

$$D x^7 = 7 x^6$$

$$D \sqrt{x} = D x^{\frac{1}{2}} = \frac{1}{2} x^{-\frac{1}{2}}$$

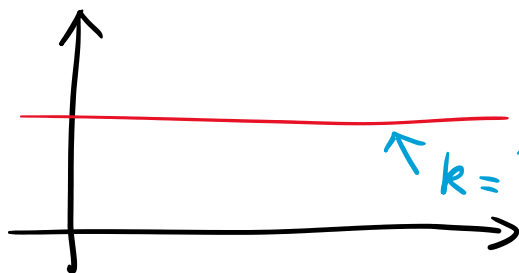
$$D \frac{3}{x^2} = D 3 x^{-2} = 3 \cdot (-2) x^{-3} = -6 x^{-3}$$

$$D e^x = e^x$$

$$D \ln(x) = \frac{1}{x}$$

$$D e^{x^5} = e^{x^5} \cdot 5x^4$$

$$D \ln(\sin(x)) = \frac{1}{\sin(x)} \cdot \cos(x)$$



$$y=7$$

KASVUNOPEUS
= KULMAKERROIN

$$D \text{ vakio} = 0$$

$$D 7 = D 7 x^0 =$$

$$D x^7 = 7 x^6$$

$$D 3 x^{-2} = 3(-2) x^{-3}$$

$$D 7 x^0 = 7 \cdot 0 = \underline{\underline{0}}$$

Potenssin derivaatta	Trigonometristen funktioiden derivaatat	Arkusfunktioiden derivaatat
$D x^n = n x^{n-1}$, missä $n \in \mathbb{Z}$	$D \sin x = \cos x$	$D \arcsin x = \frac{1}{\sqrt{1-x^2}}$
$D \sqrt{x} = \frac{1}{2\sqrt{x}}$ [2]	$D \cos x = -\sin x$	$D \arccos x = -\frac{1}{\sqrt{1-x^2}}$
$D \sqrt[3]{x} = \frac{1}{3\sqrt[3]{x^2}}$	$D \tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$	$D \arctan x = \frac{1}{1+x^2}$
$D \sqrt[n]{x} = \frac{1}{n\sqrt[n]{x^{n-1}}}$	$D \cot x = -\frac{1}{\sin^2 x} = -1 - \cot^2 x$	$D \operatorname{arccot} x = -\frac{1}{1+x^2}$
Eksponenttifunktion derivaatta	Hyperbolisten funktioiden derivaatat	Hyperbolisten kääntöfunktioiden eli areafunktioiden derivaatat
$D e^x = e^x$	$D \sinh x = \cosh x$	$D \operatorname{arsinh} x = \frac{1}{\sqrt{x^2+1}}$
$D a^x = a^x \ln a$, missä $a > 0$	$D \cosh x = \sinh x$	$D \operatorname{arcosh} x = \frac{1}{\sqrt{x^2-1}}$

$Da^x = a^x \ln a$, missä $a > 0$	$D \cosh x = \sinh x$	$D \operatorname{arccosh} x = \frac{1}{\sqrt{x^2 - 1}}$
	$D \tanh x = \frac{1}{\cosh^2 x}$	$D \operatorname{artanh} x = \frac{1}{1 - x^2}$
Logaritmifunktioiden derivaatat		
$D \ln x = \frac{1}{x}$		
$D \log_a x = \frac{1}{x \ln a}$, missä $a > 0$ ja $a \neq 1$		

Säännön nimi	Derivointisääntö
Vakion derivaatta	$Dc = 0$, kun c on vakio. ^[4]
Vakion siirto	$Dcf(x) = cDf(x)$ ^{[1][4]}
Summan derivaatta	$D(f(x) + g(x)) = Df(x) + Dg(x)$ ^{[1][10][4]}
Tulon derivaatta	$D(f(x)g(x)) = g(x)Df(x) + f(x)Dg(x)$ ^{[1][12][13][4]}
Funktion potenssin derivaatta	$Df(x)^n = n f(x)^{n-1} f'(x)$ ^{[12][14]}
Osamäärän derivaatta	$D \frac{f(x)}{g(x)} = \frac{g(x)Df(x) - f(x)Dg(x)}{g(x)^2}$ ^{[14][13][4]}
Yhdistetyn funktion derivaatta	$Dg(f(x)) = g'(f(x))f'(x)$ ^[4]
Käänteisfunktion derivaatta	$(f^{-1})'(f(x)) = \frac{1}{f'(x)}$, jossa f^{-1} on f :n käänteisfunktio. ^[4]

