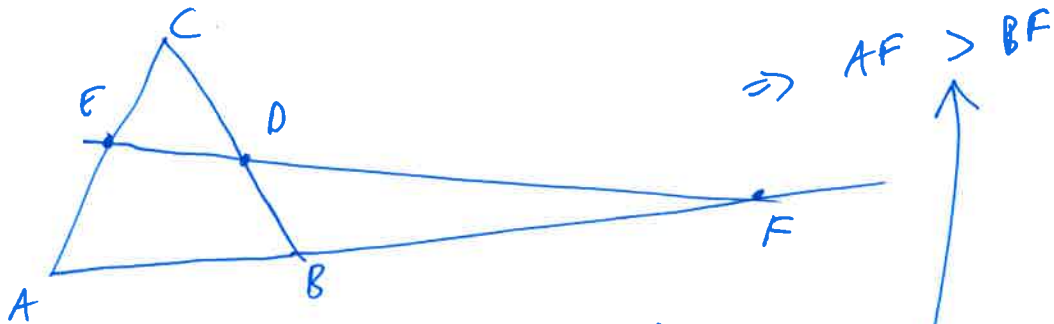


EUKLIDINEN GEOMETRIA  
 HARJOITUS 7, RATKAISUJA

1, AT



MENEFLOS :  $\frac{CF}{EA} \times \frac{AF}{FB} \times \frac{BD}{DC} = 1$

$\Rightarrow \frac{CE/EA}{CD/OB} \times \frac{AF}{FB} = 1$

$= 1$  , KOSKA  $\frac{CE}{EA} = \frac{CD}{OB}$

$\Rightarrow \frac{AF}{FB} = 1 \Rightarrow AF = FB$

$\Rightarrow AF > BF$

RR

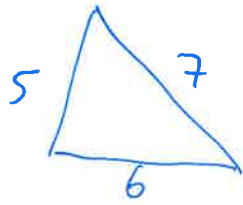
2. SIIS  $AB = BC = CA$ .

PTOLEMIOS :  $AB \times CD + BC \times AD = BD \times AC$   
 $\underbrace{\hspace{1cm}}_{=AB}$   $\underbrace{\hspace{1cm}}_{=AB}$   $\underbrace{\hspace{1cm}}_{=AB}$

$\Rightarrow AB \times CD + AB \times AD = BD \times AB \parallel : AB$

$\Rightarrow \underline{\underline{CD + AD = BD}}$

3. (a)

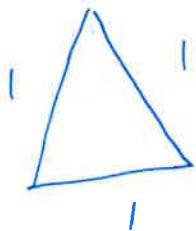


$$p = \frac{5+6+7}{2} = \frac{18}{2} = 9$$

HERON:

$$\begin{aligned} A &= \sqrt{p(p-a)(p-b)(p-c)} \\ &= \sqrt{9(9-5)(9-6)(9-7)} \\ &= \sqrt{9 \cdot 4 \cdot 3 \cdot 2} \\ &= \sqrt{36 \cdot 6} = \underline{\underline{6\sqrt{6}}} \end{aligned}$$

(b)



$$p = \frac{1+1+1}{2} = \frac{3}{2}$$

HERON: 
$$\begin{aligned} A &= \sqrt{p(p-a)(p-b)(p-c)} \\ &= \sqrt{\frac{3}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}} \\ &= \frac{\sqrt{3}}{4} \end{aligned}$$

4 korci



$$\text{ALA KORCI} = 4 \cdot \frac{\sqrt{3}}{4} = \underline{\underline{\sqrt{3}}}$$

