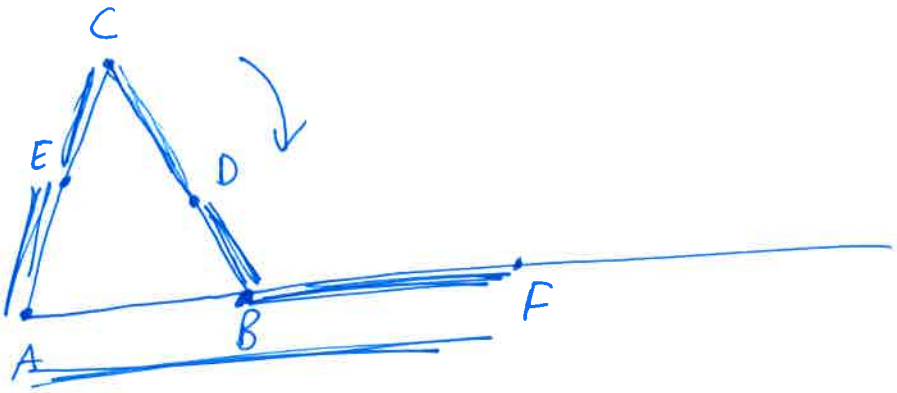
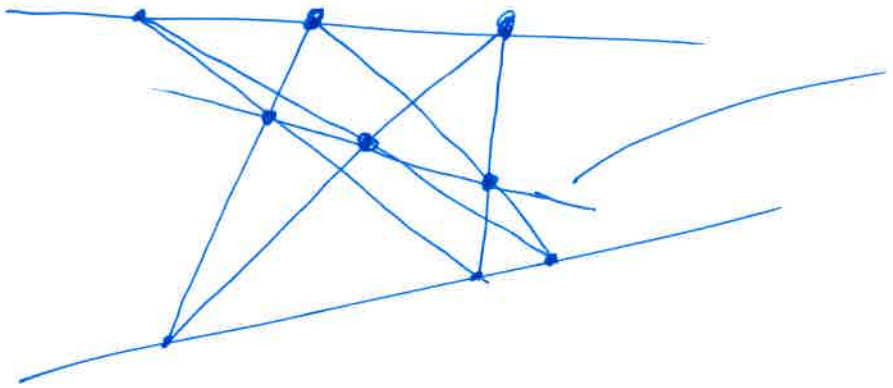
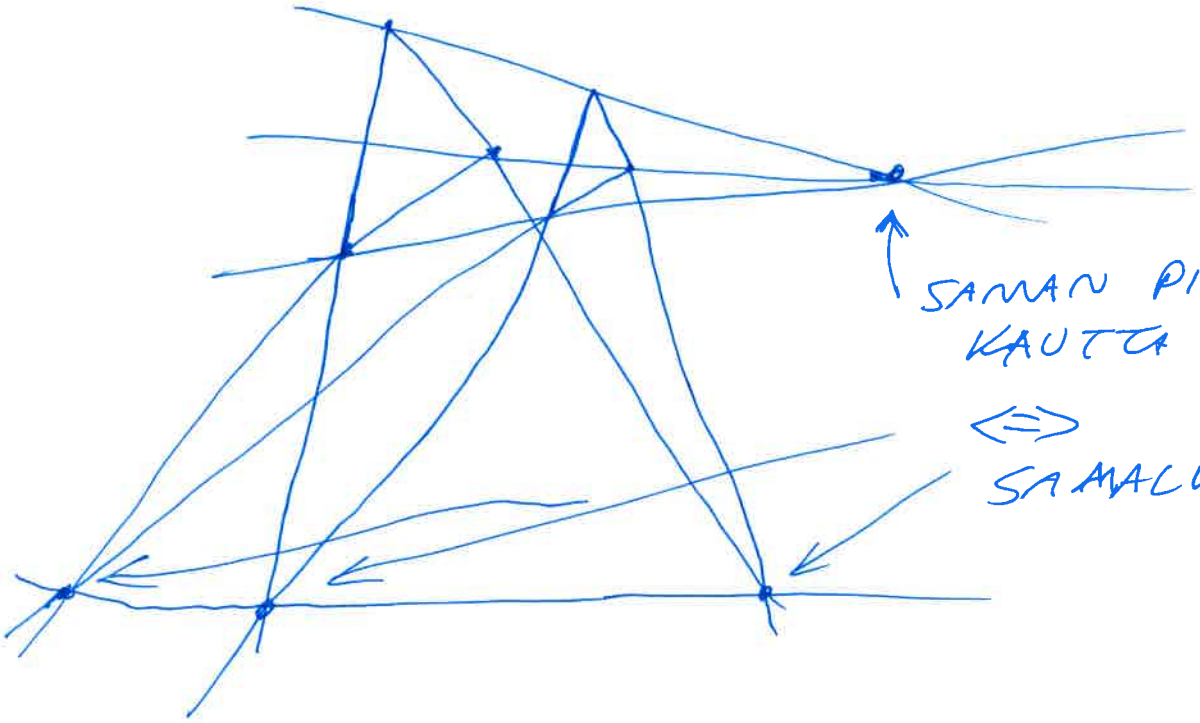


