1 Lineaariset funktiot

tiistai 12. maaliskuuta 2024 17.58

MATEMATIIKAN PERUSTEET TIETOTEKNIIKASSA 2

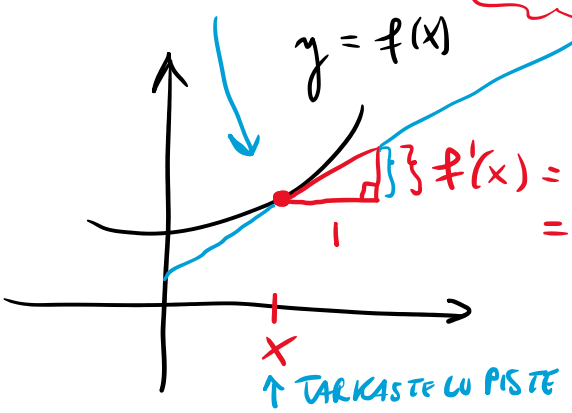
SISÄLTÖ

- DERIVOINTI = KASVUNOPEUS
- INTEGROINTI = PINTA-ALAN KERTYMÄ
- NÄIDEN SOVELLUKSET

ALOITETAAN NOIN KLO 19,05

HIPAISEE

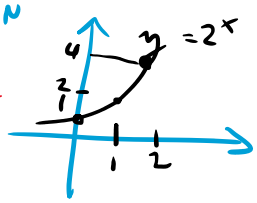
HARJOITELLAAN SUORIEN PIIRTÄMISTÄ JA KULMAKERROINTA



TANGENTTISUORAN KULMAKERROIN

$f'(x) = 0.4 = k$

= KASVUNOPEUS



$= \frac{y\text{:N MUUTOS}}{x\text{:N MUUTOS}}$

ESIMERKKI. SUORA PISTEIDEN $p = (\overset{x}{1}, \overset{y}{2})$

$q = (7, 3)$

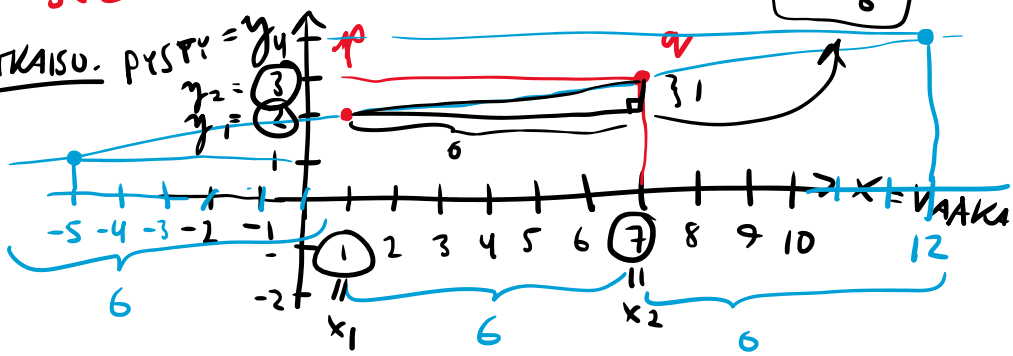
KAUTTA.

- PIIRRÄ OK
- LASKE KULMAKERROIN OK
- SELVITTÄÄ SUORAN YHTÄLÖ $\boxed{k = 1}$

- LASKE KULMAKERROIN OK
- SELVITTÄÄ SUORAN YHTÄLÖ

$\boxed{k = \frac{1}{6}}$

RATKAISU.



KULMAKERROIN $= k = \frac{y:n \text{ MUUTOS}}{x:n \text{ MUUTOS}} = \frac{y_2 - y_1}{x_2 - x_1}$

$p = (1,2)$

$= \frac{3-2}{7-1} = \boxed{\frac{1}{6}}$

SUORAN YHTÄLÖ

$y - y_1 = k(x - x_1)$

$y - 2 = \frac{1}{6}(x - 1)$

$y - 2 = \frac{1}{6}x - \frac{1}{6} \quad \| + 2 = \frac{12}{6}$

$y = \frac{1}{6}x + \frac{11}{6}$

$y = kx + b$

FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA

WolframAlpha

derivative of x^2 at x=1

NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EX

Input interpretation

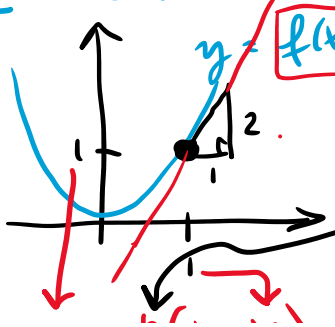
$\frac{\partial x^2}{\partial x}$ where $x = 1$

Result

2

TARKISTUS WOLFRAM ALPHALLA

ESIM. YLLÄOLEVAA TARVITAAN JATKOSSA.



$y = \boxed{f(x) = x^2}$

KASVUNOPEUS = TANGENTIN KULMAKERROIN

$= \boxed{2} = k$

$= \boxed{f'(2)}$

HARJOITELLAAN ENSI VIIKOLLA

$= \boxed{f'(2)}$ ← ENSI VIIKOLLA

$y - y_1 = k(x - x_1)$

$y - 1 = 2(x - 1)$

$y - 1 = 2x - 2 \quad \| +1$

$\boxed{y = 2x - 1}$ →

VOIDAAN KÄYTTÄÄ FUNKTION ARVIOINTIIN

$\boxed{x^2 \approx 2x - 1, \text{ KUN } x \approx 1}$

$\Rightarrow (1.01)^2 \approx 2 \cdot 1.01 - 1 = 2.02 - 1 = 1{,}02$

DERIVAATTA SOVELTUU SIIS HYVIN FUNKTIOIDEN ARVIOINTIIN

ESIM. TILILLÄ ON 1000 €. KORKO ON 2%. MONENKO VUODEN (n) PÄÄSTÄ TILILLÄ ON 2000 €?

RATKAISU. $1000 \cdot 1.02^n = 2000 \quad \| : 1000$ ($n = ?$)

TUNTEMATON POTENSSISSA ⇒ LOGARITMI!

$1.02^n = 2 \quad \| \ln(\ )$

$\ln(1.02^n) = \ln(2)$

$n \ln(1.02) = \ln(2) \quad \| : \ln(1.02)$

$n = \dfrac{\ln(2)}{\ln(1.02)} \approx \dfrac{0.7}{\frac{2}{100}} = \dfrac{70}{2} = 35$

$\ln\left(1 + \frac{2}{100}\right)$

$\boxed{\ln(1+x) \approx x, \text{ KUN } x \approx 0}$

ESIM. KAKSINKERTAISTUMINEN

VUOSIA $= \boxed{n = \dfrac{70}{p}}$ (KUN p ON PIENI)

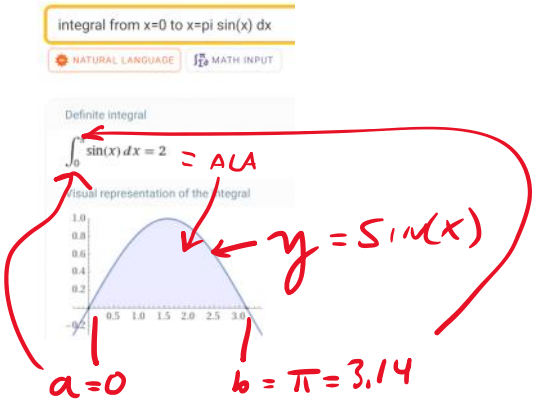
p ← KORKOPROSENTTI

ESIM. KORKO 1%, $1000€ \xrightarrow{n} 2000€$

RATKAISU. $n = \dfrac{70}{1} = 70$

MOODLE → PDF → 5 PALAUTETTAVAA SUORISTA

INTEGRAALI (= PINTA-ALAN KERTYMÄ)



$$\int_0^{\pi} \sin(x)\,dx = ?$$

KAKSI VAIHETTA

① ETSITÄÄN INTEGRAALIFUNKTIO] VOI OLLA VAIKEAA, OPETELLAAN VÄHITELLEN

② SIJOITETAAN] MEKKAANISTA, "YLÄ − ALA"

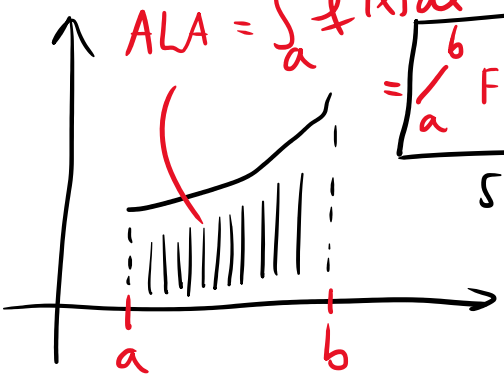
$$\int_0^{\pi} \sin(x)\,dx \overset{①}{=} \Big/_0^{\pi} -\cos(x) \overset{②}{=} -\cos(\pi) - (-\cos(0))$$

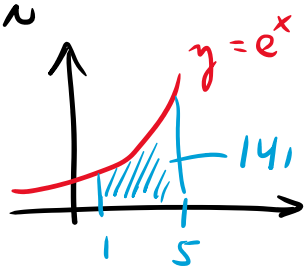
$$= 1 - (-1) = 2$$

HARJOITELLAAN TÄLLÄ VIIKOLLA

$$\text{ALA} = \int_a^b f(x)\,dx = \boxed{\Big/_a^b F(x) = F(b) - F(a)}$$

SIJOITTAMINEN





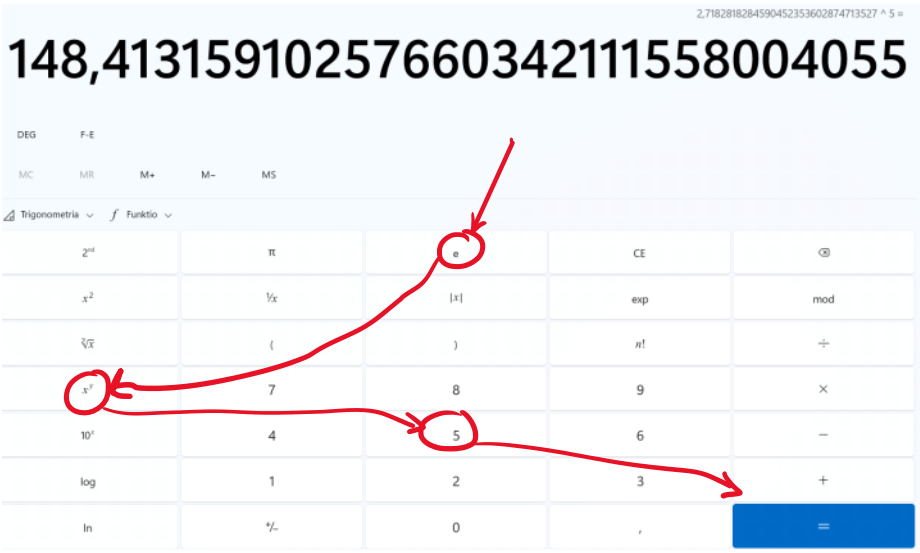
ESIM. $\Big/_2^5 e^x = e^5 - e^2 \approx 141$

MUISTA SULUT

$$2^{x} - 3x = 2^{5} - 3\cdot 5 - \left(2^{2} - 3\cdot 2\right)$$

MUISTA SULUT

LASKIN $e^5 = \exp(5)$



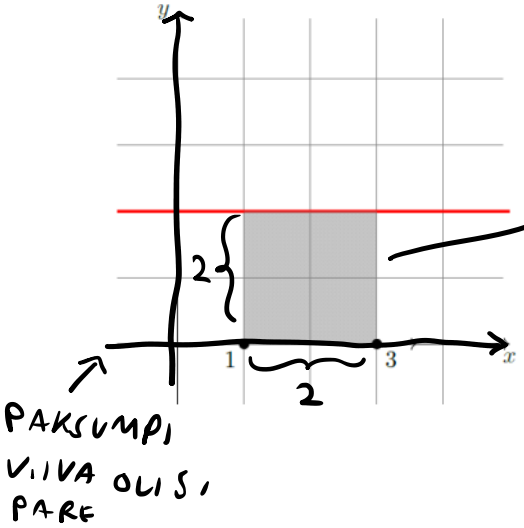
$\boxed{e}$ & $\boxed{x^y}$ & $\boxed{5}$ & $\boxed{=} \rightarrow e^5$

$\boxed{2{,}718}$ & $\boxed{x^y}$ & $\boxed{5}$ & $\boxed{=} \rightarrow e^5$

$$e^5 - e^2 = 148 - 7{,}4 \approx 141 \neq e^3 \approx 20$$

LASKIN

7. Laske alla olevien kuvien varjostetut pinta-alat. Riittää laskea sijoitukset.



KUVAN PERUSTEELLA

SUORAKULMIO
= ALA = KANTA · KORKEUS
= 2 · 2 = 4

SAMA

$\int_1^3 2dx = \Big/_1^3 2x = 2\cdot 3 - 2\cdot 1$

$= 6 - 2 = 4$

Tehtäviä. Voit myös piirtää tehtävissä kuvat, jos haluat.

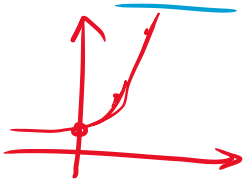
1. Tarkastellaan pisteiden $p = (1,3)$ ja $q = (4,2)$ kautta kulkevaa suoraa. Määritä suoran kulmakerroin ja suoran yhtälö normaalimuodossa. Solution $k = -1/3$, $y = -\frac{1}{3}x + \frac{10}{3}$
2. Tarkastellaan pisteiden $p = (1,2)$ ja $q = (4,2)$ kautta kulkevaa suoraa. Määritä suoran kulmakerroin ja suoran yhtälö normaalimuodossa. Solution $k = 0$, $y = 2$
3. Tarkastellaan pisteiden $p = (1,2)$ ja $q = (1.01,4)$ kautta kulkevaa suoraa. Määritä suoran kulmakerroin ja suoran yhtälö normaalimuodossa. Solution $k = 200$, $y = 200x - 198$
4. Tarkastellaan suoraa $3y = 2x+1$. Kirjoita yhtälö muodossa $y = kx+b$. Mikä on suoran kulmakerroin k? Etsi kaksi suoran pistettä. Piirrä suoran kuvaaja. Solution $y = \frac{2}{3}x + \frac{1}{3}$, esim. $p = (1,1)$ ja $q = (4,3)$

4. RATKAISU

$\frac{\cancel{3}y}{\cancel{3}} = \frac{2x+1}{3}$

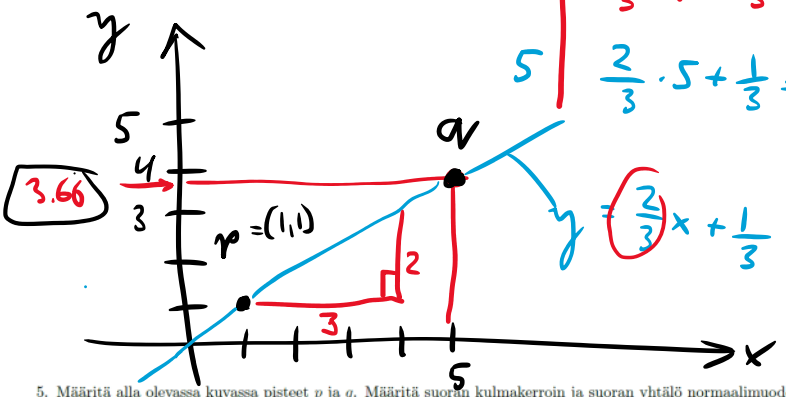
$y = \underbrace{\frac{2}{3}}_{=k} x + \frac{1}{3}$

x	2^x
0	1
1	2
2	4



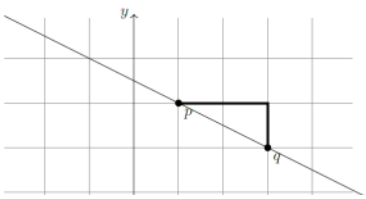
$y = \frac{2}{3}x + \frac{1}{3}$

x		(x,y)
0	$\frac{1}{3}$	
1	$\frac{2}{3}\cdot 1 + \frac{1}{3} = \frac{2+1}{3} = 1$	$= p$ $(1,1)$
5	$\frac{2}{3}\cdot 5 + \frac{1}{3} = \frac{10+1}{3} = \frac{11}{3}$ $\approx 3{,}66$	$(5, 3{,}66)$ $= q$

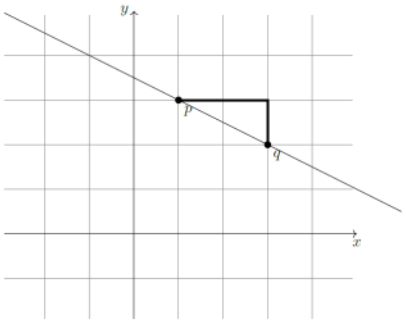


$y = \frac{2}{3}x + \frac{1}{3}$

5. Määritä alla olevassa kuvassa pisteet p ja q. Määritä suoran kulmakerroin ja suoran yhtälö normaalimuodossa. Solution $y = -\frac{1}{2}x + \frac{7}{2}$