$$e^{x^5} \ln(\sin(x))$$

$$\begin{aligned}
 & D x^5 \ln(x) \\
 &= 5x^4 \ln(x) + x^5 \cdot \frac{1}{x} \\
 &= 5x^4 \ln(x) + x^4 = \frac{x^5}{x} = x^4
 \end{aligned}$$

Osamäärän derivaatta

$$D \frac{f(x)}{g(x)} = \frac{g(x)Df(x) - f(x)Dg(x)}{g(x)^2}$$

$$D \frac{x^5}{\sin(x)} = \frac{5x^4 \sin(x) - x^5 \cos(x)}{\sin(x)^2}$$

$$\begin{aligned}
 D \frac{f(x)}{g(x)^{-1}} &= D f(x) (g(x))^{-1} & \frac{1}{x^2} &= x^{-2} \\
 &= f'(x) \underbrace{(g(x))^{-2}}_{g(x)^{-2}} g'(x) + f(x) \cdot (-1) \underbrace{(g(x))^{-2}}_{g(x)^{-2}} g'(x)
 \end{aligned}$$

$$\begin{aligned}
 &= f'(x) \underbrace{(g(x))}_{g'(x)} + f(x) \cdot (-1) \underbrace{(g(x))}_{g'(x)} \\
 &= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}
 \end{aligned}$$

INTEGRATION
ON DERIVATION
VIA ANTI-OPERATION

$$D x^{\textcircled{7}} = 7 x^{\overset{-1}{6}}$$

$$\int x^5 dx = \frac{x^6}{6}$$

$$\int x^3 dx = \frac{x^4}{4}$$

$$D e^x = e^x \quad \int e^x dx = e^x$$

$$\int x^3 dx = \frac{x^4}{4}$$

$$\int e^{2x} dx = \frac{e^{2x}}{2}$$

$$D e^{2x} = e^{2x} \cdot \underbrace{D(2x)}_{=2}$$

$$D e^{2x} = 2e^{2x}$$

$$\int e^{2x} dx = \frac{e^{2x}}{2}$$

$$\int e^{5x} dx = \frac{e^{5x}}{5}$$

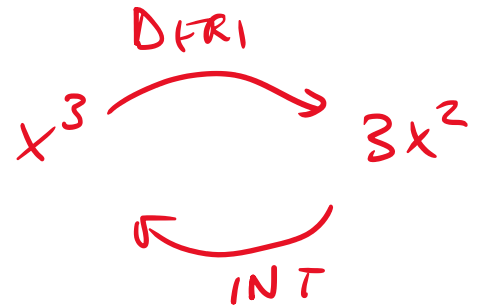
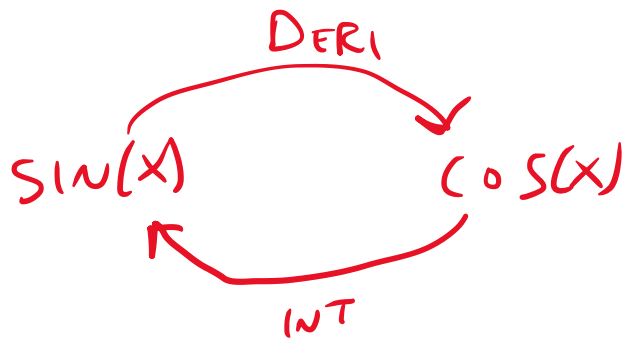
$$D \sin(x) = \cos(x)$$

