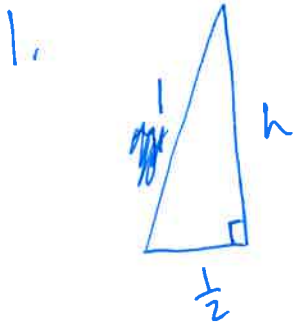


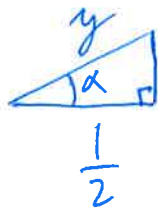
EUKLIDINEN GEOMETRIA
 HARJOITUS 2/2018
 RATKAISUT



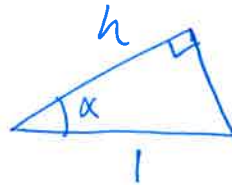
Pythagoras: $1^2 = h^2 + (\frac{1}{2})^2$

$$\Rightarrow h = \sqrt{1 - (\frac{1}{2})^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

TOISAALTA



~



OVAT YHDEN MUOTOISIA, KOSKA KAIKKI KULMAT
 OVAT YHTÄ SUURIA PAREITTAIN, SIIS

$$\frac{y}{\frac{1}{2}} = \frac{1}{h} \Rightarrow y = \frac{1}{2} \frac{1}{h} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

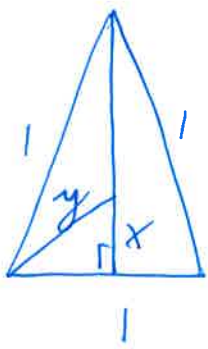
NYT SYMMETRIA $\Rightarrow h = y + x$
 $\Rightarrow x = h - y$

$$= \frac{3\sqrt{3}}{6} - \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{6}$$

$$= \frac{1}{2\sqrt{3}}$$



2. Siis

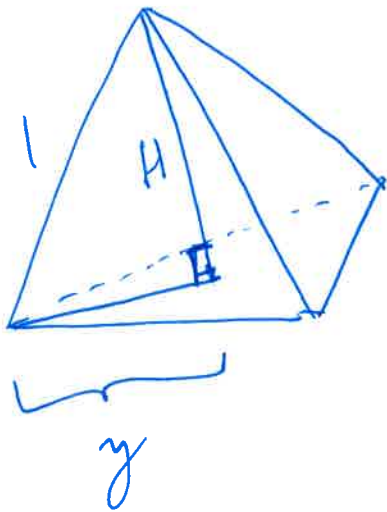


$$h = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \text{ALA} &= \frac{1}{2} \cdot 1 \cdot \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{3}}{4} = A_1 \end{aligned}$$

$$y = \frac{\sqrt{3}}{3}, \quad x = \frac{\sqrt{3}}{6}$$

TERRAEDRI



Pythagoras:

$$\begin{aligned} 1^2 &= H^2 + y^2 \\ \Rightarrow H^2 &= 1^2 - y^2 \\ &= 1 - \frac{3}{9} = \frac{2}{3} \end{aligned}$$

$$\Rightarrow H = \underline{\underline{\sqrt{\frac{2}{3}}}}$$

VAIPPA = 3 kpl TASASIVUISIA KOLMIOTTA

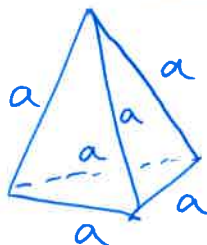
$$\Rightarrow \text{ALA (VAIPPA)} = 3 \cdot \frac{\sqrt{3}}{4} = \underline{\underline{\frac{3\sqrt{3}}{4}}}$$

KUORI = VAIPPA + POHJA

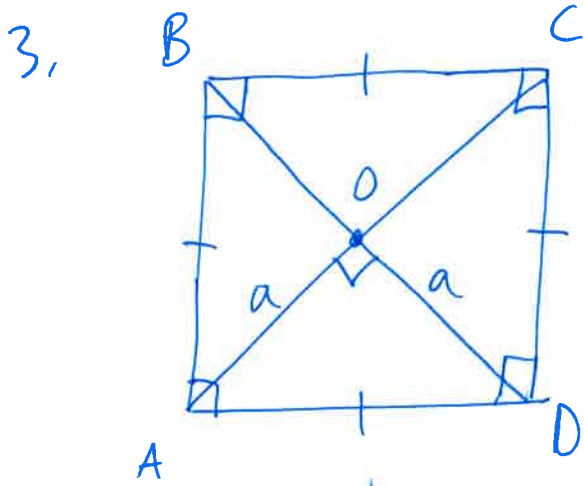
$$\text{ALA (KUORI)} = 4 \cdot \frac{\sqrt{3}}{4} = \sqrt{3}$$

$$\text{TILAVUUS} = \frac{1}{3} A_1 H = \frac{1}{3} \cdot \frac{\sqrt{3}}{4} \cdot \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{12}$$

VASTAAVASTI



$$\Rightarrow \begin{cases} H = \sqrt{\frac{2}{3}} a \\ \text{ALA (KUORI)} = \sqrt{3} a^2 \\ V = \frac{\sqrt{2}}{12} a^3 \end{cases}$$

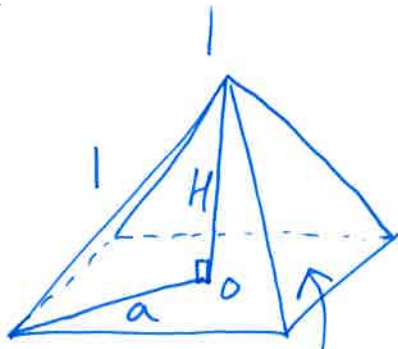


Pythagoras:

$$1^2 = a^2 + a^2 = 2a^2$$

$$\Rightarrow a^2 = \frac{1}{2}$$

$$\Rightarrow a = \frac{1}{\sqrt{2}} = A_0$$



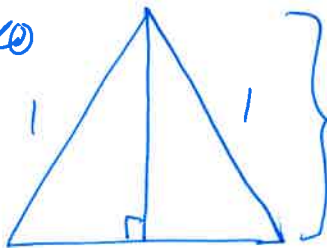
Pythagoras:

$$1^2 = H^2 + a^2$$

$$\Rightarrow H^2 = 1^2 - a^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow H = \frac{1}{\sqrt{2}}$$