5. Määritä alla olevassa kuvassa pisteet p ja q. Määritä suoran kulmakerroin ja suoran yhtälö normaalimuodossa. Solution
$y = -\frac{1}{2}x + \frac{7}{2}$



6. Laske sijoitukset.

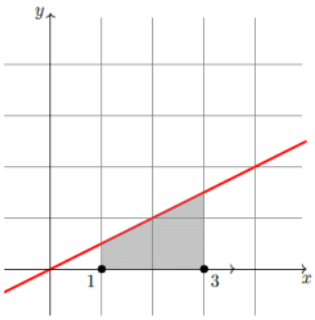
(a) $\Big/_3^4 x^2$ Solution 7

(b) $\Big/_3^4 \frac{1}{x}$ Solution $-\frac{1}{12}$

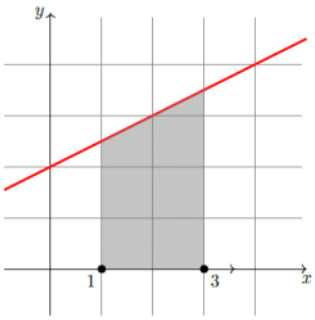
(c) $\Big/_2^3 2x^2 - x$ Solution 9

(d) $\Big/_2^6 \ln(x)$ Solution $\ln(3)$

(e) $\Big/_0^\pi \cos(x)$ Solution -2



$\int_1^3 0.5x dx = \Big/_1^3 0.25x^2 =$



$\int_1^3 2 + 0.5x dx = \Big/_1^3 2x + 0.25x^2 =$

19,00 - 20.00

- LASKETAAN
- VOIDAAN KATSOA JOKU ESIMERKKI, JOS ON TARVETTA

• VOIDAAN KATSOA JOKU ESIMERKKI, JOS ON TARVETTA

6. Laske sijoitukset.

(a) $\int_3^4 x^2$ Solution 7

(b) $\int_3^4 \frac{1}{x}$ Solution $-\frac{1}{12}$

(c) $\int_2^3 2x^2 - x$ Solution 9

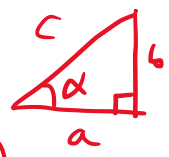
(d) $\int_2^6 \ln(x)$ Solution $\ln(3)$

(e) $\int_0^\pi \cos(x)$ Solution -2

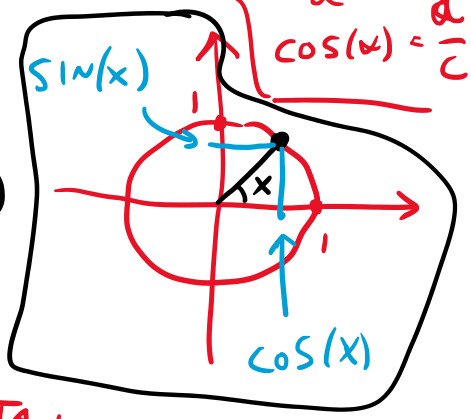
JOS KÄYTETÄÄN ASTETTA → $\cos(45^\circ)$

PELKKIÄ LUKUJA ⇒ RADIAANEJA

SIJOITUS: $\int_a^b F(x) = F(b) - F(a)$



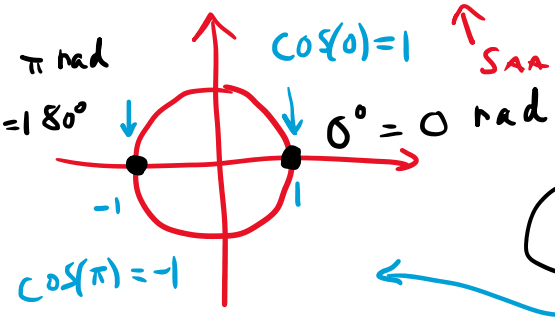
$\cos(\alpha) = \frac{a}{c}$



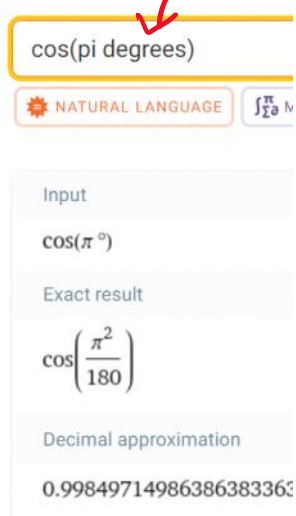
(e) $\int_0^\pi \cos(x) = \cos(\pi) - \cos(0)$

$= -1 - 1 = -2$

SAA LASKIMELLA TAI YKSIKKÖYMPYRÄSTÄ



JOS ON ASTEET, TULEE VÄÄRÄ ARVO

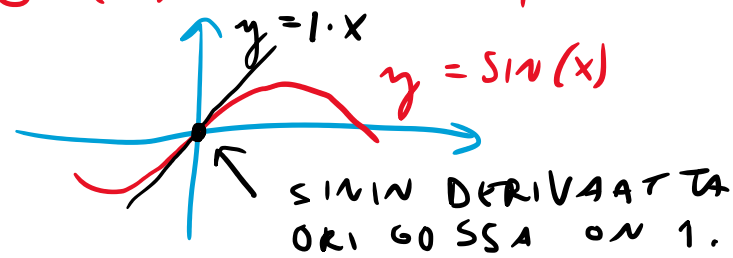


- KULMIA VOI ILMOITTAA ASTEINA JA RADIAANEINA
- OPE KOETTAA OLLA SELKEÄ
- TÄLLÄ KURSSILLA ~~RA~~ RADIAANIT OVAT HYÖDYLLISIÄ, KOSKA SAADAAN KAAVOJA, JOTKA TOIMIVAT VAIN NIILLÄ

RADIAANIEN HYÖTY, HYVIÄ KAAVOJA

$\sin(x) \approx x$, KUN $x \approx 0$

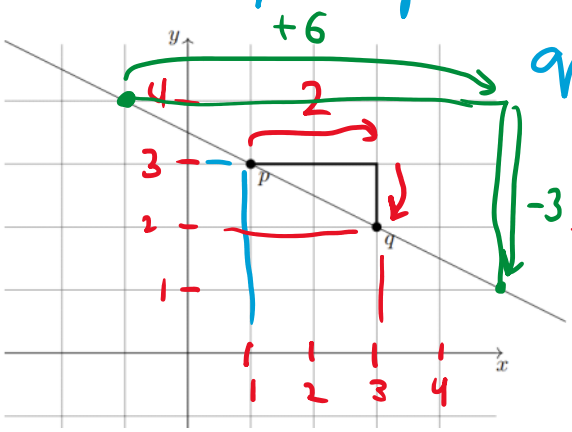
$\cos(x) \approx 1 - x^2$, KUN $x \approx 0$



5. Määritä alla olevassa kuvassa pisteet p ja q. Määritä suoran kulmakerroin ja suoran yhtälö normaalimuodossa. Solution
$y = -\frac{1}{2}x + \frac{7}{2}$

$p = (x, y) = (1, 3)$

$q = (3, 2)$



KULMAKERROIN $= k$

$= \frac{y:\text{n muutos}}{x:\text{n muutos}} = \frac{-1}{2}$

$k = -\frac{1}{2}$

TOINEN KULMIO

$k = \frac{-3}{\ } = -1$

x, n,

$$k = \frac{-3}{+6} = -\frac{1}{2}$$

$$p = (\overset{x_1}{1}, \overset{y_1}{3})$$
$$q = (\underset{x_2}{3}, \underset{y_2}{2})$$

TAPA 2 (SÖKKONA ILMAN KUVAA)

$$k = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{3-1} = \frac{-1}{2} = \underbrace{-\frac{1}{2}}$$

$p = (1, 3)$

SUORAN YHTÄLÖ

$$y - y_1 = k(x - x_1)$$

$$y - 3 = -\frac{1}{2}(x - 1)$$

$$y - 3 = -\frac{1}{2}x + \frac{1}{2} \quad \| +3$$

$$y = -\frac{1}{2}x + \underbrace{3}_{=\frac{6}{2}} + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{7}{2}$$

NORMAALI-MUOTO
$y = kx + b$

SUORIA ERI MUODOSSA

$$y = 2x + 3$$

$$y - 2x - 3 = 0$$

YLIMÄÄRÄINEN ESIMERKKI, EI TARVITSE OPETELLA

$$3y + 2x = 7$$

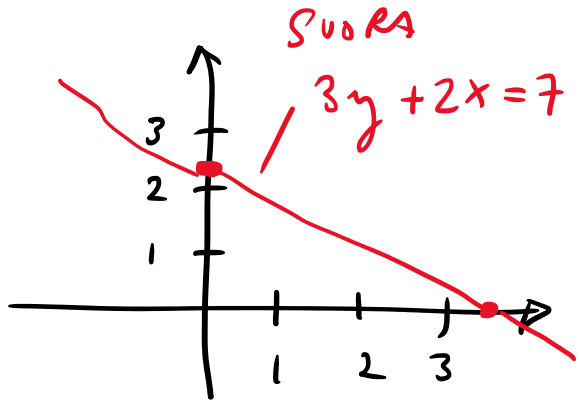
$$\frac{1}{\left(\frac{1}{a}\right)} = 1 : \frac{1}{a} = 1 \cdot \frac{a}{1} = a$$

$$3y + 2x = 7$$

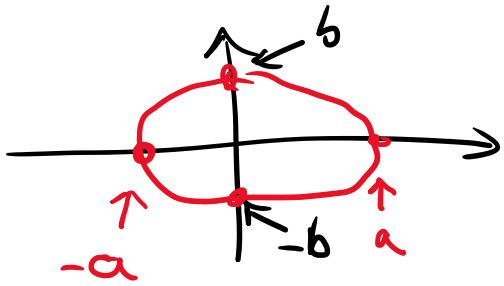
$$\frac{y}{\frac{1}{3}} + \frac{x}{\frac{1}{2}} = 7 \quad \| : 7$$

$$\frac{y}{2{,}33 = \frac{7}{3}} + \frac{x}{\frac{7}{2} = 3{,}5} = 1$$

AKSELIEN LEIKKAUSPISTEET

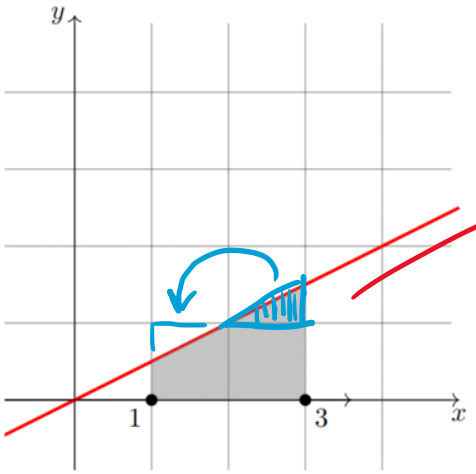


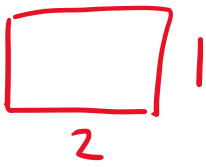
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



ENSI VIIKOLLA

$$D\, x^3 = 3x^2 \quad \& \quad \int x^7\, dx = \frac{x^8}{8}$$





PINTA-ALA $= 2 \cdot 1 = \underline{\underline{2}}$

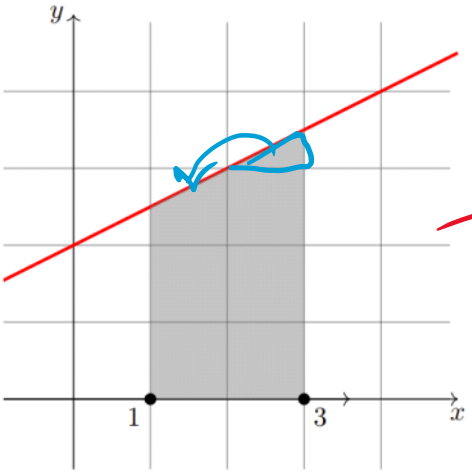
$\frac{1}{4}$

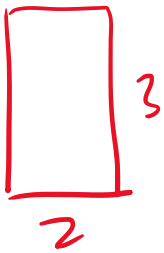
$$\int_1^3 0.5x\,dx = \Big/_1^3\, 0.25x^2 =$$

$$\Big/_1^3\, 0.25x^2 = 0.25 \cdot 3^2 - 0.25 \cdot 1^2$$

$$= 0.25(3^2 - 1^2) = 0.25(9-1)$$

$$= 0.25 \cdot 8 = \underline{\underline{2}}$$





ALA $= \underline{\underline{6}}$

$$\int_1^3 2 + 0.5x\,dx = \Big/_1^3\, 2x + 0.25x^2 =$$

$$\Big/_1^3\, 2x + 0.25x^2 = 2 \cdot 3 + 0.25 \cdot 3^2 - (2 \cdot 1 + 0.25 \cdot 1^2)$$

$$= 6 - 2 + 0.25(3^2 - 1^2)$$

$$= 4 + 2 = \underline{\underline{6}}$$

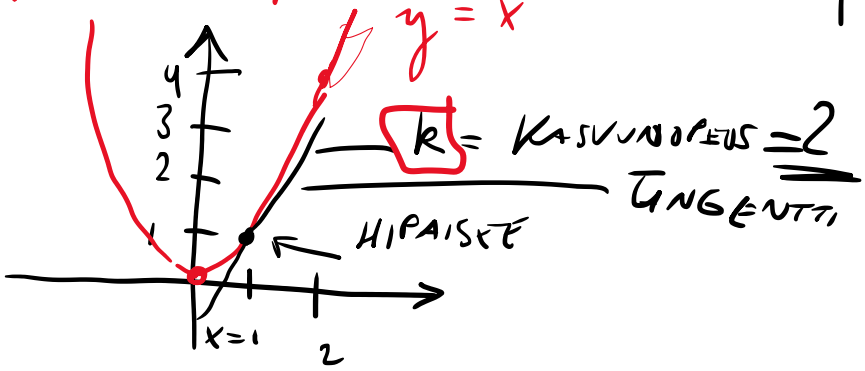
2 Polynomit

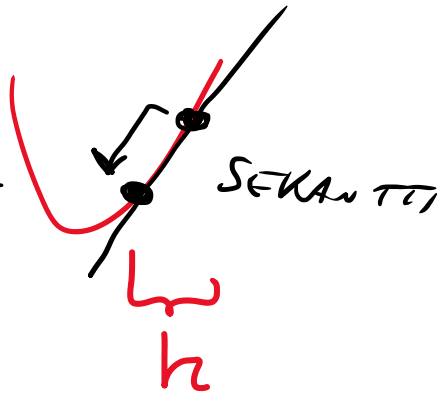
torstai 14. maaliskuuta 2024 18.49

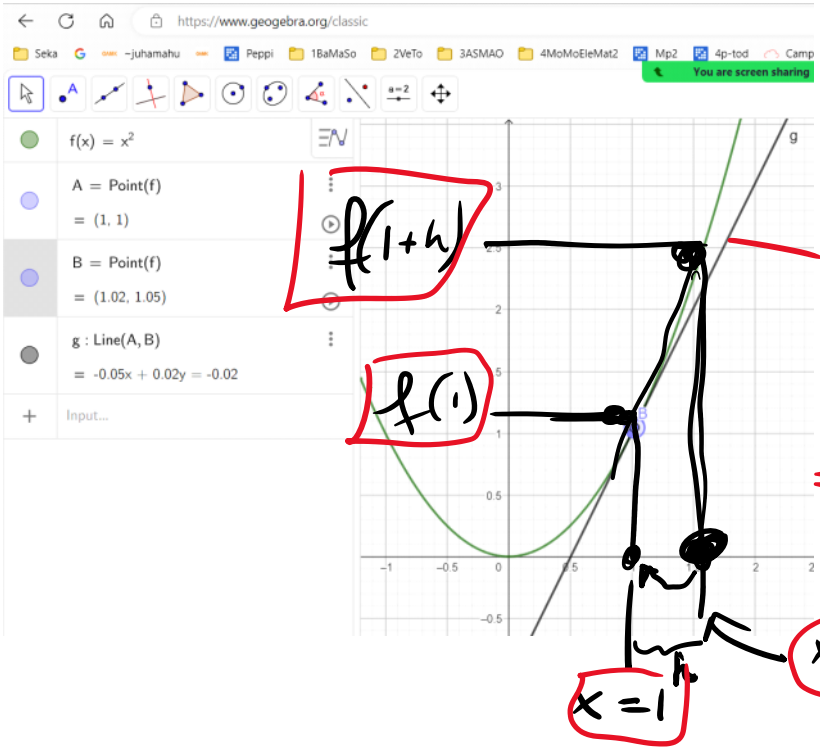
ESIM. FUNKTIO $f(x) = x^2$.