$$\int \cos(x) dx = \sin(x)$$

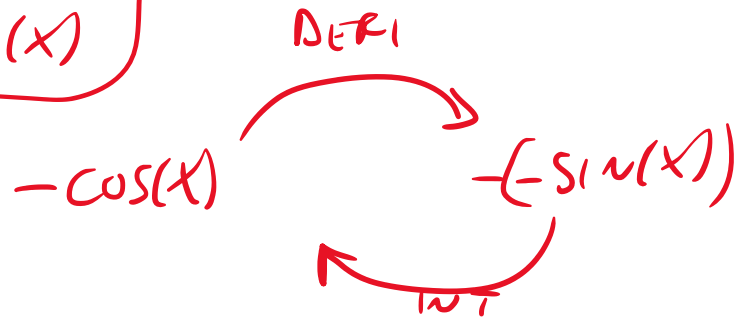
$$D \cos(x) = -\sin(x)$$

DERI

DERI

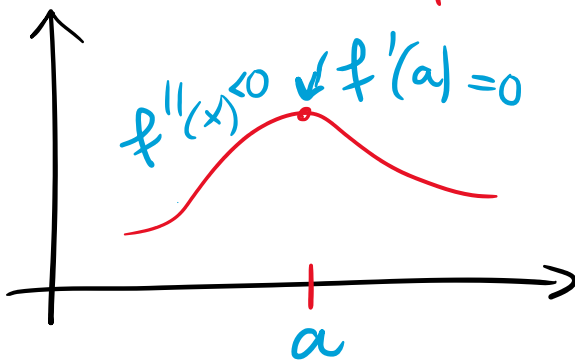
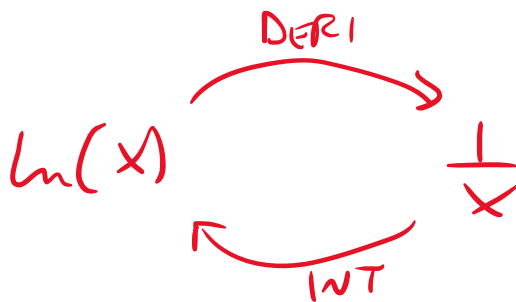


$$\int \sin(x) dx = -\cos(x)$$



$$D \ln(x) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \ln(x)$$



MISTÄ? TIETÄÄ:
ONKO
MAKSIMI/MINIMI, ?

① KUVA / LUONNOS

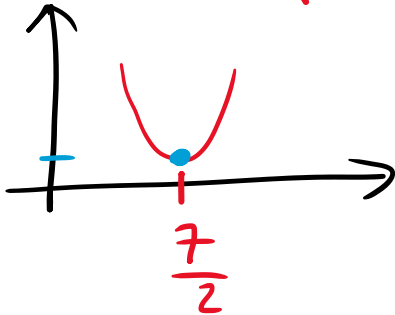
② PISTESSÄ a

$$\left[\begin{array}{l} \text{MAKSIMAL, JOS} \\ \left\{ \begin{array}{l} f'(a) = 0 \\ f''(a) < 0 \end{array} \right. \end{array} \right.$$

$$\left[\begin{array}{l} \text{MINIMAL, JOS} \\ \left\{ \begin{array}{l} f'(a) = 0 \\ f''(a) > 0 \end{array} \right. \end{array} \right.$$

ESIM. $f(x) = x^2 - 7x$

POLYNOMIAL



$$2x - 7 = 0 \quad || +7$$

$$2x = 7 \quad || :2$$

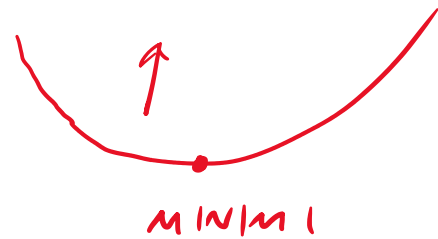
$$x = \frac{7}{2} = 3.5$$

ETS1 MINIMAL,

$$f(x) = x^2 - 7x$$

$$f'(x) = 2x - 7$$

$$f''(x) = 2 > 0$$



$$f(x) = x^2 - 7x$$

$$\begin{aligned} f\left(\frac{7}{2}\right) &= \left(\frac{7}{2}\right)^2 - 7 \cdot \frac{7}{2} = \frac{49}{4} - \frac{49}{2} \\ &= -\frac{49}{4} \end{aligned}$$

HARJOITTELETT

— MINIMAL & MAKSIMAL

— INTEGRANT

KOE