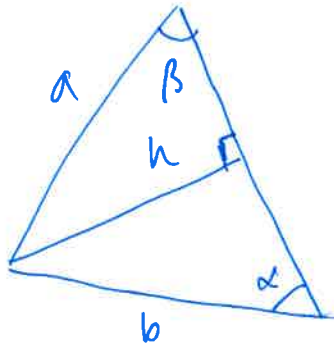


EUKLIDINEN GEOMETRIA
 HARJOITUS 6 RATKAISUJA

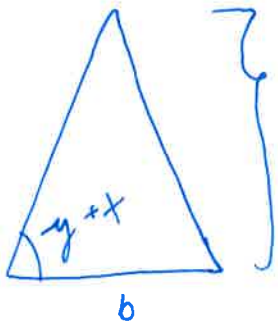
1.



$$h = a \sin \beta = b \sin \alpha$$

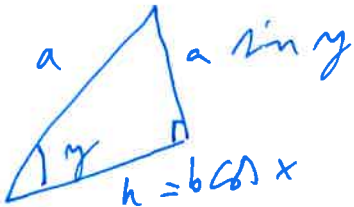
$$\Rightarrow \frac{\sin \alpha}{a} = \frac{\sin \beta}{b}$$

2.

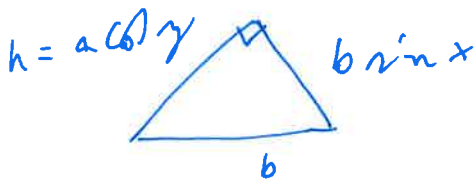


$$h = a \sin(x+y)$$

$$A_1 = \frac{1}{2} a b \sin(x+y)$$



$$A_2 = \frac{1}{2} a b \sin y \cos x$$



$$A_3 = \frac{1}{2} a b \sin x \cos y$$

$$A_1 = A_3 + A_2$$

$$\frac{1}{2} a b \sin(x+y) = \frac{1}{2} a b \sin x \cos y + \frac{1}{2} a b \sin y \cos x \quad \parallel : \frac{1}{2} a b$$

$$\Rightarrow \sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$3. \sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y) \quad \parallel \frac{d}{dx}$$

$$\frac{d}{dx} \sin(x+y) = \frac{d \sin(x+y)}{d(x+y)} \frac{d(x+y)}{dx}$$

KETJU SÄÄNTÖ

$$= \cos(x+y) \cdot 1$$

VAKIO
X:N
SUMMEEN

$$= \cos(y) \left[\frac{d \sin x}{dx} \right] + \sin(y) \left[\frac{d \cos(x)}{dx} \right]$$

$$= \cos(y) \cos(x) - \sin(y) \sin(x)$$

$$4. \begin{aligned} \sin(x+y) &= \sin(x) \cos(y) + \cos(x) \sin(y) \\ + \sin(x-y) &= \sin(x) \cos(y) - \cos(x) \sin(y) \end{aligned}$$

$$\begin{cases} \cos(-y) = \cos y \\ \sin(-y) = -\sin y \end{cases}$$

$$\sin(x+y) + \sin(x-y) = 2 \sin(x) \cos(y)$$

SIS (4) ON TOSI, SAA TÄISIIN

$$(4) \xRightarrow{\frac{d}{dy}} (5) \quad \& \quad (4) \xRightarrow{\frac{d}{dx}} (6)$$