KASVUNOPEUS = DERIVAATTA, KUN $x = 1$.

x	$y = x^2$
0	$0^2 = 0$
1	$1^2 = 1$
2	$2^2 = 4$

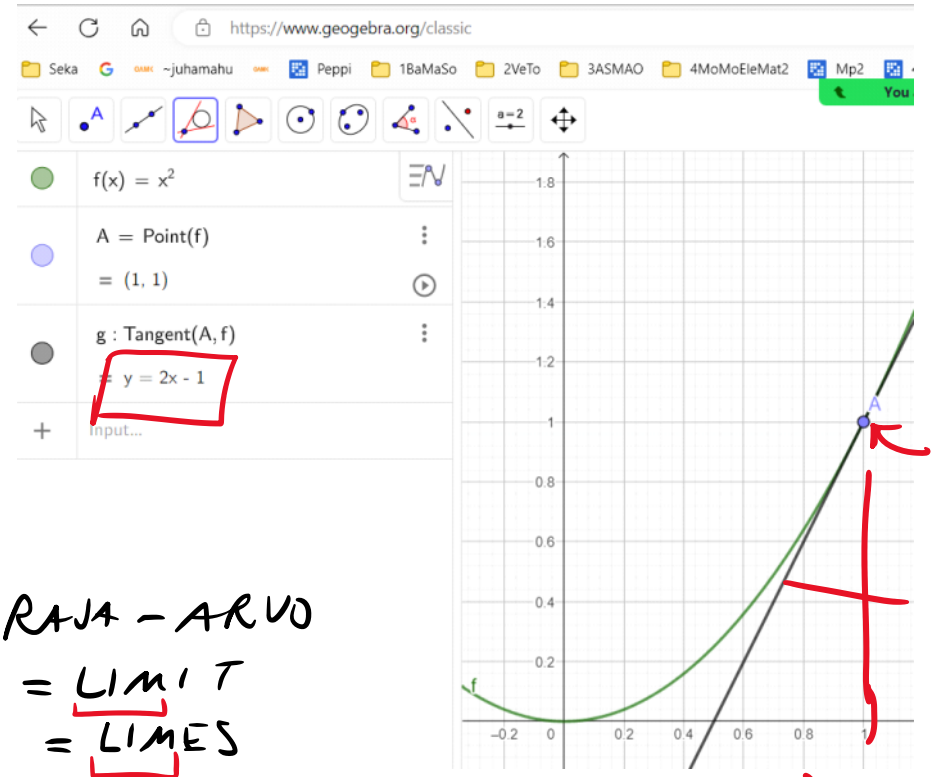






$0{,}02y = -0{,}02 + 0{,}05x \quad | : 0{,}02$

$\Rightarrow y = -1 + \underbrace{\frac{5}{2}}_{2\,?} x$



KASVUNOPEUS
$k = 2$

$y = 2x - 1$

$k = 2$

RAJA-ARVO
= LIMIT
= LIMES

LASKEMALLA

$$k = f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{(1+h) - 1}$$

$x = 1$

$(f(x) = x^2$

SEKANTIN KULMAKERROIN

$$= \lim_{h \to 0} \frac{(1+h)^2 - (1^2)}{h}$$

$(a+b)^2 = a^2 + 2ab + b^2$

$$= \lim_{h \to 0} \frac{1^2 + 2 \cdot 1 \cdot h + h^2 - 1^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2+h)}{h} = \lim_{h \to 0} 2 + h = 2 + 0 = 2$$

DERIVAATAN MATEMAATTINEN MÄÄRITELMÄ

$$\boxed{f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}}$$

EI KÄYTETÄ PALJOA KURSSILLA, DERIVOINTIKAAVAT JOHDETAAN TÄMÄN AVULLA.

$$f(x) = x^n \Rightarrow f'(x) = n x^{n-1}$$

VIIKON DERIVOINTIKAAVA:

$$\underbrace{f', \ \dot{f}, \ \frac{df}{dx}, \ \boxed{Df}}_{\text{DERIVAATAN MERKINNÄT}}$$

$$\boxed{D x^n = n x^{n-1}}$$

ESIM.

$$D\, x^7 = 7x^6$$

$$D\, x^{120} = 120 x^{119}$$

$$\sqrt{x} = \boxed{x^{\frac{1}{2}}}$$

$$\frac{1}{x^5} = \boxed{x^{-5}}$$

$$x^{2.1}$$

VAKIO SÄILYY

$$D\, 5x^3 = 5 \cdot 3x^2 = 15x^2$$

DERIVOIDAAN SUMMASSA ERIKSEEN

$$D\, 3x^4 + 4x^7 = 12x^3 + 28x^6$$

$$D\sqrt{x} = D\, x^{\frac{1}{2}} = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{2} \frac{1}{x^{\frac{1}{2}}}$$

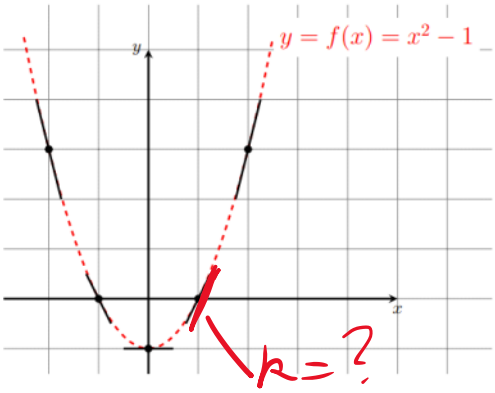
$$= \frac{1}{2\sqrt{x}}$$

$$D \frac{1}{x^3} = D\, x^{-3} = -3x^{-4} = \frac{-3}{x^4}$$

$$D\, x^{2.1} = 2.1\, x^{1.1}$$

$$D\, x^{2.1} = 2.1\, x^{1.1}$$

2.3 Polynomin derivointi ja kuvaaja



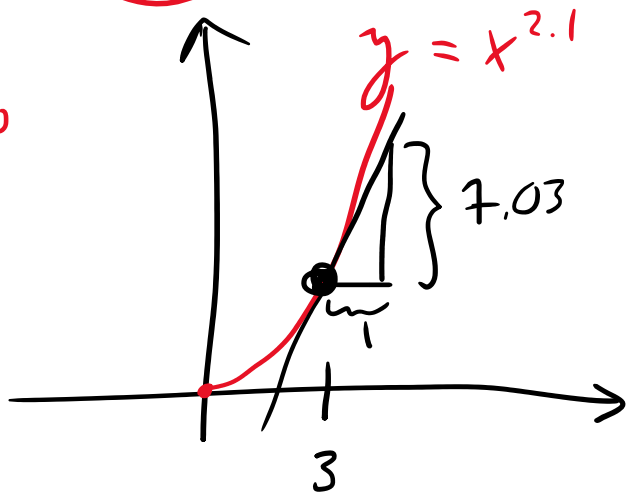
4. Olkoon $f(x) = x^2 - 1$. Laske
   (a) $f'(x)$ Solution $2x$
   (b) $f'(-2)$ Solution -4
   (c) $f'(-1)$ Solution -2
   (d) $f'(0)$ Solution 0
   (e) $f'(1)$ Solution 2
   (f) $f'(2)$ Solution 4
5. Selvitä funktion $f(x) = x^2 - 1$ kuvaajalle pisteeseen $(1, 0)$ piirretyn tangenttisuoran yhtälö.
   Solution $y - 0 = 2(x - 1)$
6. Selvitä funktion $f(x) = x^2 - 1$ kuvaajalle pisteeseen $(0, -1)$ piirretyn tangenttisuoran yhtälö.
   Solution $y = -1$
7. Selvitä funktion $f(x) = x^2 - 1$ kuvaajalle pisteeseen $(-2, 3)$ piirretyn tangenttisuoran yhtälö.
   Solution $y - 3 = -4(x - (-1))$

MELKO PALJON TEKEMISTÄ, EI PAKKO LASKEA KAIKKEA

ESIM. $f(x) = x^{2.1}$, $x \geq 0$

$f'(x) = ?$

$f'(3) = ?$



RATKAISU. $f(x) = x^{2.1}$

SIJOITETAAN $x = 3$

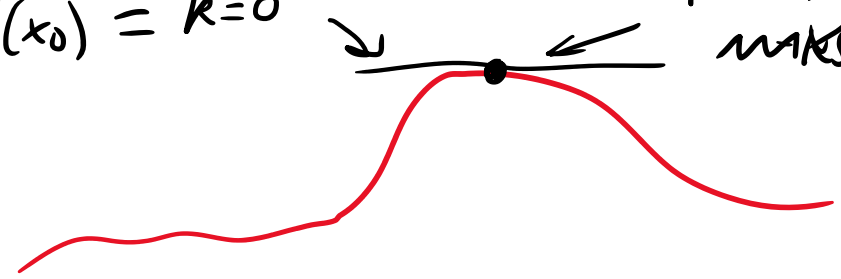
$f'(x) = D\, x^{2.1} = 2.1\, x^{1.1}$

$f'(3) = 2.1\, 3^{1.1} = 7.03158 \approx \boxed{7.03}$

SOVELLUS. MIN/MAX

$f'(x_0) = k = 0$

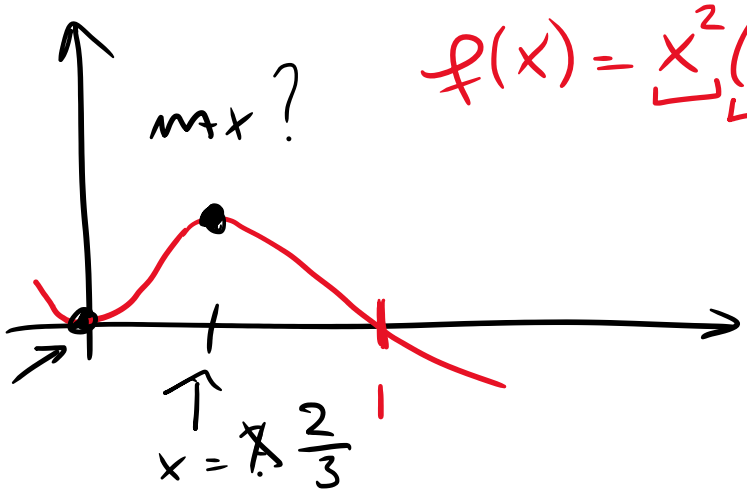
PAIKALLINEN MAKSIMI



x_0

MIN/MAX PAIKAT LÖYTÄÄ EHDOSTA $\boxed{f'(x)=0}$

ESIM.



$f(x) = \underbrace{x^2}\underbrace{(1-x)}$ $\Rightarrow f(1) = 1^2 \cdot 0 = 0$

MIN $x=0$

RATKAISU. AUKAISTAAN SULUT $f(x) = x^2 - x^3$

DERIVOIDAAN $f'(x) = 2x - 3x^2 = 0$

$2x - 3x^2 = 0$ ① RATKAISUKAAVA

② $\underbrace{x}\underbrace{(2-3x)} = 0$

$x = 0$ TAI $2 - 3x = 0$

$\Rightarrow 2 = 3x \quad || : 3$

$\Rightarrow \frac{2}{3} = x$

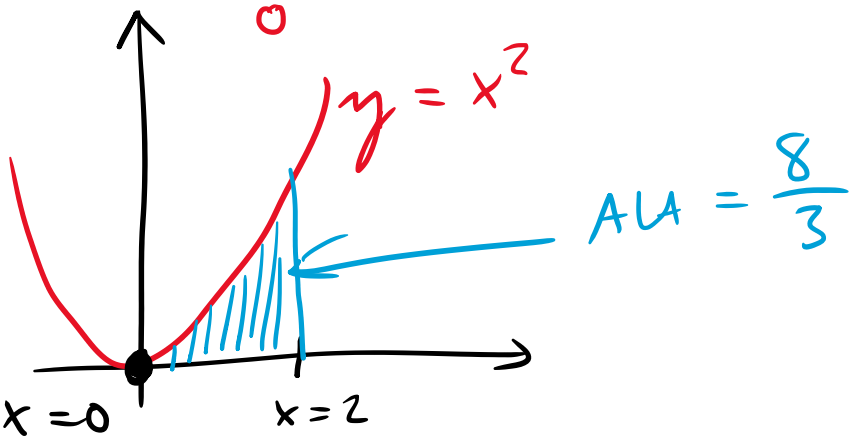
$\Rightarrow x = \frac{2}{3}$

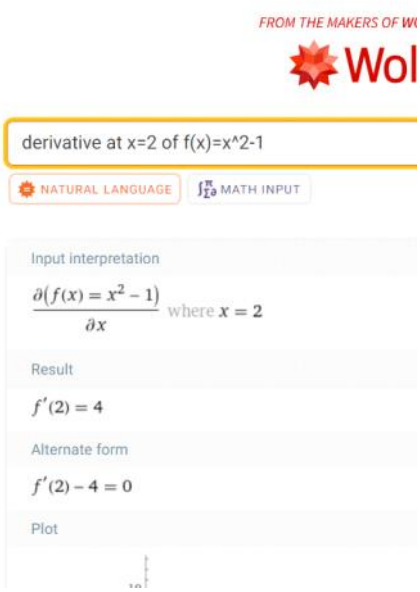
DERIVOINTI $\boxed{D\, x^{n} = n\, x^{n-1}}$

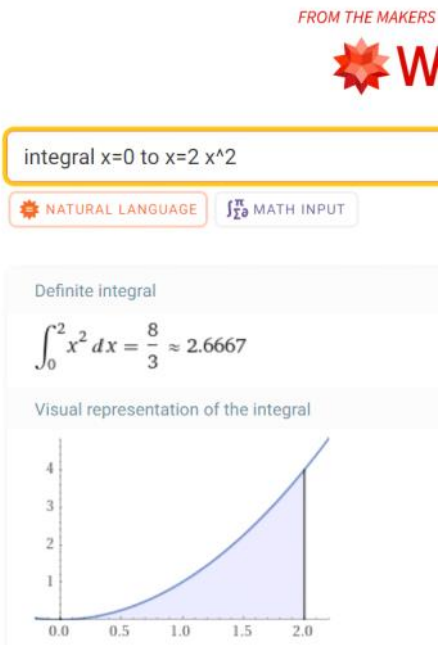
INTEGROINTI $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

INTEGRAALI-
MERKKI

ESIM. $\int_0^2 x^2 \, dx = \left. \frac{x^3}{3} \right|_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3}$







1. Derivoi monomit.

(a) $3x^5$ Solution $15x^4$

(b) $-3x^11$ Solution $-33x^{10}$

(c) $4x^1$ Solution 4

(d) 2 eli $2x^0$ Solution 0

ⓐ $D\ 3x^5 = 15x^4$

ⓑ $\frac{d}{dx} -3x^{11} = -33x^{10}$

$D\ 4x = \underline{\underline{4}}$

ⓒ $(4x^1)' = 4 \cdot 1 \cdot \underbrace{x^0}_{=1} = \underline{\underline{4}}$

$(\sin(x))' = \cos(x)$

$(fg)' = f'g + fg'$ ← HELPPO KIRJOITTAA

$D(fg) = gDf + fDg$

ⓓ $D\ 2 = D\ 2\underbrace{x^0}_{=1} = 2 \cdot 0 \cdot x^{-1} = 0$

$D\ 2 = 0$

VAKIO,

VAKIOFUUKTIO EI KASVA ⇒ KASVUNOPEUS = 0

2. Derivoi muuttujan x potenssit

(a) $\frac{3}{x^2} = 3x^{-2}$

(b) $\sqrt{x}$ $x^{\frac{1}{2}}$

(c) $x\sqrt{x}$

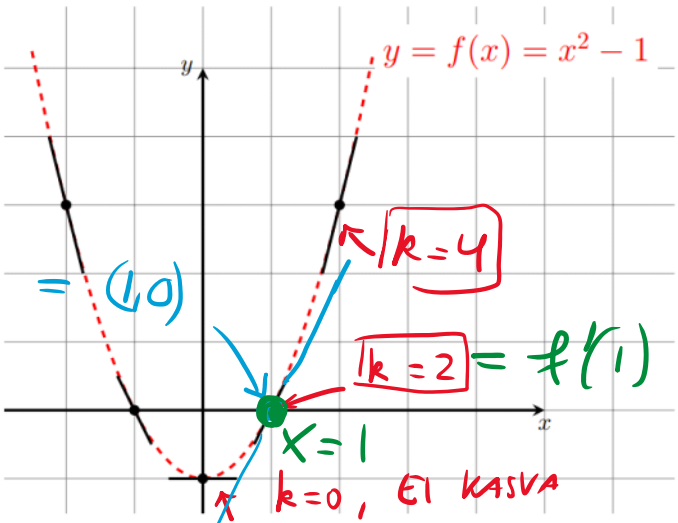
(d) $2x^{-2.3}$

$a^b a^c = a^{b+c}$

(c) $x\sqrt{x} = x^1 x^{\frac{1}{2}} = x^{1+\frac{1}{2}} = x^{\frac{3}{2}}$

$D\, x\sqrt{x} = D\, x^{\frac{3}{2}} = \frac{3}{2} x^{\frac{3}{2}-1} = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

2.3 Polynomin derivointi ja kuvaaja



$(x, y) = (1, 0)$

$\boxed{k = 4}$

$\boxed{k = 2} = f'(1)$

$x = 1$

$k = 0$, EI KASVA

YHTÄLÖ

$y - y_1 = k(x - x_1)$

SUORA
- KULMAKERROIN k
- PISTEEN (x_1, y_1) KAUTTA

YHTÄLÖ

5. NYT $\boxed{k = 2}$

JOKU PISTE ESIM. $(x, y) = \boxed{(1, 0)}$

$D\, x^2 - 1 = 2x + 0 = 2x$

4. Olkoon $f(x) = x^2 - 1$. Laske

(a) $f'(x) = 2x$ Solution $2x$

(b) $f'(-2) = 2(-2) = -4$ Solution -4

(c) $f'(-1) = 2(-1) = -2$ Solution -2

(d) $f'(0) = 2 \cdot 0 = 0$ Solution 0

(e) $\boxed{f'(1)} = 2 \cdot 1 = 2$ Solution 2

(f) $f'(2) = 2 \cdot 2 = 4$ Solution 4

5. Selvitä funktion $f(x) = x^2 - 1$ kuvaajalle pisteeseen $(1, 0)$ piirretyn tangenttisuoran yhtälö.

Solution $y - 0 = 2(x - 1)$

6. Selvitä funktion $f(x) = x^2 - 1$ kuvaajalle pisteeseen $(0, -1)$ piirretyn tangenttisuoran yhtälö.

Solution $y = -1$

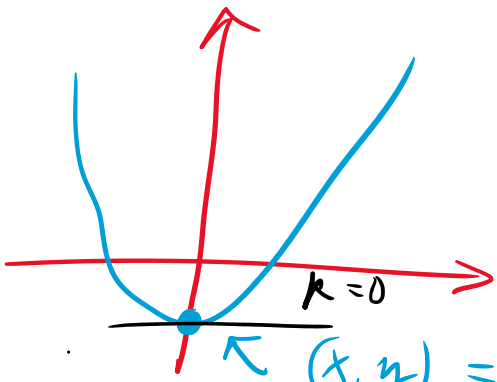
7. Selvitä funktion $f(x) = x^2 - 1$ kuvaajalle pisteeseen $(-2, 3)$ piirretyn tangenttisuoran yhtälö.