E, D F SAMALLA SUORALLA $\Leftrightarrow \frac{CD}{DB} \times \frac{BF}{FA} \times \frac{AE}{EC} = 1$

[$\Leftrightarrow$ "$\triangle$" : $\frac{CE}{EA} \times \frac{AF}{FB} \times \frac{BD}{DC} = 1$]



SAMALLA SUORALLA

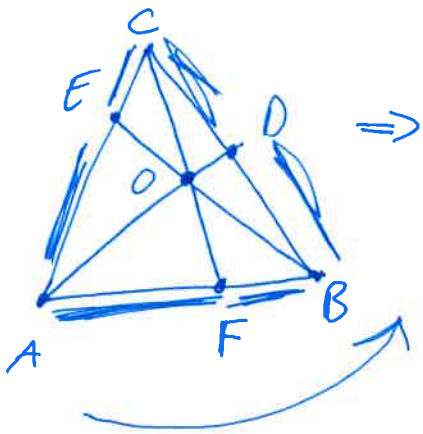


SAMAN PISTEEN KAUTTA

$\Leftrightarrow$

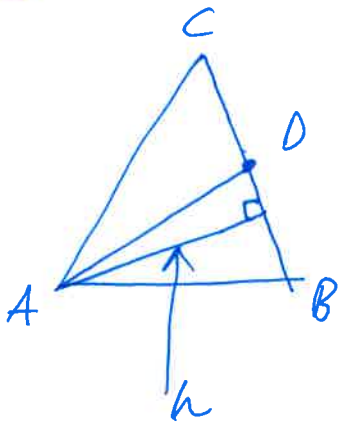
SAMALLA SUORALLA

CEVAN LAUSE



$$\Rightarrow \frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA} = 1$$

Tod. MERKITÄÄN $ALA(\Delta XYZ) = |\Delta XYZ|$.

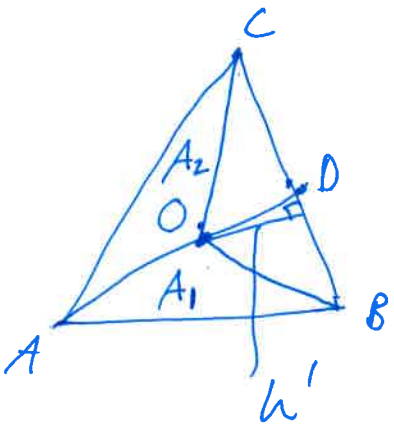


$$|\Delta ABD| = \frac{1}{2} \cdot h \cdot DB$$

$$|\Delta ADC| = \frac{1}{2} \cdot h \cdot DC$$

$$\Rightarrow \frac{|\Delta ABD|}{|\Delta ADC|} = \frac{DB}{DC} = \frac{BD}{DC}$$

$\frac{24}{16} = \frac{3}{2}$



VASTAAVASTI

$$\frac{|\Delta OBD|}{|\Delta ODC|} = \frac{BD}{DC}$$

$\frac{BD}{DC} = \frac{A_1}{A_2}$?