TOISAALTA

$$\begin{aligned}\cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) \\ + \cos(x-y) &= \cos(x)\cos(y) + \sin(x)\sin(y)\end{aligned}$$

$$\cos(x+y) + \cos(x-y) = 2\cos(x)\cos(y) \Rightarrow \textcircled{6}$$

$$\begin{aligned}\text{JA } \cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) \\ \cos(x-y) &= \cos(x)\cos(y) + \sin(x)\sin(y)\end{aligned}$$

$$\cos(x+y) - \cos(x-y) = -2\sin(x)\sin(y) \quad \text{||} \cdot (-1)$$

$$\cos(x-y) - \cos(x+y) = 2\sin(x)\sin(y) \Rightarrow \textcircled{5}$$

VALITTAVAN KAARASSA $\textcircled{5}$ $y=x \Rightarrow$

$$2\sin^2(x) = 1 - \cos(2x) \Rightarrow \textcircled{7} \quad [\cos(0) = 1]$$

VALITTAVAN KAARASSA $\textcircled{6}$ $y=x \Rightarrow$

$$2\cos^2(x) = 1 + \cos(2x) \Rightarrow \textcircled{8}$$

$$5. \quad e^{i(x+y)} = \underline{\cos(x+y)} + i \underline{\sin(x+y)}$$

$$= e^{i(x+y)} = e^{ix} e^{iy}$$

$$= (\cos x + i \sin x) (\cos y + i \sin y)$$

$$= \cos x \cos y + \cos x i \sin y$$

$$+ i \sin x \cos y + \underbrace{i^2 \sin x \sin y}_{=-1}$$

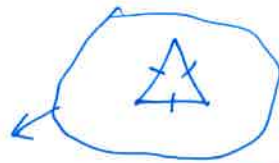
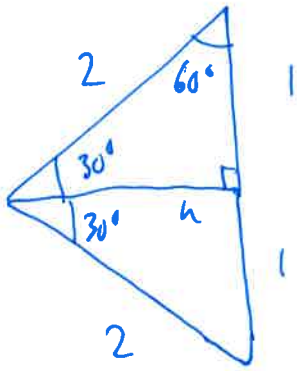
$$= \underline{\cos x \cos y - \sin x \sin y}$$

$$+ i (\cos x \sin y + \sin x \cos y)$$

$$= \text{SAMAT} \Rightarrow \textcircled{3}$$

$$\text{SAMT} \Rightarrow \textcircled{2}$$

6.



$$h = \sqrt{2^2 - 1^2} = \sqrt{3}$$

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

S115

$$(7) \Rightarrow \sin(15^\circ) = \sqrt{\frac{1}{2}(1 - \cos(30^\circ))}$$

$$= \sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\right)}$$

$$= \frac{\sqrt{2 - \sqrt{3}}}{2}$$

(8) \Rightarrow

$$\cos(15^\circ) = \sqrt{\frac{1}{2}(1 + \cos(30^\circ))}$$

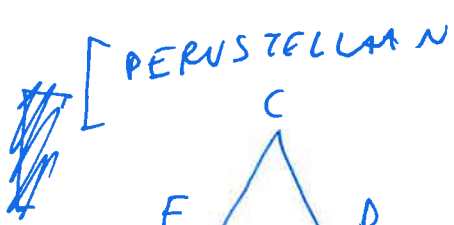
$$= \sqrt{\frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\right)} = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

7. KOLMIOLLE $\triangle FBC$ JA SUORALLE \overleftrightarrow{AO} MENELÄKSEN LAUSEEN MUKAAN

$$\frac{CO}{OF} \times \frac{FA}{AB} \times \frac{BD}{DC} = 1$$

$\underbrace{\quad}_{= \frac{1}{2}} \quad \quad \quad \underbrace{\quad}_{= 1}$

$$\Rightarrow \frac{CO}{OF} = 2 \Rightarrow \underline{\underline{CO = 2OF}}$$



PISTEEN O OLEMASKOLO:

$$\frac{CE}{EA} \times \frac{AF}{FB} \times \frac{BD}{DC} \stackrel{AM}{=} 1$$

$$= \frac{CE}{CE} \times \frac{AF}{AF} \times \frac{BD}{BD}$$

$$= 1 \cdot 1 \cdot 1 = 1$$

~~##~~ KÄÄN TÄISEN CEVAAN LAUSEEN NOJALLA

$$\boxed{\exists O \in AD \cap BE \cap CF.}$$