUKUJONO (x_n) NIIN, JOKA SUPPENEE KOHTI LUKUA $\sqrt[3]{2}$.

$x_0 = 1 \qquad f(x_0) = 1^3 - 2 = -1$

$x_1 = \frac{3}{2} \qquad f(x_1) = \left(\frac{3}{2}\right)^3 - 2 = \frac{27}{8} - \frac{16}{8} = \frac{11}{8}$

$$x_2 = \frac{3}{2} - \frac{\frac{11}{8}\left(\frac{3}{2} - 1\right)}{\frac{11}{8} + (+1)}$$

$$= \frac{3}{2} - \frac{\frac{11}{8} \cdot \frac{1}{2}}{\frac{19}{8}} = \frac{3}{2} - \frac{11}{2 \cdot 19} = \frac{3 \cdot 19 - 11}{2 \cdot 19}$$

$$= \frac{57 - 11}{2 \cdot 19} = \frac{\overset{23}{\cancel{46}}}{\cancel{2} \cdot 19} = \frac{23}{19} \approx 1.21$$

TARKKA ARVO $\sqrt[3]{2} = 1{,}2599\ldots$

HAARUKOINTI : $\varepsilon_{n+1} = \frac{\varepsilon_n}{2}$ (NOPEUS 1)

SEKANTTI-MENETELMÄ : $\varepsilon_{n+1} = \varepsilon_n^{p}$ (NOPEUS 1.618)

(INTERNET : $p = \frac{1+\sqrt{5}}{2}$)

3) NEWTONIN MENETELMÄ:

SEKANTTI

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

NEWTON

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

"$x_{n-1} \to x_n$"

Esim. LASKE $\sqrt[n]{a}$ ELI RATKAISE

$$f(x) = x^n - a = 0,$$

$$f'(x) = n x^{n-1}$$

$$\Rightarrow \boxed{x_{k+1} = x_k - \frac{x_k^n - a}{n x_k^{n-1}}}$$

$n = 2$

$$x_{k+1} = x_k - \frac{x_k^2 - a}{2x_k}$$

$$= \frac{2x_k^2 - x_k^2 + a}{2x_k}$$

$$\boxed{x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right)} \Rightarrow (x_k)_{k\in\mathbb{N}}$$

$x_k \to \sqrt{a}$