

9 December, 2016

To whom it may concern,

This is a letter of support written upon a request from Mr. *Juha-Matti Huusko*, who is applying for a position in your department. Getting a postdoctoral position in a research-active environment would be most helpful for his further mathematical development.

I have known the applicant since I started my work at the Department of Physics and Mathematics in the University of Eastern Finland, Joensuu (academic course 2013/2014). I was his teacher in the elective advanced course *Numerical Linear Algebra*. His grade in this course was among the top two best ones. Other such courses taught by Juha-Matti were, for instance, Differential Geometry, Functional Analysis, or Univalent Functions. The interest shown by Juha-Matti in these such different-nature subjects is just a sample that reveals that Mr. Huusko is one of those exceptional students who are willing to learn and develop the different interactions and relations that can occur among distinct areas of mathematics.

In addition to what is mentioned in the previous paragraph, I would like to point out that I have had the chance to attend Juha-Matti's seminar talks in Joensuu and maintain several mathematical discussions with him. These facts not only allowed me to follow the development of his thesis work, but also led to the production of our recently submitted joint paper on a topic that is not directly related to those questions considered in Mr. Huusko's Ph. D. thesis. What I can say at this point is that I believe that his progress in doing research along these years has been steady and spectacular.

More specifically, during the development of his thesis, Mr. Huusko studies different problems regarding the growth of solutions of linear differential equations with analytic coefficients that I will now describe in more detail.

A fundamental question in the study of linear differential equations with analytic coefficients is to relate the growth of the coefficients to the growth of the solutions and to the distribution of their zeros.

In his first individual article (published in *Bull. Aust. Math. Soc.*, 2016), Mr. Huusko uses conformal maps to localize higher order differential equations with analytic coefficients, which allows him to apply known results for the unit disk to obtain lower bounds for the growth of solutions.

The main purpose in the second paper included in Juha Matti's thesis (in collaboration with T. Korhonen and A. Reijonen) is to find conditions which guarantee that all the solutions of non-necessarily homogeneous linear differential equations with analytic coefficients belong to either weighted H^∞ spaces or standard weighted Bloch spaces of analytic functions in the unit disk. In particular, the authors obtain a condition on the coefficients that ensures that every solution of the corresponding differential equation belongs to the classical Bloch space.

Finally, the separation of zeros of solutions of higher order linear differential equations is considered by Mr. Huusko (in collaboration with J. Gröhn and J. Rättyä) in the third article that completes Juha-Matti's Ph. D. work. Moreover, in the second order case (and among other results) a different approach from those one in the literature is applied to obtain sufficient conditions that place all solutions of the equation in the classical Hardy spaces of analytic functions.

In addition to the three papers in Mr. Huusko's Ph. D. thesis described above, very recently we both have extended two classical criteria for bounded valence of analytic functions in the unit disk due to Gehring and Pommerenke (*Comment. Math. Helv.*, 1984) and Becker and Pommerenke (*Comput. Methods Funct. Theory*, 2016), respectively, to the cases when the mappings considered are merely harmonic. As far as we are aware, these are the first results regarding these type of criteria for harmonic functions in the unit disk. This opened a new line of research with respect to the publications from Juha-Matti Ph. D. studies.

In summary, it can be said that Juha-Matti already possesses a relatively broad knowledge of classical spaces of analytic functions, linear differential equations, and geometric function theory, as well as of different parts of the theory of harmonic mappings in the plane. I should also add that it is quite easy to collaborate with him, since he is a very pleasant and easy going person.

What seems important now for Mr. Huusko's further mathematical development is to get immersed into a very active new research environment under a supervision of well-known experts in order to broaden further his horizons and enhance his international collaborations. I think this goal could be fully achieved if he would get the chance of getting a postdoctoral position in your Department. I strongly support his application for this purpose.

If I can provide any further information, please do not hesitate to contact me.

Sincerely yours,



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