TAHUKO



$$h = \frac{\sqrt{3}}{2}$$

$$ALTA = \frac{\sqrt{3}}{4}$$

TEHT 2.
PERUSTEELLA

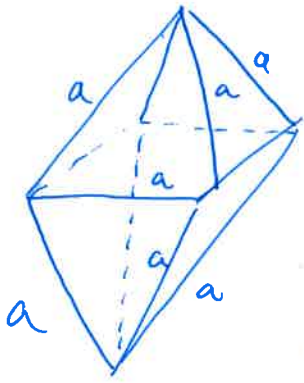
$$VAIPAN ALTA = 4 \cdot \frac{\sqrt{3}}{4} = \sqrt{3}$$

$$POHJAN ALTA = 1 \cdot 1 = 1 = A_1$$

$$KOKON ALTA = 1 + \sqrt{3}$$

$$TILAVUUS \quad V = \frac{1}{3} \cdot A_1 \cdot H = \frac{1}{3} \cdot 1 \cdot \frac{1}{\sqrt{2}} = \frac{1}{3\sqrt{2}}$$

SIIS OKTAEDRILLE

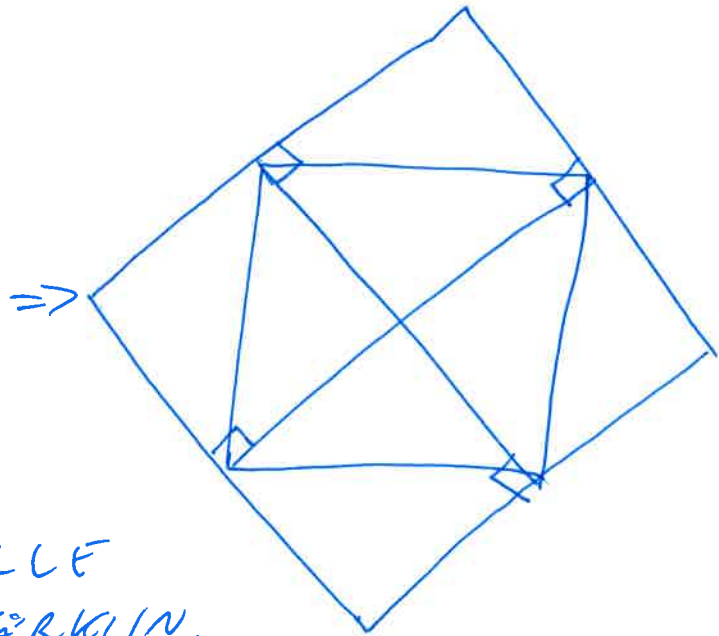
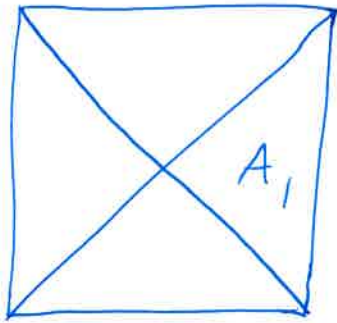


$$2H = \frac{2a}{\sqrt{2}} = \underline{\underline{\sqrt{2} a}}$$

$$K_{\text{O R I}} = a^2 \cdot 8 \cdot \frac{\sqrt{3}}{4} = \underline{\underline{2\sqrt{3} a^2}}$$

$$TILAVUUS = a^3 \cdot 2 \cdot \frac{1}{3\sqrt{2}} = \underline{\underline{\frac{\sqrt{2}}{3} a^3}}$$

4.



PIIRÄ LÄVISTÄ JILLE
KOH TI SUORTT KÄRKIIN.

⇒ TULEE ISOMPI NELIÖ.

$$A_{\text{PIENI}} = 4A_1$$

$$A_{\text{ISO}} = 8A_1 = 2 \cdot 4A_1 = 2A_{\text{PIENI}}$$

$$5, \quad \frac{1}{x} = \frac{x}{1-x} \quad || \cdot x(1-x)$$

$$\Rightarrow 1-x = x^2$$

$$\Rightarrow x^2 + x - 1 = 0 \quad x > 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{\sqrt{5}-1}{2}$$

$$\text{Siis } \varphi = \frac{1}{x} = \frac{2}{\sqrt{5}-1} = \frac{2(\sqrt{5}+1)}{\underbrace{\sqrt{5^2-1^2}}_{=4}} = \frac{1+\sqrt{5}}{2} \approx 1,618$$

$$(a-b)(a+b) = a^2 - b^2$$

AHAA!

$$6, \quad \frac{1}{2} = \cos \delta \Rightarrow AC = \frac{1}{2 \cos \delta} = \frac{1}{2 \cos 70^\circ} \approx \underline{\underline{1,618}}$$

$$\frac{AG}{\frac{1}{2}} = \tan \delta \Rightarrow AG = \frac{1}{2} \tan \delta = \frac{1}{2} \tan 70^\circ \approx \underline{\underline{1,539}}$$

$$\text{VASTAVASTI} \quad OB = \frac{1}{2 \cos 54^\circ} \approx \underline{\underline{0,851}}$$

$$OH = \frac{1}{2} \tan 54^\circ \approx \underline{\underline{0,688}}$$