

Differential geometry

Spring 2012

Exercise 3.

1. Compute the curvatures of the following curves
 - (a) The ellipse $c(t) = (a \cos(t), b \sin(t))$.
 - (b) The curve given in *polar coordinates* by $f(r, \theta) = r - \cos(2\theta) = 0$.
2. Determine the evolute of an ellipse $c(t) = (a \cos(t), b \sin(t))$.
3. Prove that if the curvature κ of curve c is at some point $\kappa'(s_0) = 0$ and $\kappa(s_0) \neq 0$ then the evolute e of the curve c is not regular at s_0 .
4. Compute the representation for a curve whose (signed) curvature is given by arc length $\kappa = 1/\sqrt{s}$.
Hint: Use the representation we got when proving theorem 2.9.
5. Compute the involute of a circle.
6. Prove that the set $\mathbb{O}(n)$ is in fact a group with respect to matrix multiplication. Is the group Abelian? That is; does the matrix product commute in $\mathbb{O}(n)$.
7. Report all the possible errors in lecture notes.