$\frac{9}{6} = \frac{3}{2}$

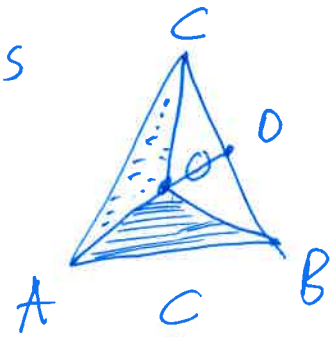
$\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

$$\frac{3}{2} = \frac{24-9}{16-6} = \frac{15}{10}$$ ok

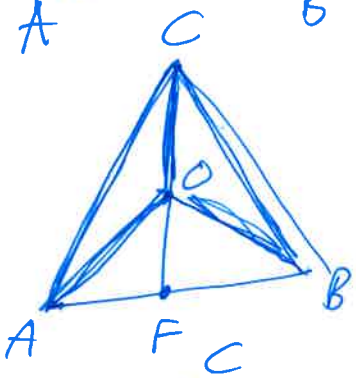
$$\Rightarrow \frac{a}{b} = \frac{c-e}{d-f}$$ (HT)

$$\Rightarrow \frac{BD}{DC} = \frac{|\Delta ABD| - |\Delta OBD|}{|\Delta ADC| - |\Delta ODC|} = \frac{|\Delta ABO|}{|\Delta AOC|}$$

SIIS

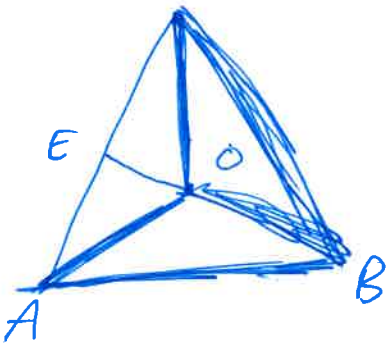


$$\frac{BD}{DC} = \frac{|\triangle ABO|}{|\triangle AOC|} =$$



VASTAAVASTI

$$\frac{AF}{FB} = \frac{|\triangle AOC|}{|\triangle COB|}$$



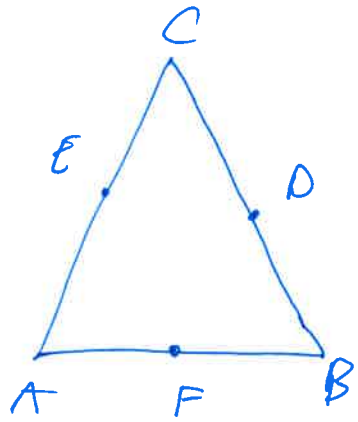
$$\frac{CE}{EA} = \frac{|\triangle COB|}{|\triangle ABO|}$$

$$\Rightarrow \frac{BD}{DC} \times \frac{AF}{FB} \times \frac{CE}{EA} = \frac{|\triangle ABO|}{|\triangle AOC|} \times \frac{|\triangle AOC|}{|\triangle COB|} \times \frac{|\triangle COB|}{|\triangle ABO|}$$

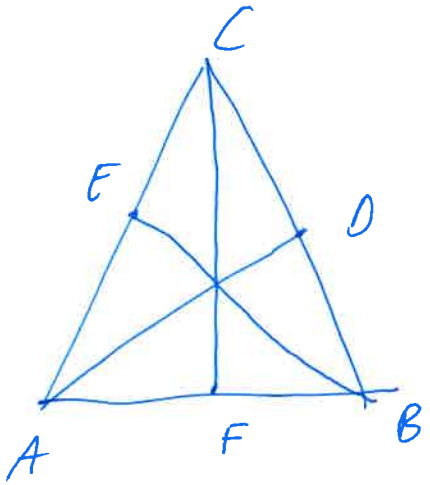
$$= 1.$$

□

KÄÄNTEINEN CEVAN LAUSE

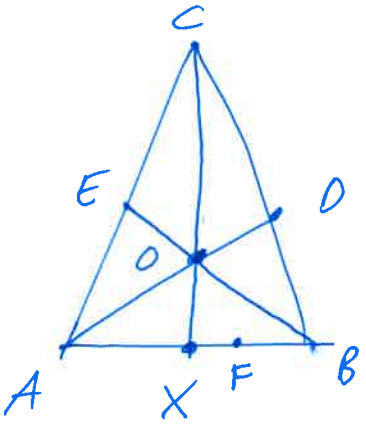


$\Rightarrow$



$$\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA} = 1$$

Todistus PIIRRETÄÄN AD JA BE JA OLKOON $AD \cap BE = O$.



