

EUKLIDINEN GEOMETRIA
 HARJOITUS 4 / 2018
 RATKAISUT

1. a) TASA KYLKINEN KOLMIO $\Rightarrow \alpha = \beta$

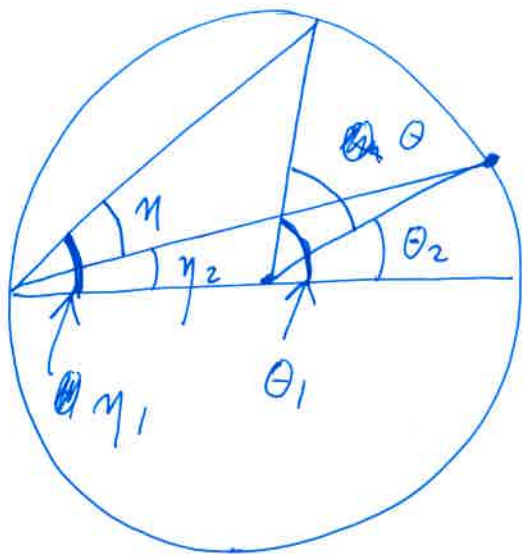
KOLMION KULMAT $\Rightarrow \alpha + \beta + \gamma = 180^\circ$

$$\left. \begin{aligned} \Rightarrow \gamma + 2\alpha &= 180^\circ \\ \Rightarrow \gamma + \delta &= 180^\circ \end{aligned} \right\} \Rightarrow \delta = 2\alpha$$

VIERUSKULMAT \Rightarrow

b) HALKAISIA JA KAVAT KULMAT Σ JA ζ
 OSIIN $\Sigma = \varepsilon_1 + \varepsilon_2$ JA $\zeta = \zeta_1 + \zeta_2$.
 NYT $\zeta = \zeta_1 + \zeta_2 \stackrel{a)}{=} 2\varepsilon_1 + 2\varepsilon_2 = 2\Sigma$.

c)

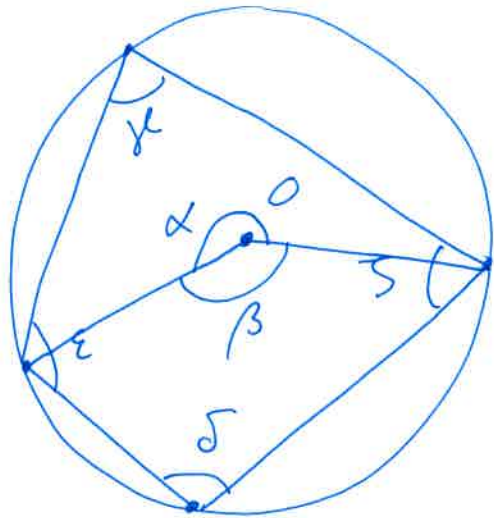


$$\begin{aligned} \theta &= \theta_1 - \theta_2 \\ &\stackrel{a)}{=} 2\eta_1 - 2\eta_2 \\ &= 2(\eta_1 - \eta_2) \\ &= 2\eta \end{aligned}$$

2. (a) O LKON KAAKKA VASTAANVA
KEHÄKULMA θ , NYT

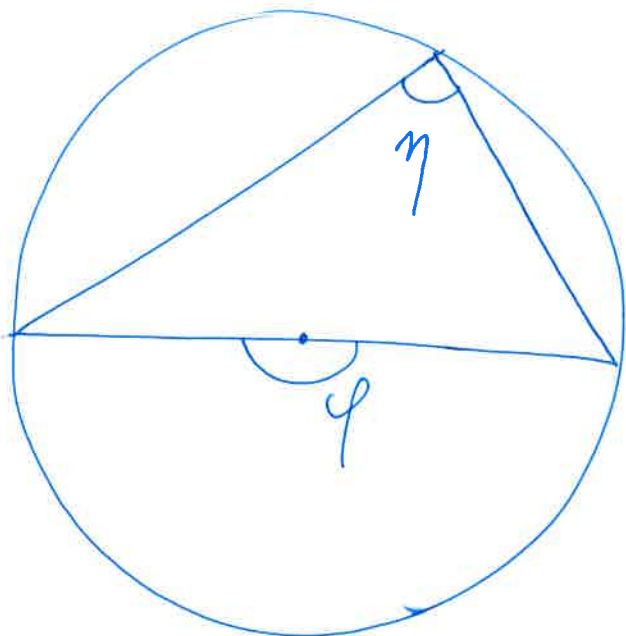
$$\alpha = \frac{\overset{\textcircled{T1}}{\theta}}{2} = \beta,$$