Solution $y - 3 = -4(x - (-1))$

Now find EJM, $(x, y) = \boxed{(1, 0)}$

$$y - 0 = 2(x - 1)$$

$$\underline{\underline{y = 2x - 2}}$$



$(x_1, y_1) = (0, -1)$

$$y - y_1 = k(x - x_1)$$

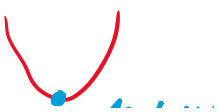
$$y - (-1) = 0(x - 0) = 0$$

$$y - (-1) = 0$$

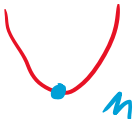
$$y + 1 = 0 \quad \| -1$$

$$y = -1$$

$+ 3x^2$



$+3x^2$



MINIMI

8. Olkoon $f(x) = 3x^2 + 5x + 13$. Etsi ääriarvokohdat.

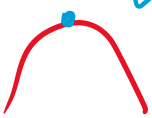
(a) Laske $f'(x)$. Solution $f'(x) = 6x + 5$

(b) Kirjoita $f'(x) = 0$. Solution $6x + 5 = 0$

(c) Ratkaise saadusta yhtälöstä x. Solution $x = -\frac{5}{6}$

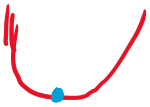
(d) Voitko päätellä, onko kyseessä minimi vai maksimi? Solution Minimi kohdassa $x = -\frac{5}{6}$

ESIM. $f(x) = -3x^2 + 7$



MAKSIMI

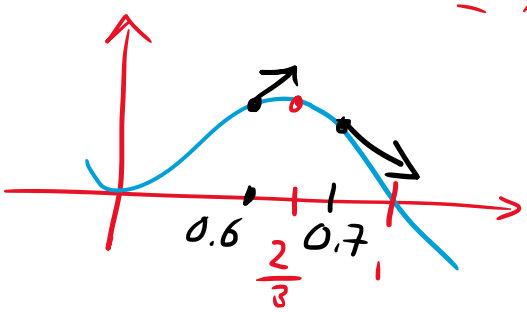
ESIM. $f(x) = +3x^4$



MINIMI

ESIM. (JATKOA) $f(x) = x^2(1-x)$

$= x^2 - x^3$



$f'(x) = \underbrace{2x - 3x^2}$

$\frac{2}{3} = 0.666$

$f'\left(\frac{2}{3}\right) = 0$

$f'(0.7) = -0.07$

$f'(0.6) = +0.12$

KULKUKAAVIO

		$\frac{2}{3}$	
$f'(x)$	+	0	−
$f(x)$	↗	→	↘

MAKSIMI PISTEESSÄ $x = \frac{2}{3}$

Kirjoitusvirheitä

keskiviikko 20. maaliskuuta 2024 17.55

KIRJOITUSVIRHETTÄ:

1. Derivoi monomit.
   (a) $3x^5$
   (b) $-3x^1 1$ — $-3x^{11}$ — ok
   (c) $4x^1$
   (d) 2 eli $2x^0$

7. Selvitä funktion $f(x) = x^2 - 1$ kuvaajalle pisteeseen $(-2, 3)$ piirretyn tangenttisuoran yhtälö.
   Solution $y - 3 = -4(x - \cancel{2}(-2))$ — ok

$x_1 = -2 \longrightarrow (-2)$

KÄYDÄÄN LÄPI AINAKIN

2. Derivoi muuttujan x potenssit
   (a) $\frac{3}{x^2}$ — Solution $-6x^{-3}$
   (b) $\sqrt{x}$ — Solution $\frac{1}{2\sqrt{x}}$
   (c) $x\sqrt{x}$ — Solution $\frac{3}{2}\sqrt{x}$
   (d) $2x^{-2.3}$ — Solution $-4.6x^{-3.3}$

(a) $D\,\frac{3}{x^2} = D\,3x^{-2} = 3\cdot(-2)x^{-2-1} = -6x^{-3} \left[= \frac{-6}{x^3}\right]$

(b) $D\sqrt{x} = D\,x^{\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} \left[= \frac{1}{2\sqrt{x}}\right]$

(c) $D\,x\sqrt{x} = D\,x^1 x^{\frac{1}{2}} = D\,x^{\overbrace{1+\frac{1}{2}}^{\frac{3}{2}}} = \frac{3}{2}x^{1+\frac{1}{2}-1} = \frac{3}{2}x^{\frac{1}{2}}$

$= \frac{3}{2}\sqrt{x}$

$\boxed{a^b a^c = a^{b+c}}$

$3 + 5\cdot 7 - 1 = 37$

ESIM. $D\,\frac{x^3 (x^5)^7}{x^1}$ $D\,x^3 \cdot x^{5\cdot 7} \cdot x^{-1} = D\,x^{37} = 37x^{36}$

ENSI VIIKOLLA $D\,(x^5)^7 = D\,x^{35} = 35x^{34}$ SAMA

2 TAPA $D\,\underbrace{(x^5)^7}_{\text{YHDISTETTY}} = 7(x^5)^6 \cdot \underbrace{Dx^5}_{5x^4} = 35\,x^{30+4}$

FUNKTIO

11. Olkoon $f(x) = x^3(1-x)^2$. Etsi ääriarvokohdat.

Solution Paikallinen maksimi kohdassa $x = \cancel{1}\ \frac{3}{5}$

ÄR

RATKAISU.

$$f(x) = x^3(1-x)^2$$

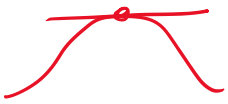
$$= x^3(1^2 - 2\cdot 1\cdot x + x^2)$$

$$= x^3(1 - 2x + x^2)$$

$$= x^3 - 2x^4 + x^5$$

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$

$k = f'(x) = 0$



$$f'(x) = 3x^2 - 8x^3 + 5x^4 = 0$$

$$x^2(3 - 8x + 5x^2) = 0$$

$$x^2 = 0 \quad \text{TAI} \quad 5x^2 - 8x + 3 = 0$$

$$ax^2 + bx + c = 0$$

$$\begin{cases} a = 5 \\ b = -8 \\ c = 3 \end{cases}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$4\cdot 5\cdot 3 = 20\cdot 3 = 60$$

$$= \frac{8 \pm \sqrt{64 - 4\cdot 5\cdot 3}}{2\cdot 5}$$

$$= \frac{8 \pm \sqrt{64 - 60}}{10} = \frac{8 \pm \sqrt{4}}{10}$$

$$= \frac{8 \pm 2}{10}$$

$$\begin{cases} x_1 = \frac{8+2}{10} = 1 \\ x_2 = \frac{8-2}{10} = \frac{6}{10} = \frac{3}{5} \end{cases}$$

← AINOA MAHDOLLINEN HUIPUN PAIKKA

$f(x) = x^3(1-x)^2$

$f(0) = 0$

$f(1) = 0$

VASTAUS: MAKSIMI KOHDASSA $x = \frac{3}{5}$

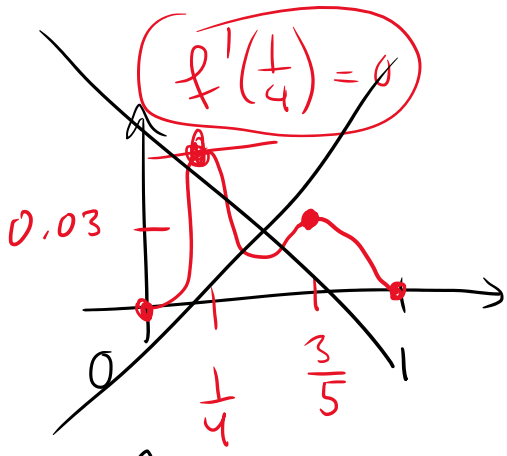
$$f(x) = x^3(1-x)^2$$

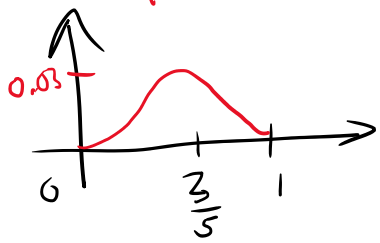
$$= x^3(1^2 - 2\cdot 1\cdot x + x^2)$$

$$= x^3(1 - 2x + x^2)$$

$$= x^3 - 2x^4 + x^5$$

$$f\left(\frac{3}{5}\right) = \left(\frac{3}{5}\right)^3\left(1 - \frac{3}{5}\right)^2 \approx 0{,}03$$





(3/5)^3*(1-3/5)^2

NATURAL LANGUAGE

Input

$\left(\frac{3}{5}\right)^3\left(1-\frac{3}{5}\right)^2$

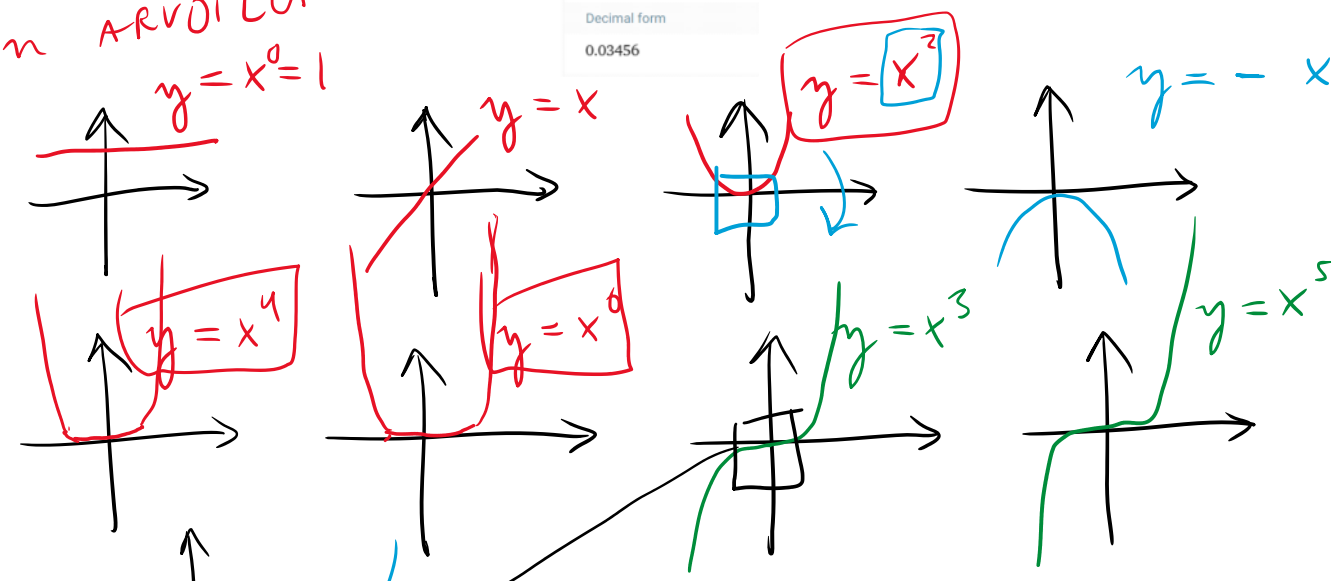
Exact result

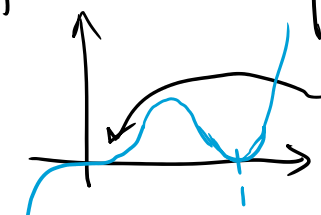
$\frac{108}{3125}$

Decimal form

0.03456

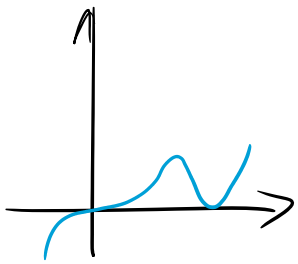
x^n ERI n ARVOILLA



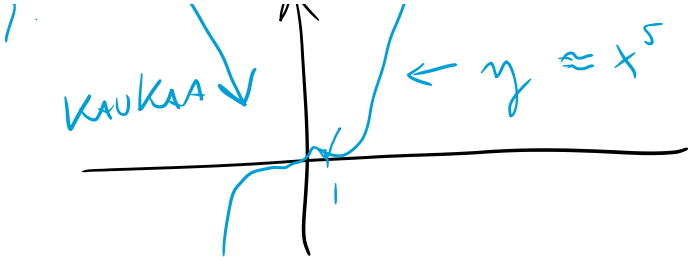


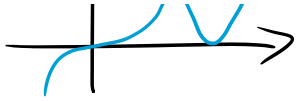
$$y = x^3(1-x)^2$$

"$x = 0$" "$x = 1$"



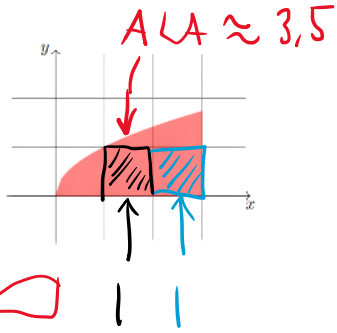
$\leftarrow y \approx x^5$





(d) $\int_0^3 \sqrt{x}dx$

Solution $2\sqrt{3} \approx 3.464$.



≈ 1.5

RATKAISU.

$$\int_a^b x^n dx = \Big/_a^b \frac{x^{n+1}}{n+1}$$

$$\int_0^3 \sqrt{x}\, dx$$

$$= \int_0^3 x^{\frac{1}{2}} dx$$

$$= \Big/_0^3 \frac{x^{\overbrace{\frac{1}{2}+1}^{\frac{3}{2}}}}{\frac{1}{2}+1} \left[= \Big/_0^3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]$$

$$\boxed{\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}}$$

$$= \Big/_0^3 \frac{x\sqrt{x}}{\frac{3}{2}} = \Big/_0^3 x\sqrt{x} \cdot \frac{2}{3}$$

$$= \cancel{3}^{1}\sqrt{3} \cdot \frac{2}{\cancel{3}_{1}} - \underbrace{0\sqrt{0} \cdot \frac{2}{3}}_{=0}$$

$$= 2\sqrt{3} \approx 3.464$$

4. Olkoon $k(x) = \cancel{0.2^x}$ $0{,}5^x$.

(a) Laske $k'(x)$. Solution $0.5^x \ln(0.5)$

(b) Laske $k'(0)$. Solution -0.69

a^x, $a > 0$

$D \frac{d}{dx} (\ )'$

RATKAISU.