AT $\overrightarrow{CO} \cap AB = X \neq F$

$$\frac{AF}{FB} > \frac{AX}{FB} > \frac{AX}{XB}$$

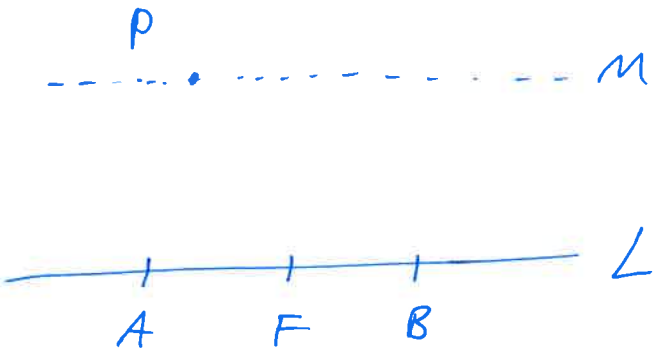
$$\Rightarrow \frac{AF}{FB} > \frac{AX}{XB} \longrightarrow RR$$

CEVAN LAUSE $\longrightarrow$ OLETUS

$$\frac{AX}{XB} \times \frac{BD}{DC} \times \frac{CE}{EA} = 1 = \frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA}$$

$\square$

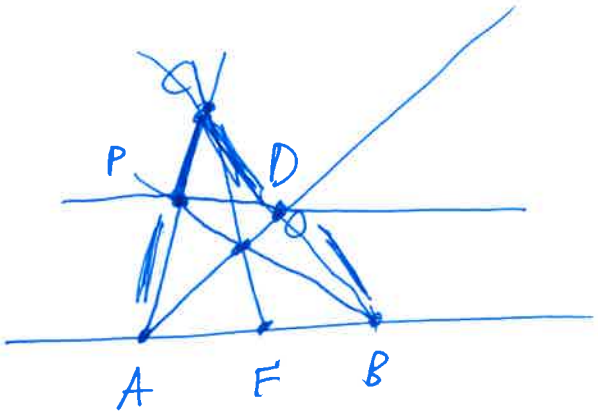
CEVAN LAUSEEN SOVELLUS



$L \parallel M$

$AF = FB$

YHDENSUUNTAINEN $M \parallel L$ OSATAAN PIIRTÄÄ PELKÄSTÄÄN VIIVOTINTA KÄYTTÄMÄLLÄ:



PIIRRÄ $\overrightarrow{AP}$, OTA $C \in \overrightarrow{AP}$

PIIRRÄ $\overrightarrow{FC}$

$\overrightarrow{BP}$

OLKOON $O = \overrightarrow{FC} \cap \overrightarrow{BP}$

PIIRRÄ $\overrightarrow{AO}$

$\overrightarrow{BC}$

OLKOON $D = \overrightarrow{AO} \cap \overrightarrow{BC}$

PIIRRÄ $\overleftrightarrow{PD}$.

VÄITE: $\overleftrightarrow{PD} \parallel \overleftrightarrow{AB}$

Tod. CEVAN LAUSE $\Rightarrow \frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CP}{PA} = 1$

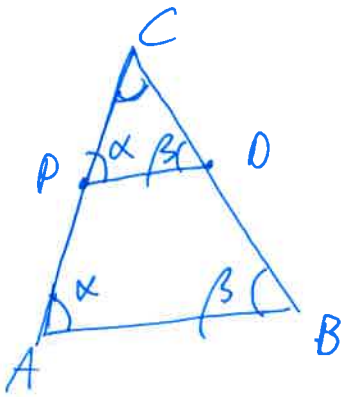
$AF = FB \Rightarrow \frac{BD}{DC} \times \frac{CP}{PA} = 1 \quad \| \cdot \frac{PA}{BD}$