(b)



$$\begin{aligned} \text{TÄYSI KULMA} &= 360^\circ \\ &= \alpha + \beta \\ &= \cancel{\delta + \gamma} \quad 2\delta + 2\gamma \\ \Rightarrow \quad \delta + \gamma &= 180^\circ \end{aligned}$$

(c)



$$\begin{aligned} \varphi &= 180^\circ = 2\overset{\textcircled{T1}}{\eta} \\ \Rightarrow \eta &= 90^\circ \end{aligned}$$

3. TAPAK 1

$$(a+b)^2 = \underbrace{c^2}_{c^2 = a^2 + h^2} + \underbrace{d^2}_{d^2 = b^2 + h^2}$$

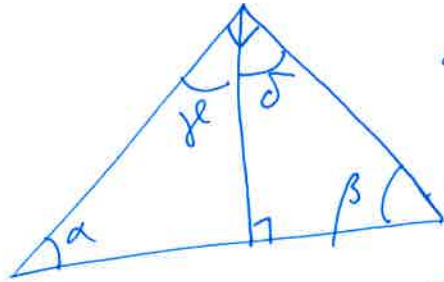
$$\Rightarrow (a+b)^2 = (a^2 + h^2) + (b^2 + h^2)$$

$$\Rightarrow a^2 + 2ab + b^2 = a^2 + h^2 + b^2 + h^2$$

$$\Rightarrow 2ab = 2h^2$$

$$\Rightarrow \underline{ab = h^2}$$

TAPAK 2

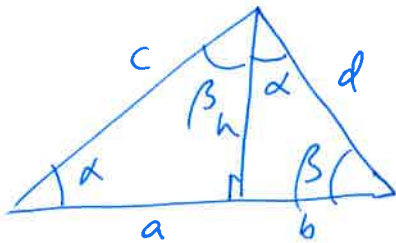


$$\begin{cases} \alpha + \beta + 90^\circ = 180^\circ \\ \alpha + \gamma + 90^\circ = 180^\circ \\ \beta + \delta + 90^\circ = 180^\circ \end{cases}$$

$$\begin{cases} \textcircled{1} \alpha + \beta = 90^\circ \\ \textcircled{2} \alpha + \gamma = 90^\circ \\ \textcircled{3} \beta + \delta = 90^\circ \end{cases} \Rightarrow \beta = \gamma$$

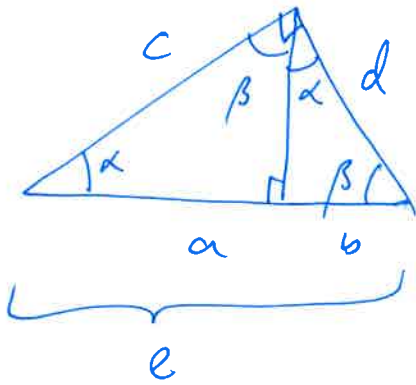
$$\textcircled{1} \ \& \ \textcircled{3} \Rightarrow \alpha = \delta$$