$k(x) = 0{,}5^x$

$k(x) = 0{,}5^x$

ⓐ $k'(x) = \underline{0{,}5^x \ln(0{,}5)}$

$\boxed{D\ a^x = a^x \ln(a)}$

LASKIN

$\ln(0{,}5) \approx -0{,}69$

$\ln(0{,}5) = \ln\left(\frac{1}{2}\right) = \ln(2^{-1}) = -\ln(2) \approx -0{,}69$

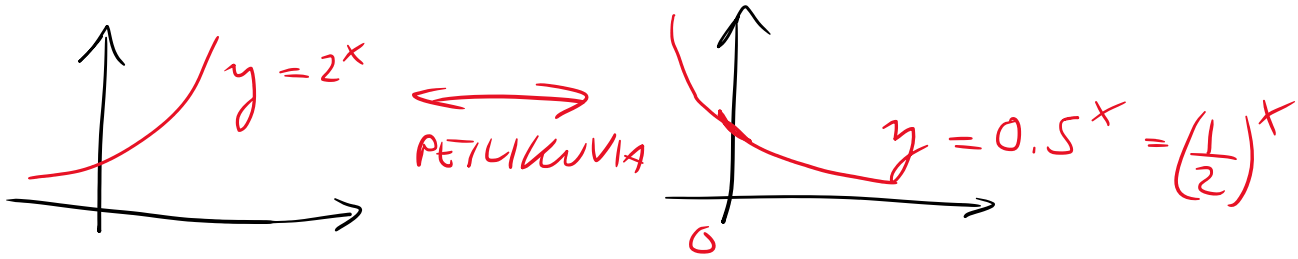
ⓑ $k'(0) = 0{,}5^{(0)} \ln(0{,}5) = \underbrace{0{,}5^0}_{=1} \ln(0{,}5) = \ln(0{,}5) = \underline{-0{,}69}$

VÄHENEVÄ KUN $x = 0$

1. $f(x) = 2^x$

ⓐ $f'(x) = \underline{2^x \ln(2)} \approx 0{,}69 \cdot (2^x)$

ⓑ $f'(0) = 2^0 \ln(2) = 1 \cdot \ln(2) = 0{,}69$



$\boxed{D\ a^x = a^x \ln(a)}$

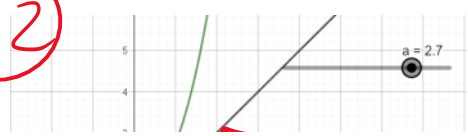
$D\ e^x = e^x \cancel{\ln(e)} = e^x \longrightarrow \boxed{D e^x = e^x}$

2,7

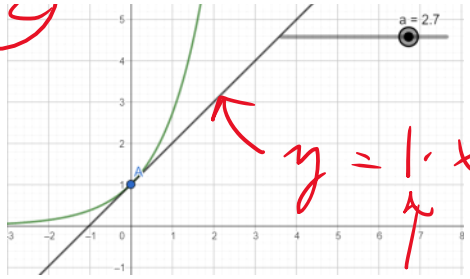
MIKÄ LUKU ON $e = 2{,}7$?

1

2



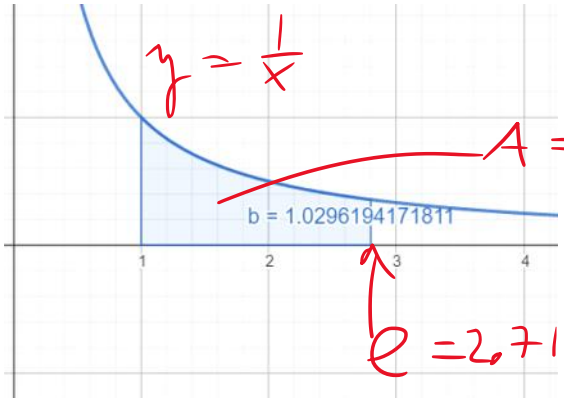
$e = 2{,}7$?



$y = 1 \cdot x + 1$

KULMAKERROIN 1

3

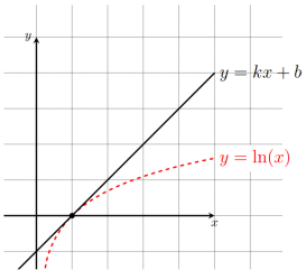


$y = \frac{1}{x}$

$A = 1$

$e = 2{,}718\ldots$

3.2 Logaritmifunktion derivointi ja kuvaaja



3. Olkoon $f(x) = \ln(x)$. Laske

(a) $f'(x)$	Solution $\frac{1}{x}$
(b) $f'(1)$	Solution 1
(c) $f'(2)$	Solution 0.5
(d) $f'(3)$	Solution 0.33
(e) $f'(4)$	Solution 0.25
(f) $f'(5)$	Solution 0.2

4. Selvitä funktion $f(x) = \ln(x)$ kuvaajalle pisteeseen $(1, 0)$ piirretyn tangenttisuoran

(a) kulmakerroin	Solution 1
(b) yhtälö.	~~Solution $y = x + x$~~

$y = x - 1$ — ok

4. Laske integraalit.

3^x

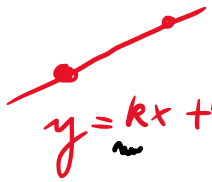
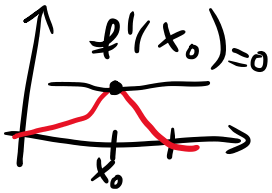
(a) $\int_0^2 2^x dx$	Solution 4.3281
(b) $\int_0^2 3^2 dx$	Solution 7.2819
(c) $\int_0^2 0.5^x dx$	Solution 1.082

Talking:

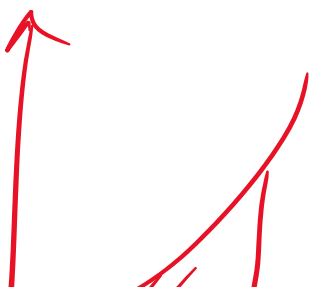
3 Eksponenttifunktio

keskiviikko 27. maaliskuuta 2024 19.56

KURSSIN SISÄLLÖN JÄRJESTÄMINEN

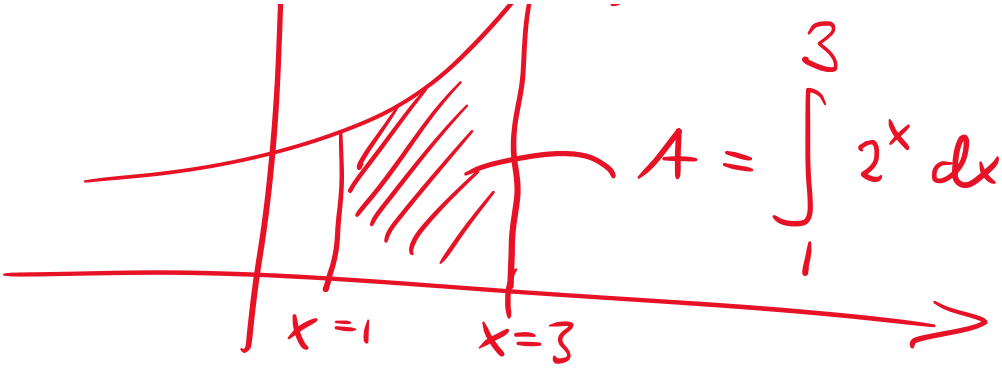
ALKEIS FUNKTIO DERIVOINTIA	YLEISIÄ DERIVOINTIKAAVOJA	SOVELLUS	INTEGROINTIA
 $y = kx + b$			$\int_a^b f(x) = F(b) - F(a)$
$D\,x^7 = 7x^6$ POLYNOMI		 $f'(x_0) = 0$ $x^3(1-x)^2$	$\int x^7 dx = \frac{x^8}{8}$
$D\,7^x = 7^x \ln(7)$ EKSPONENTTI F.	$D\,7^{x^3} = 7^{x^3}\ln(7) \cdot \underbrace{Dx^3}_{=3x^2}$ $D\,f(g(x)) = f'(g(x))\underbrace{g'(x)}_{\text{SISÄFUNKTION DERIVAATTA}}$	$f''(x_0) = 0$ JYRKKENEE MÄKI LOIVENEE JYRKIN KOHTA = KÄÄNNEPISTE	$\int 7^x dx = \frac{7^x}{\ln(7)}$
$D\,\sin(x)$ $D\,\cos(x)$	?	?	?

ESIM.



$y = 2^x$

3 3



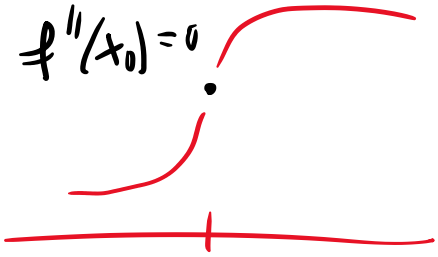
$$A = \int_1^3 2^x \, dx = \left. \frac{2^x}{\ln(2)} \right|_1^3$$

$$= \frac{2^3 - 2^1}{\ln(2)}$$

$$= \frac{7}{\ln(2)} \approx \frac{7}{0.7} \approx 10$$

$\ln(2) \approx 0{,}69 \approx 0{,}7$

KÄÄNNEPISTE $f''(x_0) = 0$



x_0
JYRKIN KOHTA

ESIM. $f(x) = x^{10} + x^9$

ETSI KÄÄNNEPISTEET.

RATKAISU. (a) $f(x) = x^{10} + x^9$

$$f'(x) = 10x^9 + 9x^8$$

$$f''(x) = 90x^8 + 72x^7$$

$$= 9x^7(10x + 8) = 0$$

JOKO $x^7 = 0$ TAI $10x + 8 = 0$

$x = 0$ $\quad 10x = -8$

VASTAUS $\quad x = -0{,}8$

VASTAUS $x = -0{,}8$

YHDISTETYN FUNKTION DERIVOINTI

"DOO"

∂ - Wikipedia
Merkinnän keksi ranskalainen
Vuonna 1770

$$\boxed{D\, f(g(x)) = f'(g(x))\, g'(x)}$$

$$\underbrace{D}\ \underbrace{\frac{d}{dx}}\ \underbrace{(\ )'}\ \underbrace{\frac{\partial}{\partial x}}$$

$$\frac{d\, f}{dx} = \frac{d\, f}{d\, t}\ \frac{d\, t}{dx}$$

ESIM.

$$\frac{d}{dx}\, e^{x^3} = \frac{d}{dx}\, e^{t} = \boxed{\frac{d\, e^{t}}{dt}} \cdot \frac{dt}{dx}$$

$x^3 = t$

$e^{t} = e^{x^3}$

$\frac{dt}{dx} = \boxed{\frac{d\, x^3}{dx}} = 3x^2$

$= e^{x^3} \cdot 3x^2$

ESIM.

$$D\, x^7 = 7x^6$$

$$D\, (x^3 + 2x^5)^7 = 7(\quad)^6 D$$

$$= 7(x^3 + 2x^5)^6 \cdot (3x^2 + 10x^4)$$

ESIM.

$$D\, 2^{3^x} = D\, 2^{(3^x)} = 2^{3^x} \cdot \ln(2) \cdot \underbrace{D\, 3^x}_{3^x \ln(3)}$$

$$= 2^{3^x} \cdot 3^x \cdot \ln(2) \ln(3)$$

$$= 2^{3^x} \cdot 3^x \cdot \ln(2)\ln(3)$$

ESIM. $D2^{x^3} = 2^{x^3}\ln(2) \cdot \underbrace{D\,x^3}_{3x^2}$

ESIM. $D\,(2^x)^7 = 7(2^x)^6 \cdot \underbrace{D\,2^x}_{2^x\ln(2)}$

Eksponenttifunktion derivointia ja integrointia, tehtäviä

torstai 28. maaliskuuta 2024 19.01

7. (e) De^{5x+1} $= e^{5x+1} D(5x+1)$

$= e^{5x+1}(5+0)$

$= 5e^{5x+1}$

(f) $D2^{3x}$

$D2^{3x} = 2^{3x} \cdot \ln(2) \cdot D\,3x$

$= 2^{3x} \ln(2) \cdot 3$

$De^x = e^x$

$Dx^n = nx^{n-1}$

$D5x^1 = 5 \cdot 1 \cdot \underbrace{x^{1-1}}_{=x^0=1} = 5$

- $D5x = 5$
- $D\underbrace{1}_{\text{VAKIO}} = 0$

Solution $5e^{5x+1}$

Solution $2^{3x} \cdot 3 \cdot \ln(2)$

8. Laske

(a) $De^{\sqrt{x}}$

(b) $De^{(1/x^2)}$

(c) $D2^{(1/\sqrt{x})}$

Solution $e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

Solution $e^{(1/x^2)} \cdot \frac{-2}{x^3}$

Solution $2^{(1/\sqrt{x})} \cdot \frac{-1}{2x\sqrt{x}}$

8. Laske

(a) $De^{\sqrt{x}}$

(b) $De^{(1/x^2)}$

(c) $D2^{(1/\sqrt{x})}$

Solution $e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

Solution $e^{(1/x^2)} \cdot \frac{-2}{x^3}$

Solution $2^{(1/\sqrt{x})} \cdot \frac{-1}{2x\sqrt{x}}$ $\cdot \ln(2)$

8.(a) $De^{\sqrt{x}} = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$De^x = e^x$

$D\sqrt{x} = Dx^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

(b) $De^{\frac{1}{x^2}}$

$= e^{\frac{1}{x^2}} \cdot \frac{-2}{x^3}$

$D\frac{1}{x^2} = Dx^{-2} = -2x^{-3}$

$= \frac{-2}{x^3}$

$D\,2^{\frac{1}{\sqrt{x}}}$

$= 2^{\frac{1}{\sqrt{x}}} \ln(2)\, \underbrace{D\frac{1}{\sqrt{x}}}_{= \frac{-1}{2x\sqrt{x}}}$

$= 2^{\frac{1}{\sqrt{x}}} \ln(2) \frac{-1}{2x\sqrt{x}}$ $x^{3/2}$

$= -2^{\frac{1}{\sqrt{x}} - 1} \frac{\log(2)}{x^{3/2}}$

$D\sqrt{x} = \frac{1}{2\sqrt{x}}$

$D\frac{1}{\sqrt{x}} = Dx^{-\frac{1}{2}}$

$= -\frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{-1}{2\,x^{\frac{3}{2}}} = \frac{-1}{2x\sqrt{x}}$

$= x\sqrt{x}$

$\boxed{D2^x = 2^x \ln(2)}$

derivative of 2^(1/sqrt(x))

NATURAL LANGUAGE | MATH INPUT

Derivative

$$\frac{d}{dx}\left(\sqrt[\sqrt{x}]{2}\right) = -\frac{2^{1/\sqrt{x}-1}\log(2)}{x^{3/2}}$$

9. Laske (Lähde: Kari Jyrkkä, 295s

(a) $D2x^3$ Solution $6x^2$

(b) $D(2x^3)^5$ Solution $5\cdot(2x^3)^4\cdot 6x^2$

(c) $\frac{\partial}{\partial k}(2x^3+2k^2)^5$ Solution $5(2x^3+2k^2)^4\cdot 4k^1$

VAKIO MUUTTUJAN k SUHTEEN

DERIVOIDAANKIN k:N SUHTEEN

10. Olkoon $f(x) = 2^x$. Laske

(a) $f'(x)$ Solution $2^x\ln(2)$

(b) $f'(0)$ Solution $\ln(2)\approx 0.7$

(c) $f'(1)$ Solution 1.4

(d) $f'(2)$ Solution 2.8

(e) $f'(-1)$ Solution 0.35

(f) $f'(-2)$ Solution 0.175

10. (a) $f(x)=2^x$

$f'(x) = D\,2^x = 2^x\ln(2)$

(e) $f'(-1) = 2^{(-1)}\ln(2) = 2^{-1}\ln(2) = 0.3465$

≈ 0.35

$D\,2^{-1}\ln(2) = 0$

$\frac{d}{dx}$

EI RIIPU x:STÄ, VAKIO

$D\,2^{-1}\ln(2) \overset{?}{=} \cancel{-1\,2^{-2}\ln(2)}$ 0

D VAKIO = 0

+ DERI 0

DERI + SIJ. + DERI 0 1

LASKIN DERI + DERI + SIJ.

10. (f) $f(x) = 2^x$

$f'(x) = 2^x \ln(2)$

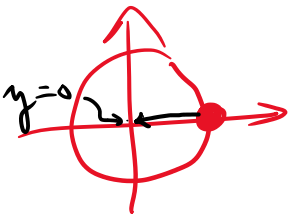
$f'(-2) = 2^{-2} \ln(2) \approx \frac{0.35}{2} = 0.175$

$\frac{\ln(2)}{2^2} = \frac{\ln(2)}{4} \approx \frac{0.7}{4} = 0.175$

ARVO KÄSIN

EI TARVITSE LASKEA KÄSIN, MUTTA OPETTE:

- $2^{-2} = \frac{1}{2^2}$ POT. LASKUSÄÄNNÖT
- $\ln(2^3) = 3\ln(2)$
- $\ln(e) = 1$
- $\sin(0) = 0$



KOE (1) SATUNNAISTUKSIA

D 2^x

D 3^x

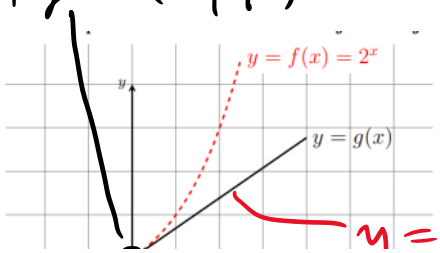
D 7^x

(2) ~~TOIVOTTAVASTI EI VÄLIVAIHETTA KUMA TARVITSE PALAUTTAA~~

PALAUTETAAN VÄLIVAIHEET

→ JOS SATTUI LASKUVIRHE, KERRO OPETTAJALLE

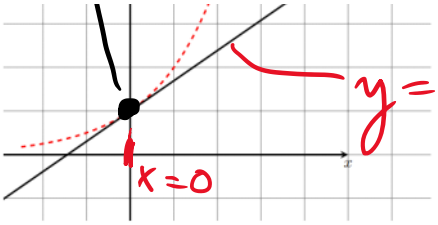
$(x_1, y_1) = (0, 1)$



$y = kx + b$

11. (a) $k = ?$

(b)



$y = kx + b$

(b)

$y = kx + b$?

RATKAISU.