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} = \frac{a+c}{b+d}$ (HT)

$\Rightarrow \frac{CP}{CD} = \frac{PA}{BD} = \frac{CA}{CB} = \frac{CP+PA}{CP+BD}$

$\frac{3}{2} = \frac{6}{4} \Rightarrow \frac{3}{2} = \frac{3+6}{2+4} = \frac{9}{6}$ ok

SIIS

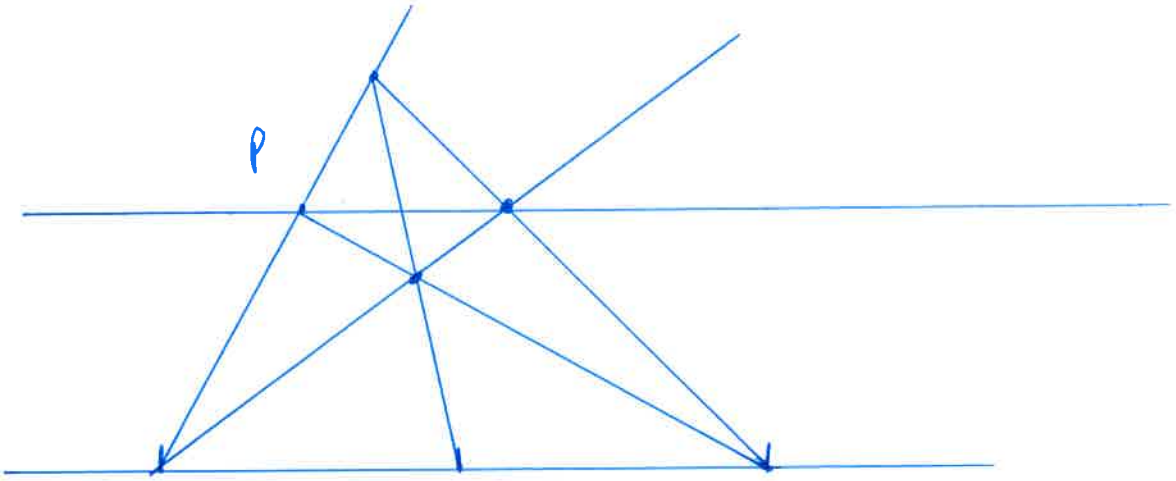


$$\begin{cases} \frac{CP}{CD} = \frac{CA}{CB} \\ \text{KULMA YHTEINEN} \end{cases}$$

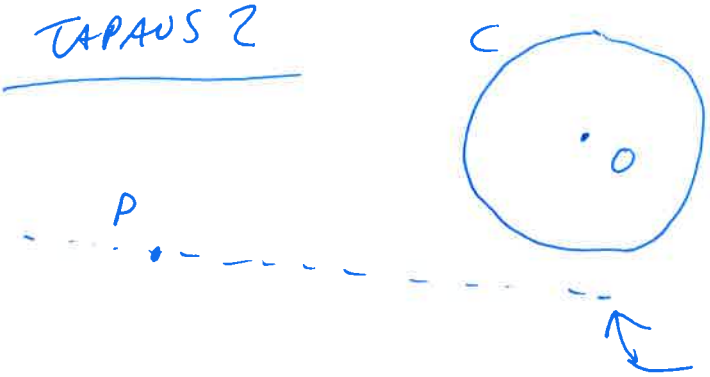
$\Rightarrow \triangle ABC \sim \triangle PDC$