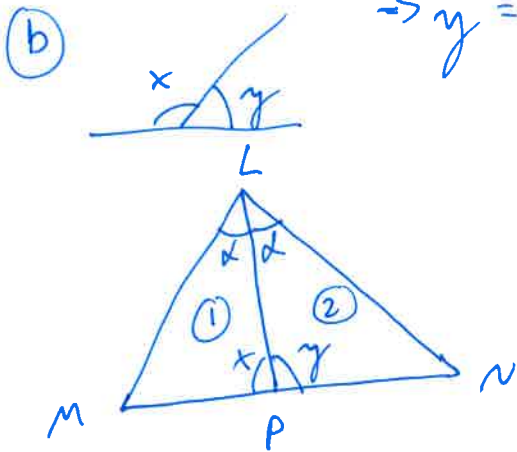
$$4. (a) \sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta) \quad \left| \begin{array}{l} \alpha = 180^\circ \\ \beta = -x \end{array} \right.$$

$$\sin(180^\circ - x) = \underbrace{\sin(180^\circ)}_{=0} \cos(-x) + \underbrace{\cos(180^\circ)}_{=-1} \underbrace{\sin(-x)}_{=-\sin(x)}$$

$$\Rightarrow \sin(180^\circ - x) = \sin(x)$$

VIERECKLMAT

$$\Rightarrow y = 180^\circ - x \Rightarrow \sin(x) = \sin(y)$$



SINUSLAUSE:

$$(1) \frac{\sin(x)}{ML} = \frac{\sin(\alpha)}{MP} \Rightarrow \frac{MP}{ML} \sin(x) = \sin \alpha$$

$$(2) \frac{\sin(y)}{NL} = \frac{\sin(\alpha)}{PN} \Rightarrow \frac{PN}{NL} \sin(y) = \sin \alpha$$

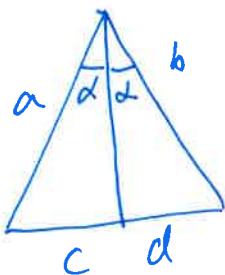
$$\Rightarrow \frac{MP}{ML} \sin(x) = \sin \alpha = \frac{PN}{NL} \sin(y)$$

$$\left. \begin{array}{l} \sin(x) \\ = \sin(y) \end{array} \right|$$

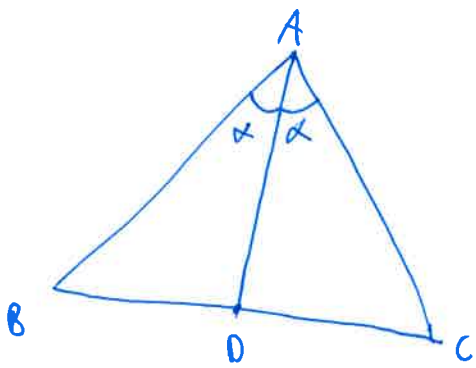
$$\Rightarrow \frac{MP}{ML} = \frac{PN}{NL}$$

$$\Rightarrow \boxed{\frac{MP}{PN} = \frac{ML}{LN}}$$

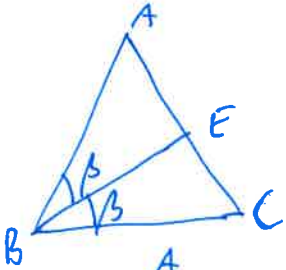
S115



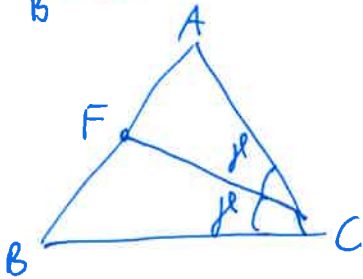
$$\Rightarrow \frac{c}{d} = \frac{a}{b}$$



$$\frac{BD}{DC} = \frac{BA}{AC}$$

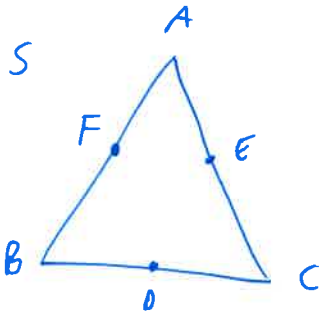


$$\frac{CE}{EA} = \frac{CB}{BA}$$



$$\frac{AF}{FB} = \frac{AC}{CB}$$

SIIS



$$\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA} = \frac{AC}{CB} \times \frac{BA}{AC} \times \frac{CB}{BA} = 1$$

KÄÄNTÄINEN CEVA =>

$$\exists O \in AD \cap BE \cap CF.$$

SIIS KOLMION KULMANPUOLITTAJAT LEIKKAAVAT SAMASSA PISTEISSÄ.