(a) $k = f'(0)$ = KASVUNOPEUS

$f(x) = 2^x$

$f'(x) = 2^x \ln(2)$

$= \underbrace{2^0}_{=1} \ln(2) = \ln(2)$

$\boxed{\text{SIIS } k = \ln(2)}$

(b) $y - \overset{1}{y_1} = \overset{\ln(2)}{k}(x - \overset{0}{x_1})$

MISSÄ k = KULMAKERROIN

(x_1, y_1) SUORAN PISTE

$y - 1 = \ln(2)(x - 0)$

$y - 1 = \ln(2)\, x$

$y = 1 + \underbrace{\ln(2)}_{\approx 0.7} x \rightarrow \boxed{y = 1 + 0.7x}$

13. Etsi funktion $f(x) = x^3 + x^2$ käännepisteet.

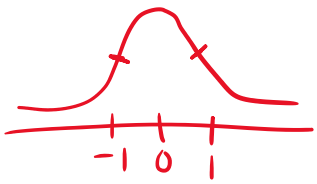
(a) Laske $f'(x)$. — Solution $3x + 2$

(b) Laske $f''(x)$. — Solution $x = -2/3$

(c) Merkitse $f''(x) = 0$ ja ratkaise x.

OH 1

14. Etsi funktion $f(x) = e^{-x^2/2}$ käännepisteet.



(a) Laske $f'(x)$.

(b) Laske $f''(x)$. — Solution $e^{-x^2/2}(x^2 - 1)$

(c) Merkitse $f''(x) = 0$ ja ratkaise x. — Solution $x = \pm 1$

EI VIELÄ OSATA LASKEA KÄSIN

JYRKIN KOHTA

13. Etsi funktion $f(x) = x^3 + x^2$ käännepisteet.

(a) Laske $f'(x)$. — Solution $3x + 2$

(b) Laske $f''(x)$. — Solution $x = -2/3$

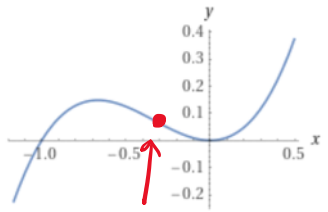
(c) Merkitse $f''(x) = 0$ ja ratkaise x.

RATKAISU.

Input

$y = x^3 + x^2$

Plots



(x from −1.2 to 0.5)

JYRKIN ALAMÄKI AINOA JYRKIN KOHTA

$f(x) = x^3 + x^2$

ⓐ $f'(x) = 3x^2 + 2x$

ⓑ $f''(x) = 6x + 2$

ⓒ $6x + 2 = 0$

$6x = -2$

$x = -\frac{1}{3}$

$x \approx -0{,}33$

16. Laske sijoitukset. Integraalit kertovat eräiden tasoalueiden pinta-alat. Tasoalueet on piirretty alla oleviin kuviin.

(a) $\int_0^2 e^x = \Big/_0^2 e^x$ Solution 6.3891

(b) $\int_0^1 e^{2x} dx = \Big/_0^1 \frac{e^{2x}}{2}$ Solution ~~2.8~~ 3,19

(c) $\int_0^1 2x \cdot e^{x^2} = \Big/_0^1 e^{x^2}$ Solution 1.7183

16.ⓑ $\int_0^1 e^{2x}\,dx = \Big/_0^1 \frac{e^{2x}}{2}$

INT

$e^{2x} \rightleftarrows \frac{e^{2x}}{2}$

DERI

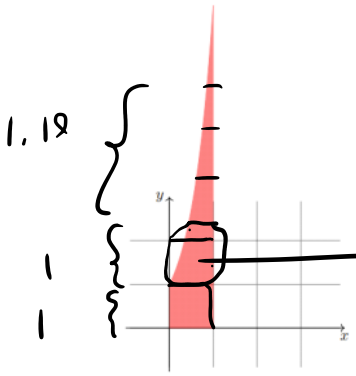
TARKISTUS. $D\,\frac{e^{2x}}{2} = \frac{e^{2x} \cdot \cancel{2}}{\cancel{2}} = e^{2x}$

$D\,2^x = 2^x \ln(2)$

$\boxed{De^x = e^x}$ $\boxed{\int e^x\,dx = e^x}$ $\int 2^x\,dx = \frac{2^x}{\ln(2)}$

16.ⓑ $\int_0^1 e^{2x}\,dx = \Big/_0^1 \frac{e^{2x}}{2} = \frac{e^{2\cdot 1}}{2} - \frac{e^{2\cdot 0}}{2}$

$= \frac{e^2}{2} - \frac{1}{2} \overset{\text{LASKIN}}{=} 3{,}19$



ALA = 3,19

OK ALA NOIN 3,19

VIIKON AIHEET KIRJASSA

14. Etsi funktion $f(x) = e^{-x^2/2}$ käännepisteet.

(a) Laske $f'(x)$.

(b) Laske $f''(x)$.

(c) Merkitse $f''(x) = 0$ ja ratkaise x.

$$f(x) = e^{-x^2/2}$$

$$f'(x) = e^{-\frac{x^2}{2}} \cdot \underbrace{\frac{-2x}{2}}_{=-x}$$

$$f'(x) = \underbrace{-x}\, \underbrace{e^{-\frac{x^2}{2}}}$$

$$(fg)' = \underbrace{f'g} + \underbrace{fg'}$$

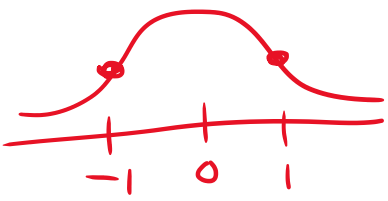
$$f''(x) = -1\, e^{-\frac{x^2}{2}} + \underbrace{(-x)(-x)}\, e^{-\frac{x^2}{2}}$$

$$= e^{-\frac{x^2}{2}}\left(x^2 - 1\right) = 0$$

$$= \underbrace{e^{-\frac{x^2}{2}}}_{\neq 0}\left(x^2 - 1\right) = 0$$

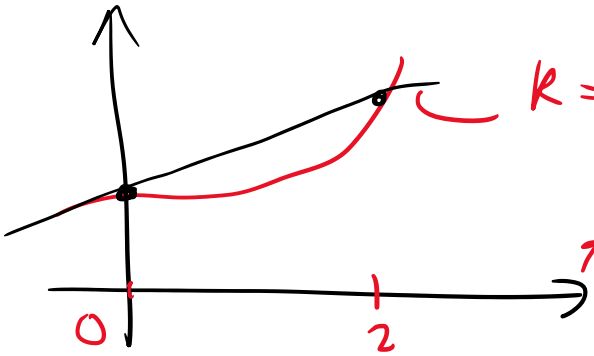
$$x^2 = 1 \quad \| \sqrt{\ }$$

$$x = \pm 1$$



LÄPIPÄÄSY TENTTI

1. Laske funktion $f(x) = 4\sin(x) - \cos(2x)$ keskimääräinen muutosnopeus välillä $[0, 2]$.



$$k = \frac{f(2) - f(0)}{2 - 0}$$

$$= 1{,}64$$

LASKIN

ok

$$\frac{f'(0) + f'(2)}{2}$$

2. Laske $f'(x)$, kun

KAAVAT

(a) $f(x) = 4x^5 - \sqrt{2x} + \frac{1}{x^3}$

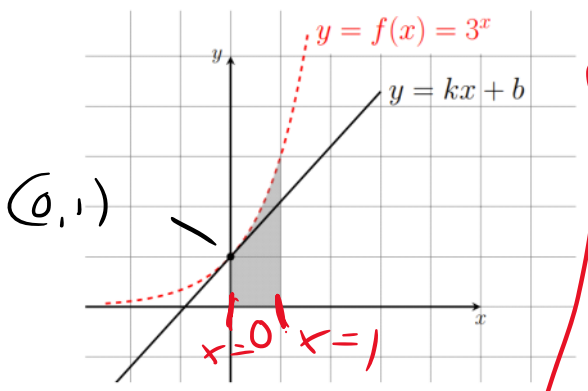
$$20x^4 - \frac{\sqrt{2}}{2\sqrt{x}} - 3\frac{1}{x^4}$$

(b) $f(x) = 2\sin(3x) - 7e^{x^4}$

$$6\cos(3x) - 7e^{x^4} \cdot 4x^3$$

(c) $f(x) = e^x \ln(x)$

$$f'(x) = e^x \ln(x) + e^x \frac{1}{x}$$



$$k = f'(0)$$
$$= 3^0 \ln(3)$$
$$= \ln(3)$$

$$y - y_1 = k(x - x_1)$$

$$y = 1 + \ln(3)x$$

Kuva 1: Funktion kuvaaja, tangenttisuora ja pinta-ala. Ruudun leveys ja korkeus ovat 1.

3. Selvitä Kuvassa 1 olevan suoran $y = kx + b$ yhtälö.

4. Selvitä Kuvassa 1 olevan variostetun alueen pinta-ala.

3. Selvitä Kuvassa 1 olevan suoran $y = kx + b$ yhtälö.

4. Selvitä Kuvassa 1 olevan varjostetun alueen pinta-ala.

$$\int_0^1 3^x\,dx = \Big/_0^1 \frac{3^x}{\ln(3)} = \frac{3^1 - 3^0}{\ln(3)} = \boxed{\frac{3}{\ln(3)}}$$

$\approx 2{,}73$

5. Uima-altaan tilavuuden tulee olla 256 kuutiometriä. Altaan pohja on neliön muotoinen ja seinät pystysuorat. Suunnittele altaan mitoitus niin, että kaakelia kuluu mahdollisimman vähän, kun seinät ja pohja kaakeloidaan. Toisin sanoen minimoi seinien ja pohjan yhteenlaskettu pinta-ala.

6. Laske

(a) $\int 4x + \sqrt{x}dx$ $\longrightarrow$ $2x^2 + \frac{2}{3}x^{\frac{3}{2}}$

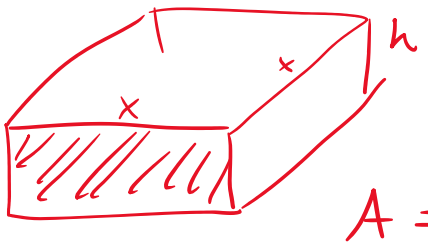
(b) $\int \cos(2x)dx$ $\longrightarrow$ $\frac{\sin(2x)}{2}$

(c) $\int_2^3 e^{-x} + 1dx$

$$\Big/_2^3 -e^{-x} + x = \underbrace{-e^{-3} + 3 - (-e^{-2} + 2)}$$

5. Uima-altaan tilavuuden tulee olla 256 kuutiometriä. Altaan pohja on neliön muotoinen ja seinät pystysuorat. Suunnittele altaan mitoitus niin, että kaakelia kuluu mahdollisimman vähän, kun seinät ja pohja kaakeloidaan. Toisin sanoen minimoi seinien ja pohjan yhteenlaskettu pinta-ala.

?



$$V = \underbrace{x^2 h = 256}$$

$$\Rightarrow h = \frac{256}{x^2}$$

$$A = 4xh + x^2$$

$$A(x) = 1024x^{-1} + x^2$$

$A(x)$

$A'(x) = 0 \Rightarrow x = \ldots \Rightarrow h = \ldots$

$$A'(x) = -1024x^{-2} + 2x = 0$$

$$2x^{-2}(\cancel{-1024} \; 512 + x^3) = 0$$

$$x^3 = \cancel{1024} \; 512 =$$

VAS

$$x = 512^{\frac{1}{3}} = 8$$

$$h = \frac{256}{x^2}$$

256

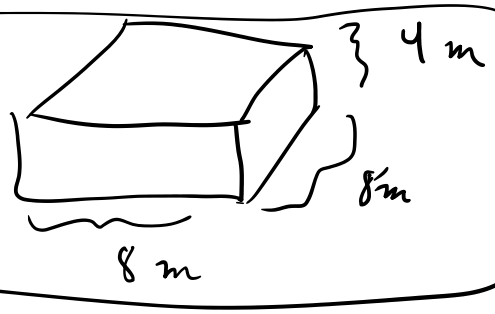
$h = \frac{\quad}{x^2}$

$= \frac{256}{64} = 4$

$x = 512^{\frac{1}{3}} = 8$

LISKU

VASTAUS



Koe

keskiviikko 3. huhtikuuta 2024 15.27

$$x^2 - 2x + 3 = 0$$

$a = x^2$?

$b = -2$

$c = 3$

VIRHE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$

VIRHE

$$= \frac{-2 \pm \sqrt{-2^2 - 4 \cdot x^2 \cdot 3}}{2 \cdot x^2}$$

VIRHE

4 Logaritmifunktio

keskiviikko 3. huhtikuuta 2024 18.57

KERTAUSTA

$$D\,x^2 = 2x^{2-1} = 2x$$

$$D\,x = 1$$

$$D\,\text{VAKIO} = 0$$

$$D\sqrt{x} = D\,x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$D\,\frac{1}{x^2} = D\,x^{-2} = -2x^{-3}$$

$$\int 2x^1\,dx = \frac{\cancel{2}x^2}{\cancel{2}} = x^2$$

$+1$

$$x^2 \xrightarrow{D} 2x$$

$$\xleftarrow{\int dx}$$

$$D\,e^x = e^x \underbrace{\ln(e)}_{=1}$$

$$\boxed{D\,a^x = a^x \ln(a)}$$

$$D\,e^{2x} = e^{2x}\,D(2x)$$

$$= e^{2x}\cdot 2$$

$$D\,e^{\overbrace{9}^{\text{VAKIO}}x} = 9e^{9x}$$

ESIM.

$$\boxed{D\,2^x \overset{\text{KAAVALLA}}{=} 2^x \ln(2)}$$

$$\ln(e^x) = x$$

$$e^{\ln(x)} = x$$

VAIHDETAAN KANTALUKU

$$= D\,e^{\ln(2^x)}$$

$$= D\,e^{x\overbrace{\ln(2)}^{\text{VAKIO}}} = e^{x\ln(2)}\cdot\ln(2)$$

$$\underbrace{e^{x\ln(2)}}_{2^x} = 2^x\ln(2)$$

2^x KIRJOITETTUNA ERI TAVALLA

JOS EI TYKKÄÄ JOISTAKIN KANTALUVUISTA, ESIM.

$$7^x = e^{x\ln(7)}$$

$$\log_7(x) = \frac{\ln(x)}{\ln(7)}$$

NIIN NÄMÄ VOI KIRJOITTAA VAIKKAPA $\boxed{e \;\&\; \ln(\;)}$ RIITTÄÄ AVULLA

LOGARITMIN DERIVOINTI JA INTEGROINTI

$$\boxed{x > 0}$$

$$\boxed{D\ln(x) = \frac{1}{x}} \quad \text{JA} \quad \boxed{\int \frac{1}{x}\,dx = \ln(x)}$$

$$\left(\text{JA} \int \ln(x)\,dx = x\ln(x) - x\right)$$

MUUT LOGARITMIT $\quad \log_a(x) = \dfrac{\ln(x)}{\ln(a)}$

ESIM. $\quad D\ln(x^2+3x) = \dfrac{1}{(\quad)} D(x^2+3x)$

DERIVAATTA

$$D\ln(f(x)) = \frac{f'(x)}{f(x)} = \frac{2x+3}{x^2+3x}$$

$$D\ln(x^7) = \frac{Dx^7}{x^7} = \frac{7x^6}{x^7} = \frac{7}{x}$$

TAPA 2

$$D\ln(x^7) = 7\,D\ln(x) = 7\cdot\frac{1}{x} = \frac{7}{x}$$

$$D\ln(\sqrt{x}) = \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}} \qquad \boxed{D\sqrt{x} = \frac{1}{2\sqrt{x}}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{x}}$$

$$= \frac{1}{2x}$$

TAPA 2

$$D\ln\sqrt{x} = D\ln x^{\frac{1}{2}}$$

$$= \frac{1}{2}D\ln(x) = \frac{1}{2}\cdot\frac{1}{x} = \frac{1}{2x}$$

$$D\log_2(x^2+1)$$

$$= D\frac{\ln(x^2+1)}{\ln(2)} = \frac{1}{\ln(2)}D\ln(x^2+1) = \frac{1}{\ln(2)}\cdot\frac{2x+0}{(x^2+1)}$$

FUNKTION POTENSSIN INTEGROINTI $\quad D\,x^n = n\,x^{n-1}$

$$\boxed{D\,f(g(x)) = f'(g(x))\,g'(x)} \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$D\,(f(x))^n = n\,f(x)^{n-1}\,f'(x)$$

$$\boxed{\int f'(x)\,\underbrace{f(x)^n}\,dx = \frac{f(x)^{n+1}}{n+1}}$$

VOI KÄYTTÄÄ JOS $n+1 \neq 0$

ELI $n = -1$

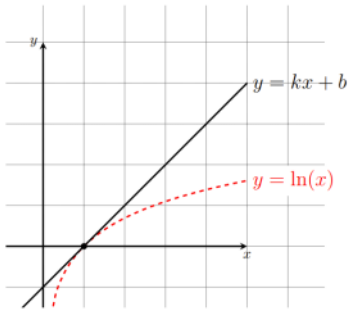
$$\neq f(x)^{-1} = \frac{1}{f(x)}$$

$$\boxed{\int f'(x)\,\frac{1}{f(x)}\,dx = \ln(f(x))}$$

ESIM. $\int \underbrace{2x}_{f'(x)}\,(\underbrace{x^2+2}_{f(x)})^7\,dx = \frac{(x^2+2)^8}{8}$

$$\int \frac{\overset{=f'(x)}{2x}}{\underset{=f(x)}{x^2+2}}\,dx = \ln(x^2+2)$$

3.2 Logaritmifunktion derivointi ja kuvaaja



3. Olkoon $f(x) = \ln(x)$. Laske

(a) $f'(x)$	Solution $\frac{1}{x}$
(b) $f'(1)$	Solution 1
(c) $f'(2)$	Solution 0.5
(d) $f'(3)$	Solution 0.33
(e) $f'(4)$	Solution 0.25
(f) $f'(5)$	Solution 0.2

4. Selvitä funktion $f(x) = \ln(x)$ kuvaajalle pisteeseen $(1, 0)$ piirretyn tangenttisuoran

(a) kulmakerroin	Solution 1
(b) yhtälö.	~~Solution $y = -1 + x$~~

$$y = x - 1$$

1. Laske

(a) $D\ln(x^5)$

(b) $D\log_2(x^5)$

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

$$\log_a(x) = \frac{\log_e(x)}{\log_e(a)} = \frac{\ln(x)}{\ln(a)}$$

$$D\log_2(x^5) = D\,\frac{\ln(x^5)}{\ln(2)} = \frac{1}{\ln(2)}\,D\ln(x^5) = \frac{1}{\ln(2)}\cdot\frac{5}{x}$$

(a)-KOHTA

① MUUTETAAN $\log_2 \to \ln(\,)$

VAKIO, VOI VIEDÄ DERIVOINNIN ULKOPUOLELLE

LASKUA

MUISTA KOKEILLA LASKUJA MYÖS TIETOKONEELLA

FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA

WolframAlpha

integrate 2*x*(x^2+1)^3 dx

NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EX

Indefinite integral

$$\int 2x(x^2+1)^3\,dx = \frac{1}{4}(x^2+1)^4 + \text{constant}$$

2. Laske

(a) $D\ln(\sqrt{x})$

(b) $D\ln(1/x^2)$

(c) $D\log_2(1/\sqrt{x})$

STRATEGIA

① KIRJOITA MUODOSSA x^p, POTENSSI p NÄKYVISSÄ!