VUOROKULMALAUSE

$\Rightarrow \overleftrightarrow{PD} \parallel \overleftrightarrow{AB}$

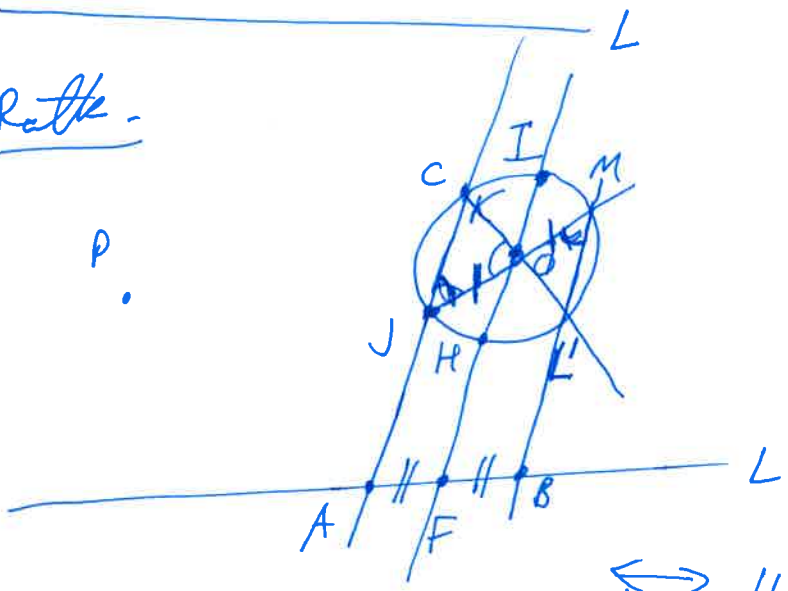


TAPAUS 2



OSATAAN PIIRTÄÄ PELKÄSTÄÄN VIIVOTTIMELLA

Ratkaisu.



OTA $F \in L$

PIIRRÄ $\overleftrightarrow{FO}$

OLKOON $\{H, I\} = L \cap C$

OTA $J \in C \setminus \{H, I\}$

KOSKA $HO = OI$,

OSATAAN PIIRTÄÄ $\overleftrightarrow{JK} \parallel \overleftrightarrow{FO}$ JOLLE $K \in C$.

OLKOON $A = \overleftrightarrow{JK} \cap L$.

OLKOON $\begin{cases} \overrightarrow{JO} \cap C = M \\ \overrightarrow{KO} \cap C = L' \end{cases}$

PIIRRÄ $\overleftrightarrow{ML'}$ JA OLKOON $B = \overleftrightarrow{ML'} \cap L$

KOSKA $JO = KO = L'O = MO$

JA $\sphericalangle KOJ = \sphericalangle L'OM$

$\Rightarrow \triangle JOK \cong \triangle MOL'$

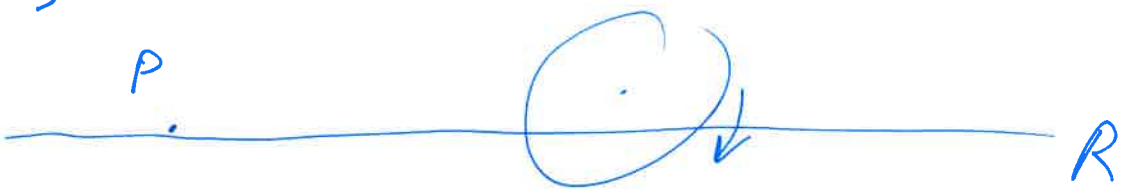
$\Rightarrow \sphericalangle KJO = \sphericalangle OML'$

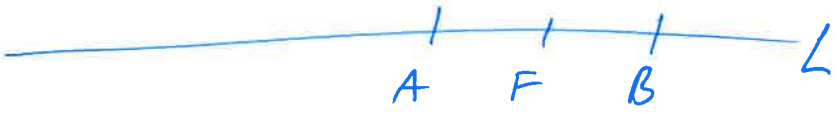
VUOROKULMALAUSE

$\Rightarrow$ OLLA $\overleftrightarrow{FO} \parallel \overleftrightarrow{JK} \parallel \overleftrightarrow{ML'}$

KOSKA JO = OM, NIIN AF = FB.

SIIS SAATIIN TAAS TILANNE





AF = FB

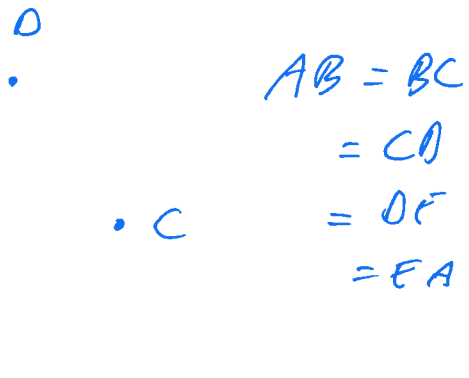
OSATAAN PIIRTÄÄ R ∥ L, JOLLE P∈R.