SUIS



$$\frac{a}{h} = \frac{h}{b} \Rightarrow \underline{ab = h^2}$$

4. TEHT 3 \Rightarrow



$$\frac{c}{e} = \frac{a}{c} \quad \& \quad \frac{d}{e} = \frac{b}{d}$$

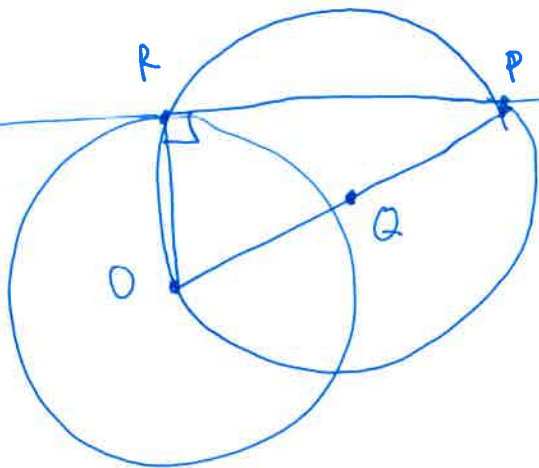
$$\text{Siiis} \quad \left(\frac{c}{e}\right)^2 + \left(\frac{d}{e}\right)^2 = \frac{c}{e} \cdot \frac{c}{e} + \frac{d}{e} \cdot \frac{d}{e} = \frac{c}{e} \cdot \frac{a}{c} + \frac{d}{e} \cdot \frac{b}{d} \\ = \frac{a+b}{e} = 1 \quad \parallel \cdot e^2$$

$$\Rightarrow c^2 + d^2 = e^2$$

5. Piiiräi OP, olkoon Q ∈ OP, jolle OQ = QP.
 Piiiräi C(Q, QO) ja olkoon
 "ympyrä" ∩ C(Q, QO) = R.
 Piiiräi \overleftrightarrow{PR} .

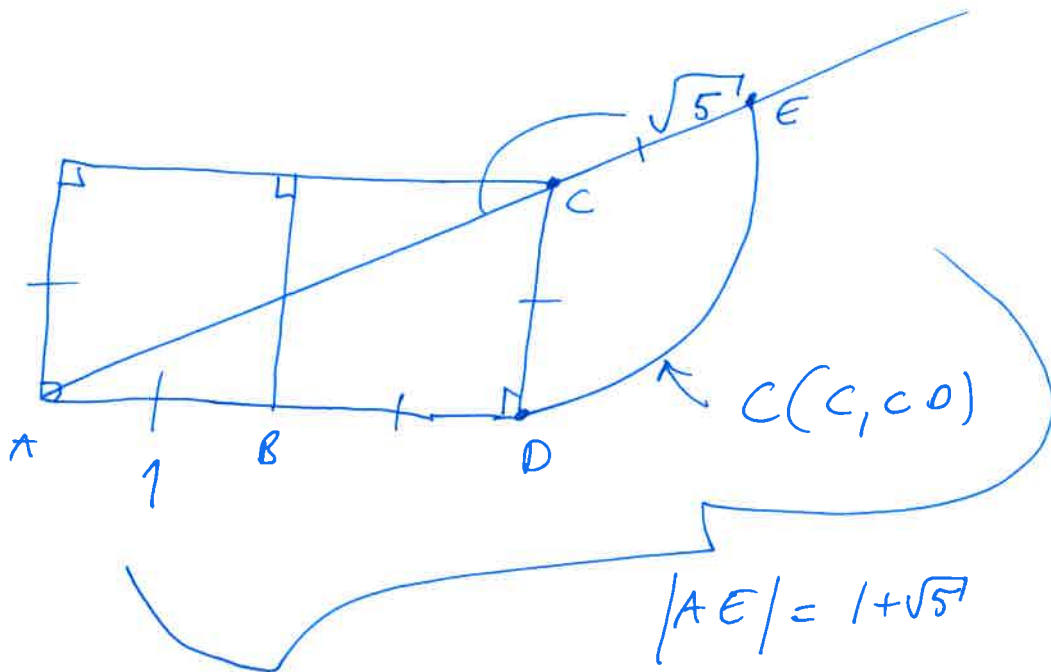
NYT \overleftrightarrow{PR} on haluttu tangentti,
 eli $\overleftrightarrow{PR} \perp OR$

PERUSTELU:

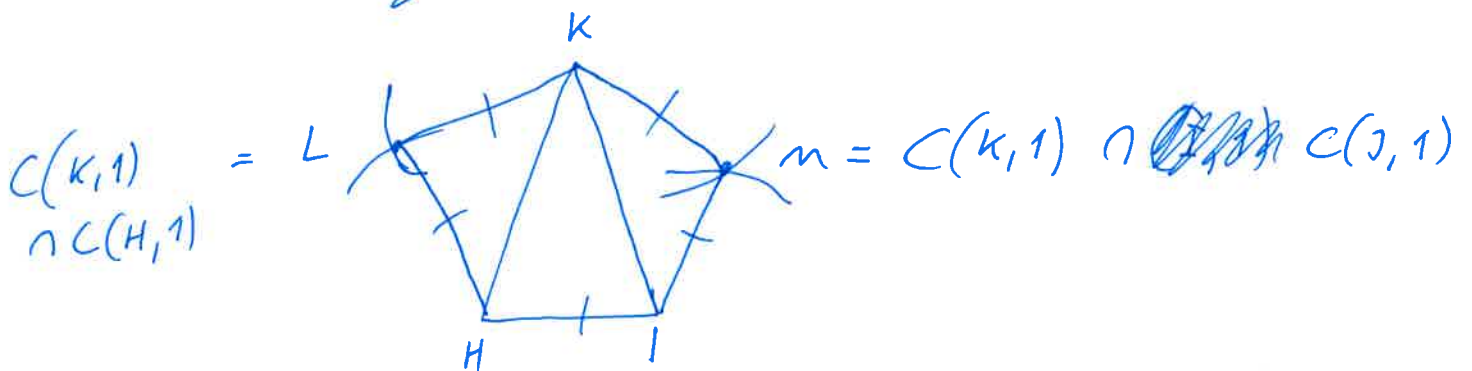
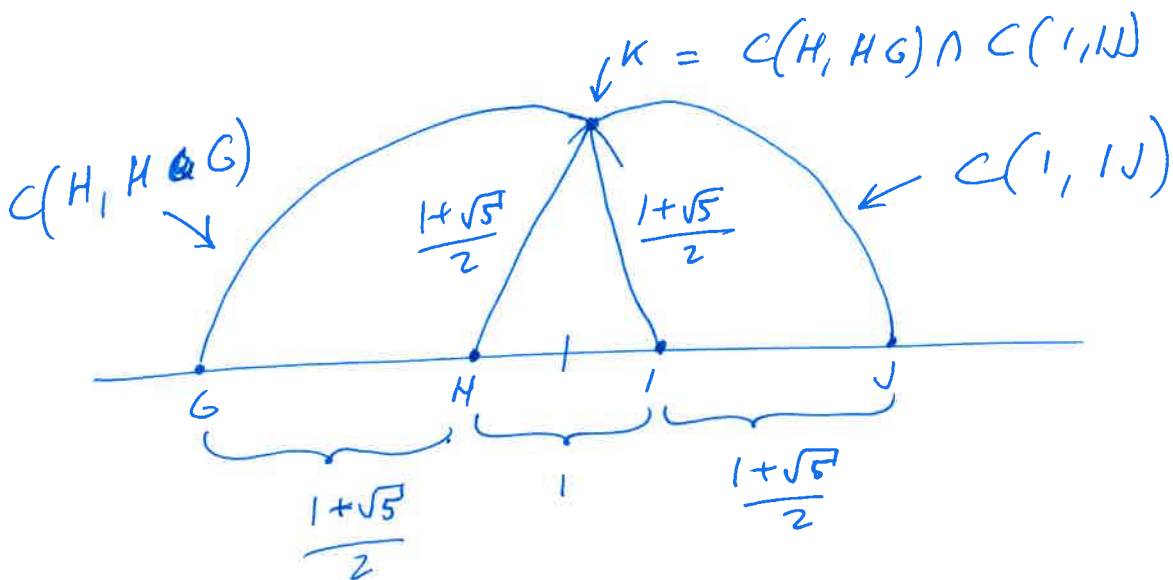


THALEEN LAUSEEN
 (HYT2©) NOJALLA
 $OR \perp RP$

6.



$AF = FE \Rightarrow AF = \frac{1 + \sqrt{5}}{2}$



8.

	V	E	F	χ
TETRAEDRI	4	6	4	$2 = 4 - 6 + 4$
KUBUTIO	8	12	6	$2 = 8 - 12 + 6$
OKTAEDRI	6	12	8	$2 = 6 - 12 + 8$
DODEKAEDRI	20	30	12	$2 = 20 - 30 + 12$
IKOSAEDRI	12	30	20	$2 = 12 - 30 + 20$