② $D\ln(x^p)$

VIE POTENSSI ULOS

③ $\boxed{\log_2(x) = \frac{\ln(x)}{\ln(2)}}$

④ $\underbrace{\frac{p}{\ln(2)}}_{\text{SÄLÄÄ}}\ \underbrace{D\ln(x)}_{=\frac{1}{x}}$

ESIM.

$D\log_7(\sqrt[3]{x})$

① $= D\log_7(x^{\frac{1}{3}})$

② $= \frac{1}{3}D\log_7(x)$

③ $= \frac{1}{3\ln(7)}D\ln(x)$

$= \frac{1}{3\ln(7)}\cdot\frac{1}{x}$

KUVAAJA -ESIMERKKI

FUNKTION $f(x) = \log_5(\sqrt{x})$ $\quad x^{\frac{1}{2}}$

TANGENTIN YHTÄLÖ

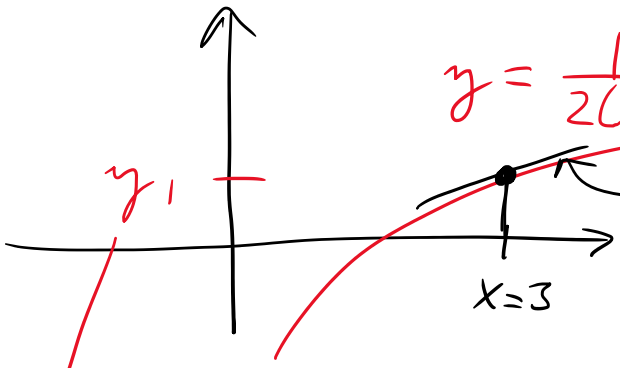
$= \frac{1}{2\ln(5)}\cdot\ln(x)$

KOHDASSA $x = 3$.

$$f'(x) = \frac{1}{2\ln(5)} \cdot \frac{1}{x}$$

$$k = f'(3) = \frac{1}{2\ln(5)} \cdot \frac{1}{3} \approx 0{,}10355$$

$$y = \frac{1}{2\ln(5)} \ln(x)$$



$$\boxed{y - y_1 = k(x - x_1)}$$

$0{,}10355$ $\quad$ 3

$$y = kx + b \ ?$$

$$y_1 = f(3) = \log_5(\sqrt{3}) = 0{,}3413$$

$$y - 0{,}3413 = 0{,}10355(x - 3)$$

$$y = 0{,}3413 + 0{,}10355x \underbrace{- 0{,}10355 \cdot 3}_{-0{,}31}$$

$$y = 0{,}10355x + 0{,}03065$$

$$\int x^n \, dx$$

$$\boxed{\int f'(x) \, f(x)^n \, dx = \frac{f(x)^{n+1}}{n+1}}$$

5. Laske integraalit

5. Laske integraalit

(a) $\int 2x(x^2+1)^3 dx$

(b) $\int \frac{2x}{(x^2+1)^3} dx = \int \underbrace{2x}_{f'(x)} \underbrace{(x^2+1)}_{f(x)}{}^{-3} dx = \frac{(x^2+1)^{\cancel{-3+1}\ -2}}{-2}$

$= \frac{(x^2+1)^{-2}}{-2}$

$= \frac{1}{-2(x^2+1)^2}$

6. Laske integraalit

(a) $\int \frac{5x^4+1}{x^5+x} dx$ → Ratk. $\ln(x^5+x)$

(b) $\int (5x^4+1)(x^5+x)^7 dx$

(c) $\int (5x^4+1)\sqrt{x^5+x} dx$

6.(b) $\int \overbrace{(5x^4+1)}^{f'(x)} \overbrace{(x^5+x)}^{f(x)}{}^7 dx = \frac{(x^5+x)^8}{8}$

JOS $f(x) = x^5 + x^1$

NIIN $f'(x) = 5x^4 + 1 \cdot \overbrace{x^0}^{=1} = 5x^4 + 1$

6.(c) $\int (5x^4+1)\sqrt{x^5+x}\, dx$

$= \int \underbrace{(5x^4+1)}_{f'(x)} \underbrace{(x^5+x)}_{f(x)}{}^{\frac{1}{2}} dx = \frac{(x^5+x)^{\cancel{\frac{1}{2}+1}\ \frac{3}{2}}}{\frac{3}{2}}$

$= \frac{2}{3}(x^5+x)^{\frac{3}{2}}$

$$= \frac{2}{3}(x^5+x)^{\frac{3}{2}}$$

TARKISTUS

$$D\,\frac{2}{3}(x^5+x)^{\frac{3}{2}} = \underbrace{\frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}}_{=1}(x^5+x)^{\frac{3}{2}-1}\,\underbrace{D(x^5+x)}_{5x^4+1}$$

$$\frac{3}{2}-1 = \frac{1}{2}$$

$$D\ln(f(x)) = \frac{f'(x)}{f(x)}$$

(f) $D\log_2(3x)$

$$D3x = 3$$

$$= D\,\frac{\ln(3x)}{\ln(2)} = \frac{1}{\ln(2)}\,D\ln(3x)$$

$$= \frac{1}{\ln(2)}\,\frac{\cancel{3}}{\cancel{3}x} = \frac{1}{\ln(2)}\,\frac{1}{x}$$

$\boxed{\text{TAPA 2}}$

$$\ln(ab) = \ln(a) + \ln(b)$$

$$D\log_2(3x) = \frac{1}{\ln(2)}\,\underbrace{D\ln(3x)}$$

$$= D\,\underbrace{\ln(3)}_{\text{VAKIO}} + \ln(x)$$

$$= 0 + \frac{1}{x}$$

$$= \frac{1}{\ln(2)}\left(0+\frac{1}{x}\right)$$

$$= \frac{1}{\ln(2)}\cdot\frac{1}{x}$$

Koevirheitä

keskiviikko 3. huhtikuuta 2024 19.15

ETSI SEURAAVASTA LASKUSTA KOLME VIRHETTÄ

$ax^2+bx+c=0$

VIRHE 1 $a \neq x^2$ VAAN $a=1$

VIRHE 2 MIINUS UNOHTUI

VIRHE 3 SULUT UNOHTUIVAT

$a = \cancel{x^2}\ 1$

$b = -2$

$c = 3$

a, b JA c OVAT LUKUJA, TÄNNE EI TULE X:ÄÄ

$$1 \cdot x^2 - 2x + 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot x^2 \cdot 3}}{2x^2}$$

HUOM. $-2^2 = -(2^2) = -4$

$(-2)^2 = (-2)(-2) = +4$

5 trigonometriset funktiot

tiistai 9. huhtikuuta 2024 18.58

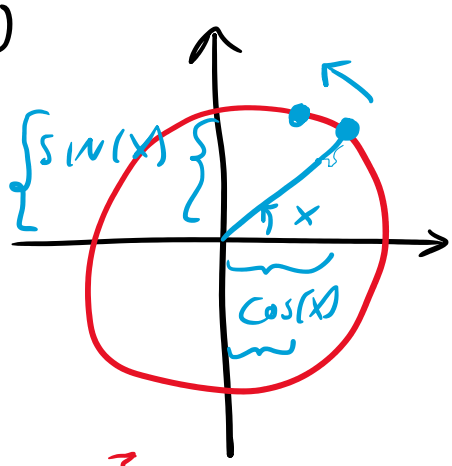
TÄNÄÄN

SUURENEE (RADIAANIT)

$$\boxed{\begin{cases} D\sin(x) = \cos(x) & >0 \\ D\cos(x) = -\sin(x) & <0 \end{cases}}$$

OK

KUVAN MUKAAN PIENENEE, KUN x KASVAA



HARJOITELLAAN KOHTA LISÄÄ

$$\frac{3}{4} > \frac{3}{5}$$

$$D\, f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

$$D\,\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$\boxed{\begin{aligned} D\, fg &= f'g + fg' \\ D\,\frac{f}{g} &= \frac{f'g - fg'}{g^2} \end{aligned}}$$

Potenssin derivaatta	Trigonometristen funktioiden derivaatat	Arkusfunktioiden derivaatat
$Dx^n = nx^{n-1}$, missä $n \in \mathbb{Z}$	$D\sin x = \cos x$	$D\arcsin x = \frac{1}{\sqrt{1-x^2}}$
$D\sqrt{x} = \frac{1}{2\sqrt{x}}$ [2]	$D\cos x = -\sin x$	$D\arccos x = -\frac{1}{\sqrt{1-x^2}}$
$D\sqrt[3]{x} = \frac{1}{3\sqrt[3]{x^2}}$	$D\tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$	$D\arctan x = \frac{1}{1+x^2}$
$D\sqrt[n]{x} = \frac{1}{n\sqrt[n]{x^{n-1}}}$	$D\cot x = -\frac{1}{\sin^2 x} = -1 - \cot^2 x$	$D\,\mathrm{arccot}\, x = -\frac{1}{1+x^2}$
Eksponenttifunktion derivaatta	**Hyperbolisten funktioiden derivaatat**	**Hyperbolisten käänteisfunktioiden eli areafunktioiden derivaatat**
$De^x = e^x$	$D\sinh x = \cosh x$	$D\ \mathrm{arsinh}\ x = \frac{1}{\sqrt{x^2+1}}$
$Da^x = a^x \ln a$, missä $a > 0$	$D\cosh x = \sinh x$	$D\ \mathrm{arcosh}\ x = \frac{1}{\sqrt{x^2-1}}$
	$D\tanh x = \frac{1}{\cosh^2 x}$	$D\ \mathrm{artanh}\ x = \frac{1}{1-x^2}$
Logaritmifunktioiden derivaatat		
$D\ln\lvert x\rvert = \frac{1}{x}$		
$D\log_a\lvert x\rvert = \frac{1}{x\ln a}$, missä $a > 0$ ja $a \neq 1$		

2

3

4

5

Säännön nimi	Derivointisääntö
Vakion derivaatta	$Dc = 0$, kun c on vakio.[4]
Vakion siirto	$Dcf(x) = cDf(x)$ [1][4]
Summan derivaatta	$D(f(x) + g(x)) = Df(x) + Dg(x)$ [1][10][4]
Tulon derivaatta	$D(f(x)g(x)) = g(x)Df(x) + f(x)Dg(x)$ [11][12][13][4]
Funktion potenssin derivaatta	$Df(x)^n = nf(x)^{n-1}f'(x)$ [12][14]
Osamäärän derivaatta	$D\frac{f(x)}{g(x)} = \frac{g(x)Df(x) - f(x)Dg(x)}{g(x)^2}$ [14][13][4]
Yhdistetyn funktion derivaatta	$Dg(f(x)) = g'(f(x))f'(x)$ [4]
Käänteisfunktion derivaatta	$(f^{-1})'(f(x)) = \frac{1}{f'(x)}$, jossa f^{-1} on f :n käänteisfunktio.[4]

TÄNÄÄN

SININ JA KOSININ DERIVOINTIA

$$D\sin(x) = \cos(x)$$

$$D e^{x^7} = e^{x^7} \cdot Dx^7 = e^{x^7} \cdot 7x^6$$

$$D\sin(x^7) = \cos(x^7) \cdot Dx^7 = \underline{\underline{\cos(x^7) \cdot 7x^6}}$$

$$D(\sin(x))^7 = 7(\sin(x))^6 \cdot D\sin(x)$$

$$= \underline{\underline{7(\sin(x))^6 \cos(x)}}$$

$$D\sin(x) = \cos(x)$$
$$D\cos(x) = -\sin(x)$$
$$De^x = e^x$$
$$D\ln(x) = \frac{1}{x}$$

$$D2^x = 2^x \ln(2)$$
$$D\log_2(x) = \frac{1}{\ln(2)} D\ln(x)$$

$$De^{\sin(x)} = e^{\sin(x)} \underbrace{D\sin(x)}_{=\cos(x)}$$

$$De^{\cos(x)} = e^{\cos(x)} D\cos(x) = e^{\cos(x)}(-\sin(x))$$

$$D\ \sin(e^x) = \cos(e^x)\cdot De^x = \cos(e^x)\, e^x$$

$$D\ \cos(e^{x^3}) = -\sin(e^{x^3})\cdot \underbrace{De^{x^3}}_{e^{x^3}\cdot 3x^2} = -3x^2 e^{x^3}\sin(e^{x^3})$$

$$D\ \ln(\sin(x)) = \frac{\cos(x)}{\sin(x)}$$

$$D\ \ln(\cos(x)) = \frac{-\sin(x)}{\cos(x)}$$

$$\boxed{D\ fg = f'g + fg'}$$

$$D\ x^7\cos(3x)$$

$$= 7x^6\cos(3x) + x^7\left(-\sin(3x)\cdot \underbrace{D3x}_{=3}\right)$$

$$\boxed{D\ \frac{f}{g} = \frac{f'g - fg'}{g^2}}$$

$$D\ \frac{x^7}{\sin(4x)}$$

$$= \frac{7x^6\sin(4x) - x^7\cdot\cos(4x)\cdot \overbrace{D\,4x}^{=4}}{(\sin(4x))^2}$$

$$= \frac{7x^6\sin(4x) - 4x^7\cos(4x)}{\sin^2(4x)}$$

3. Laske derivaatta.

(a) $f(x) = 2e^{\sin(7x)}$

(b) $g(x) = 3e^{2\cos(2x)}$

(c) $h(x) = -2e^{-\cos(5x)}$

Solution $f'(x) = 14e^{\sin(7x)}\cos(7x)$

Solution $g'(x) = -12e^{2\cos(2x)}\sin(2x)$

Solution $h'(x) = -10e^{-\cos(5x)}\sin(5x)$

3. b $\quad g(x) = 3e^{2\cos(2x)}$

$g'(x) = D\,3e^{2\cos(2x)} = 3e^{2\cos(2x)} \cdot D\,2\cos(2x)$

$= 3 \cdot 2\, e^{2\cos(2x)}\, \underbrace{D\cos(2x)}$

$= -\sin(2x)\, \underbrace{D(2x)}_{=2}$

$= -2\sin(2x)$

$= -12\, e^{2\cos(2x)} \sin(2x)$

4. Derivoi

(a) $D\sin^7(x) = D(\sin(x))^7$

(b) $D\cos^3(x)$

(c) $D\sin^5(2x)$

(d) $D\frac{1}{\sin^2(3x)}$

(e) $D\sqrt{\cos(4x)}$

Solution $7\sin^6(x)\cos(x)$

Solution $-3\cos^2(x)\sin(x)$

Solution $10\sin^4(2x)\cos(2x)$

Solution $-6(\sin(3x))^{-3}\cos(3x)$

Solution $-2(\cos(4x))^{-1/2}\sin(4x)$

4. c $\quad D(\sin(2x))^5$

$= 5(\sin(2x))^4 \cdot \underbrace{D\sin(2x)}$

$= \cos(2x) \cdot \underbrace{D(2x)}_{=2}$

$= 5(\sin(2x))^4 \cdot \cos(2x) \cdot 2$

$D\sqrt{x} = D\,x^{\frac{1}{2}}$

4. Derivoi

(a) $D\sin^7(x)$

(b) $D\cos^3(x)$

(c) $D\sin^5(2x)$

4. d $\quad D\frac{1}{\quad} = D\frac{1}{\quad}$

(a) $D\sin^{-1}(x)$
(b) $D\cos^3(x)$
(c) $D\sin^5(2x)$
(d) $D\frac{1}{\sin^2(3x)}$
(e) $D\sqrt{\cos(4x)}$

4.d $D\dfrac{1}{\sin^2(3x)} = D\dfrac{1}{(\sin(3x))^2}$

$= D\,(\sin(3x))^{-2}$

$= -2(\sin(3x))^{-3}\cos(3x)\cdot 3$

4.e $D\sqrt{\cos(4x)} = D(\cos(4x))^{\frac{1}{2}}$

$= \frac{1}{2}(\cos(4x))^{-\frac{1}{2}}(-\sin(4x))\cdot 4$

$= -2(\cos(4x))^{-\frac{1}{2}}\sin(4x)$

5. Derivoi

(a) $Dx^7\cos(2x)$
(b) Dx^3e^x
(c) $Dx^5\ln(x)$
(d) $D\sin(x)\ln(x)$
(e) $De^{2x}\cos(3x)$

RATKAISUJA

$7x^6\cos(2x) - 2x^7\sin(2x)$

$3x^2e^x + x^3e^x$

$5x^4\ln(x) + x^4$

$\cos(x)\ln(x) + \dfrac{\sin(x)}{x}$

$2e^{2x}\cos(3x) - 3e^{2x}\sin(3x)$

6. Derivoi

(a) $D\dfrac{x^7}{\cos(2x)} = \dfrac{7x^6\cos(2x) + 2x^7\sin(2x)}{\cos(2x)^2}$
(b) $D\dfrac{x^3}{e^x}$
(c) $D\dfrac{x^5}{\ln(x)}$
(d) $D\dfrac{\sin(x)}{\ln(x)}$
(e) $D\dfrac{e^{2x}}{\cos(3x)}$

ESIM.