[KONSTRUKTIO ON OSA PONCELET-STEINER LAUSETTA]

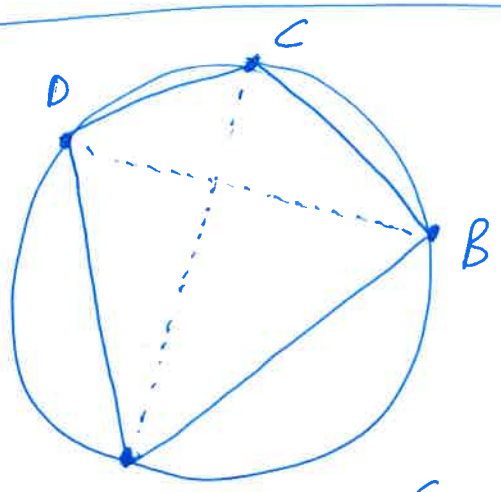
↕

[MOHR-MASCHERONI]

HARPILLA

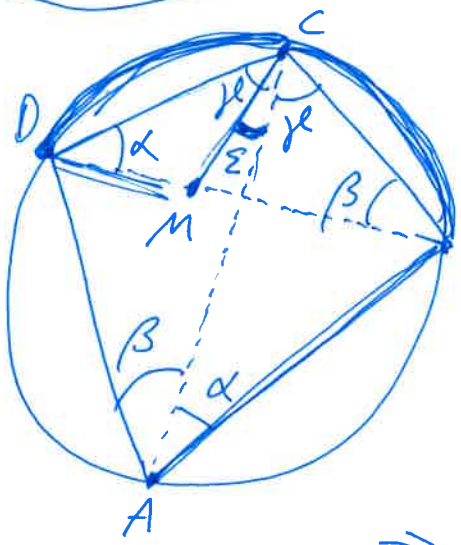


PTOLEMAIOKSEN LAUSE



$$\Rightarrow AB \cdot CD + BC \cdot DA = AC \cdot DB$$

Tod.



KEHÄKULMALAUSE

2 kpl α

2 kpl β

OTA $M \in DB$, JOLLE

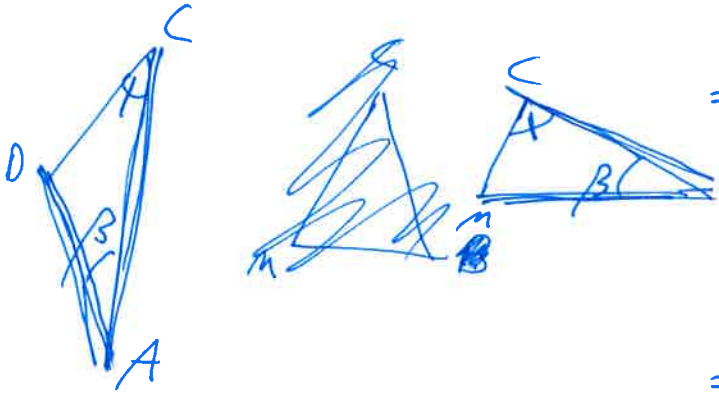
$\sphericalangle DCM = \sphericalangle ACB$

$$\Rightarrow \triangle DCM \sim \triangle ACB$$

$$\Rightarrow \frac{AB}{DM} = \frac{AC}{DC}$$

$$\Rightarrow AB \cdot CD = AC \cdot DM$$

$$\sphericalangle DCA = \gamma + \varepsilon = \sphericalangle MCB$$



$$\Rightarrow \triangle BCM \sim \triangle ACD$$

$$\Rightarrow \frac{AC}{BC} = \frac{AD}{BM}$$

$$\Rightarrow AD \cdot BC = AC \cdot MB$$

$$AB \cdot CD + BC \cdot DA = AC \cdot DM + AC \cdot MB$$

$$= AC(DM + MB) = AC \cdot DB. \quad \square$$