Kompleksianalyysi a
Syksy 2015
Harjoitus 7

1. Laske arvot: (a) $\log i$, (b) $\log(1-i)$, (c) $\operatorname{Log}(-i)$, (d) $\operatorname{Log}(\sqrt{3}+i)$.

2. Ratkaise yhtälöt: (a) $e^z = 2i$, (b) $\operatorname{Log}(z^2-1) = \frac{i\pi}{2}$, (c) $e^{2z}+e^z+1=0$.

3. Missä alueessa $f(z) = \operatorname{Log}(4+i-z)$ on analyyttinen. Laske $f'(z)$.

4. Määrää funktion $f(z) = \log(z^2+2z+3)$ se haara, joka on analyyttinen pisteessä $z=-1$. Laske $f'(-1)$.

5. Laske kaikki arvot: (a) i^i, (b) $(-1)^{\frac{2}{3}}$, (c) $2^{\pi i}$, (d) $(1+i)^{1-i}$.

6. Johda kaavat

$$\sinh^{-1} z = \log\left[z + (z^2+1)^{1/2}\right], \qquad \cosh^{-1} z = \log\left[z + (z^2-1)^{1/2}\right].$$

7. Etsi seuraavien moniarvoisten funktioiden haarat, jotka ovat analyyttisiä annetuissa alueissa.

   (a) $(z^2-1)^{\frac{1}{2}}$ yksikkökiekossa.

   (b) $(4+z^2)^{\frac{1}{2}}$ kompleksitasossa, josta on leikattu pois jana pisteestä $-2i$ pisteeseen $2i$.

   (c) $(z^3-1)^{\frac{1}{3}}$ joukossa $\{z : |z| > 1\}$.

8. Määrää funktion

$$w = q(z) := 2e^z + e^{2z}$$

   käänteisfunktio kompleksisen logaritmin avulla. Etsi ne pisteet z, joissa $q(z) = 3$.

Complex Analysis a
Autumn 2015
Exercise 7

1. Calculate: (a) $\log i$, (b) $\log(1-i)$, (c) $\operatorname{Log}(-i)$, (d) $\operatorname{Log}(\sqrt{3}+i)$.
2. Solve: (a) $e^z = 2i$, (b) $\operatorname{Log}(z^2-1) = \frac{i\pi}{2}$, (c) $e^{2z} + e^z + 1 = 0$.
3. Where is $f(z) = \operatorname{Log}(4+i-z)$ analytic? Calculate $f'(z)$.
4. Determine a branch of $f(z) = \log(z^2+2z+3)$ which is analytic at $z=-1$. Calculate $f'(-1)$.
5. Calculate all the values of: (a) i^i, (b) $(-1)^{\frac{2}{3}}$, (c) $2^{\pi i}$, (d) $(1+i)^{1-i}$.
6. Find the equalities

$$\sinh^{-1} z = \log\left[z + (z^2+1)^{1/2}\right], \qquad \cosh^{-1} z = \log\left[z + (z^2-1)^{1/2}\right].$$

7. Find a branch of each of the following multiple-valued functions that is analytic in the given domain:
   (a) $(z^2-1)^{\frac{1}{2}}$ in the unit disc.
   (b) $(4+z^2)^{\frac{1}{2}}$ on the complex plane with the line from $-2i$ to $2i$ removed.
   (c) $(z^3-1)^{\frac{1}{3}}$ in $\{z : |z| > 1\}$.
8. Determine the inverse of the function

$$w = q(z) := 2e^z + e^{2z}$$

   explicitly in terms of the complex logarithms. Use your formula to find all values of z for which $q(z) = 3$.