$D\,(\sin(2x))^5 = 5(\sin(2x))^4\cdot\cos(2x)\cdot 2$

$n=4$ (*)

ESIM

ESIM.

$n = 4$

$$\int \cos(2x)\,(\underbrace{\sin(2x)}_{f(x)})^4\,dx$$

$$\boxed{\int f'(x)\, f(x)^n\,dx = \frac{f(x)^{n+1}}{n+1}}$$

$$= \frac{1}{2}\int \underbrace{2\cos(2x)}_{f'(x)}\,(\underbrace{\sin(2x)}_{f(x)})^4\,dx$$

$f(x) = \sin(2x)$

$f'(x) = \cos(2x)\cdot 2$

AHAA

TAR NTAAN 2

$$\circledast = \frac{1}{2}\cdot\frac{(\sin(2x))^5}{5} = \frac{(\sin(2x))^5}{10}$$

5. Derivoi

(a) $Dx^7\cos(2x)$

(b) Dx^3e^x

(c) $Dx^5\ln(x)$

(d) $D\sin(x)\ln(x)$

(e) $De^{2x}\cos(3x)$

$$D\,fg = f'g + fg'$$

5.c) $D\,x^5\ln(x)$

$$= 5x^4\ln(x) + x^5\cdot\frac{1}{x}$$

$$= 5x^4\ln(x) + x^4$$

$$\frac{x^5}{x^1} = x^4$$

5 trigonometriset funktiot ratkaisut

keskiviikko 10. huhtikuuta 2024 10.17

1. Laske derivaatta.

(a) $f(x) = \sin(7x)$ Solution $f'(x) = 7\cos(7x)$

(b) $g(x) = \cos(7x)$ Solution $g'(x) = -7\sin(7x)$

(c) $h(x) = \sin(2-3x)$ Solution $h'(x) = -3\cos(2-3x)$

1 (a) $D\sin(7x) = \cos(7x)\cdot D(7x)$

$= \cos(7x)\cdot 7$

$D\sin(x) = \cos(x)$

$D\cos(x) = -\sin(x)$

(b) $D\cos(7x) = -\sin(7x)\cdot 7$

(c) $D\sin(2-3x) = \cos(2-3x)\cdot \overbrace{D(2-3x)}^{=0-3\cdot 1=-3}$

$= -3\cos(2-3x)$

2. Laske derivaatta.

(a) $f(x) = \sin(x^7)$ Solution $f'(x) = 7x^6\cos(x^7)$

(b) $g(x) = \cos(x^7)$ Solution $g'(x) = -7x^6\sin(x^7)$

(c) $h(x) = \sin(x^{-3})$ Solution $h'(x) = -3x^{-4}\cos(x^{-3})$

2. (a) $D\sin(x^7) = \cos(x^7)\cdot \underbrace{Dx^7}_{=7x^6} = 7x^6\cos(x^7)$

(b) $D\cos(x^7) = -7x^6\sin(x^7)$

(c) $D\sin(x^{-3}) = \cos(x^{-3})\cdot \underbrace{Dx^{-3}}_{=-3x^{-4}}$

$\left[= -3\,\dfrac{\cos\left(\frac{1}{x^3}\right)}{x^4}\right]$

3. Laske derivaatta.

(a) $f(x) = 2e^{\sin(7x)}$ Solution $f'(x) = 14e^{\sin(7x)}\cos(7x)$

(b) $g(x) = 3e^{2\cos(2x)}$ Solution $g'(x) = -12e^{2\cos(2x)}\sin(2x)$

(c) $h(x) = -2e^{-\cos(5x)}$ Solution $h'(x) = -10e^{-\cos(5x)}\sin(5x)$

3 (a) $D\,2e^{\sin(7x)}$

$= 2e^{\sin(7x)} \cdot \underbrace{D\sin(7x)}_{= 7\cos(7x)}$

$= 14e^{\sin(7x)}\cos(7x)$

$D\sin(x) = \cos(x)$

$D\cos(x) = -\sin(x)$

$De^x = e^x$

(b) $D\,3e^{2\cos(2x)} = -12e^{2\cos(2x)}\sin(2x)$

(c) $D\,-2e^{-\cos(5x)} = (-2)(-1)\cdot 5\,e^{-\cos(5x)}(-\sin(5x))$

$= -10e^{-\cos(5x)}\sin(5x)$

derivative of -2*exp(-cos(5*x))

NATURAL LANGUAGE MATH INPUT

Derivative

$$\frac{d}{dx}(-2\exp(-\cos(5x))) = -10\sin(5x)\,e^{-\cos(5x)}$$

$[\sin(ax)]^p$

4. Derivoi

(a) $D\sin^7(x)$ Solution $7\sin^6(x)\cos(x)$

(b) $D\cos^3(x)$ Solution $-3\cos^2(x)\sin(x)$

(c) $D\sin^5(2x)$ Solution $10\sin^4(2x)\cos(2x)$

(d) $D\frac{1}{\sin^2(3x)}$ Solution $-6(\sin(3x))^{-3}\cos(3x)$

(e) $D\sqrt{\cos(4x)}$ Solution $-2(\cos(4x))^{-1/2}\sin(4x)$

(e) $D\sqrt{\cos(4x)}$ Solution $-2(\cos(4x))^{-1/2}\sin(4x)$

4. a) $D(\sin(x))^7 = 7(\sin(x))^6 \cdot D\sin(x)$

$= 7(\sin(x))^6\cos(x)$

b) $D(\cos(x))^3 = -3(\cos(x))^2\sin(x)$

$D\,2x^1 = 2\cdot 1\cdot \underbrace{x^0}_{=1} = 2$

c) $D(\sin(2x))^5 = 5(\sin(2x))^4 \cdot \underbrace{D\sin(2x)}_{=\cos(2x)\cdot \underbrace{D(2x)}_{=2}}$

$= 10(\sin(2x))^4\cos(2x)$

d) $D(\sin(3x))^{-2} = -2(\sin(3x))^{-3}\cdot\cos(3x)\cdot 3$

e) $D\cos(4x)^{\frac{1}{2}} = \frac{1}{2}(\cos(4x))^{-\frac{1}{2}}\cdot(-\sin(4x))\cdot 4$

$= \frac{-2\sin(4x)}{\sqrt{\cos(4x)}}$

Kahden funktion tulon derivaatan voi laskea kaavalla[1]

$$D(fg) = gDf + fDg.$$

Kahden funktion osamäärän derivaatan voi laskea kaavalla[2]

$$D\frac{f}{g} = \frac{gDf - fDg}{g^2}.$$

Esimerkki. $Dx^3\sin(5x) = 3x^2\cdot\sin(5x) + x^3\cdot 5\cos(5x)$

5. Derivoi

(a) $Dx^7\cos(2x)$ Solution $7x^6\cos(2x) - 2x^7\sin(2x)$

(b) Dx^3e^x Solution $3x^2e^x + x^3e^x$

(c) $Dx^5\ln(x)$ Solution $5x^4\ln(x) + x^4$

(d) $D\sin(x)\ln(x)$ Solution $\cos(x)\ln(x) + \frac{\sin(x)}{x}$

(e) $De^{2x}\cos(3x)$ Solution $2e^{2x}\cos(3x) - 3e^{2x}\sin(3x)$

5.a) $D\,x^7\cos(2x)$

5.ⓐ $D\, x^7 \cos(2x)$

$= 7x^6 \cos(2x) + x^7(-\sin(2x))\cdot 2$

ⓑ $D\, x^3 e^x$

$= 3x^2 e^x + x^3 e^x$

$\boxed{D\ln(x) = \frac{1}{x}}$

ⓒ $D\, x^5 \ln(x)$

$= 5x^4 \ln(x) + x^{\cancel{5}\,4} \cdot \frac{1}{\cancel{x}} = 5x^4 \ln(x) + x^4$

ⓓ $D \sin(x) \ln(x)$

$= \cos(x)\ln(x) + \frac{\sin(x)}{x}$

ⓔ $D\, e^{2x} \cos(3x)$

$= 2e^{2x}\cos(3x) + e^{2x}(-\sin(3x))\cdot 3$

6. Derivoi

(a) $D\dfrac{x^7}{\cos(2x)}$ Solution $\dfrac{7x^6\cos(2x) + 2x^7\sin(2x)}{\cos^2(2x)}$

(b) $D\dfrac{x^3}{e^x}$ Solution $\dfrac{3x^2e^x - x^3e^x}{e^{2x}}$

(c) $D\dfrac{x^5}{\ln(x)}$ Solution $\dfrac{5x^4\ln(x) - x^4}{\ln(x)^2}$

(d) $D\dfrac{\sin(x)}{\ln(x)}$ Solution $\dfrac{\cos(x)\ln(x) - \frac{\sin(x)}{x}}{\ln(x)^2}$

(e) $D\dfrac{e^{2x}}{\cos(3x)}$ Solution $\dfrac{2e^{2x}\cos(3x) + 3e^{2x}\sin(3x)}{\cos^2(3x)}$

$\cdot\, \underbrace{D3x}_{=3}$

$D\, e^{2x} = 2e^{2x}\cos(3x) - e^{2x}(\sin(3x))\cdot 3$

$$D\,\frac{e^{2x}}{\cos(3x)} = \frac{2e^{2x}\cos(3x) - e^{2x}(-\sin(3x))\cdot 3}{(\cos(3x))^2}$$

7. Laske $D\tan(x) = D\dfrac{\sin(x)}{\cos(x)}$. Solution $\dfrac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$

8. Laske $D\cot(x) = D\dfrac{\cos(x)}{\sin(x)}$. Solution $\dfrac{-\cos^2(x)-\sin^2(x)}{\cos^2(x)} = -\frac{1}{\sin^2(x)}$

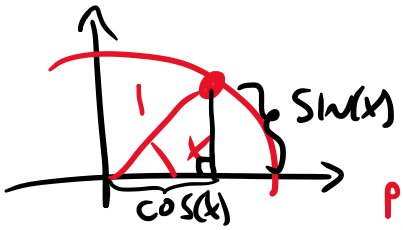
9. Laske $D\ln(\sin(2x))$. Solution $\dfrac{2\cos(2x)}{\sin(2x)}$

10. Laske $D\ln(2\cos(2x))$. Solution $\dfrac{-\sin(2x)}{\cos(2x)}$

$$D\sqrt{x} = D\,x^{\frac{1}{2}}$$

$$\boxed{D\,x^n = n\,x^{n-1}}$$

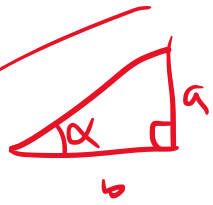
7. $D\tan(x) = D\dfrac{\sin(x)}{\cos(x)}$



$$= \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{(\cos(x))^2}$$

PYTHAGORAS

$$\frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1^2}{\cos^2(x)} = \frac{1}{\cos(x)^2}$$



$$\tan(\alpha) = \frac{a}{b}$$

$$\cot(\alpha) = \frac{b}{a}$$

$$\boxed{D\tan(x) = \frac{1}{\cos(x)^2}}$$

$$\boxed{D\cot(x) = -\frac{1}{\sin(x)^2}}$$

KOTANGENTTI

8. $D\cot(x) = D\dfrac{\cos(x)}{\sin(x)}$

TANGENTTI

$$= \frac{-\sin(x)\cdot\sin(x) - \cos(x)\cdot\cos(x)}{}$$

$$= \frac{-\sin(x)\cdot\sin(x) - \cos(x)\cdot\cos(x)}{\sin^2(x)}$$

$$= -\frac{\sin^2(x)+\cos^2(x)}{\sin^2(x)} = -\frac{1}{\sin^2(x)}$$

7. Laske $D\tan(x) = D\dfrac{\sin(x)}{\cos(x)}$. Solution $\dfrac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$

8. Laske $D\cot(x) = D\dfrac{\cos(x)}{\sin(x)}$. Solution $\dfrac{-\cos^2(x)-\sin^2(x)}{\cos^2(x)} = -\frac{1}{\sin^2(x)}$

9. Laske $D\ln(\sin(2x))$. Solution $\dfrac{2\cos(2x)}{\sin(2x)}$

10. Laske $D\ln(2\cos(2x))$. Solution $\dfrac{-\sin(2x)}{\cos(2x)}$

9. $D\ln(\sin(2x))$

$$= \frac{2\cos(2x)}{\sin(2x)}$$

$$D\ln(x) = \frac{1}{x}\,D$$

$$D\ln(x^7) = \frac{7x^6}{x^7}$$

$$= \frac{7}{x}$$

10. $D\ln(2\cos(2x))$

$$= \frac{D\,\cancel{2}\cos(2x)}{\cancel{2}\cos(2x)} = \frac{-\sin(2x)\cdot 2}{\cos(2x)}$$

11. Ison kondensaattorin[4] (kapasitanssi $C = 0.001$ F) napojen välinen jännite on

$$u(t) = 15e^{-t} \text{ mV}.$$

Laske kondensaattorin läpi kulkeva virta hetkellä $t = 1$ s. Solution $0.015e^{-1} \approx 0.0055$ mA

12. Vaihtovirtapiirissä on virta $i(t) = 5\sin(5000\cdot 2\pi t)$ mA. Virta kulkee käämin (induktanssi $L = 0.0002$ H) läpi. Laske käämin napojen välinen maksimijännite olettaen, että käämin resistanssi on nolla. Solution 10 V

13. Laske integraalit

(a) $\int \sin(x)(\cos(x))^3 dx$ Solution $-\frac{1}{4}(\cos(x))^4$

(b) $\int \frac{\cos(x)}{(\sin(x))^3}$ Solution $\frac{1}{-2}\frac{1}{(\sin(x))^2}$

(c) $\int \frac{\cos(x)}{\sin(x)} dx$ Solution $\ln(\sin(x))$

$n = 3$

13. (a) $-\int \underbrace{-\sin(x)}_{f'(x)} \underbrace{(\cos(x))}_{f(x)}^3 dx$

$f'(x) = -\sin(x)$

$$\int f'(x) f(x)^n dx = \frac{f(x)^{n+1}}{n+1}$$

$n \neq -1$

$$= -\frac{\cos(x)^4}{4}$$

$$\int \frac{\cos(x)}{\sin(x)^3} dx = \int \underbrace{\cos(x)}_{f'(x)} \underbrace{\sin(x)}_{f(x)}^{-3} dx$$

$$= \frac{\sin(x)^{-2}}{-2} = -\frac{1}{2\sin(x)^2}$$

$$\int \frac{\overbrace{\cos(x)}^{f'(x)}}{\underbrace{\sin(x)}_{f(x)}} dx = \ln(\sin(x))$$

$$\int \frac{f'(x)}{f(x)} dx = \ln(f(x))$$

14. Laske integraalit

(a) $\int (\cos(x))^3(\sin(x))^7 dx$ Hint. Sijoita $(\cos(x))^3 = \cos(x) - \cos(x)(\sin(x))^2$. Solution $\frac{\sin^8(x)}{8} - \frac{\sin^{10}(x)}{10}$

(b) $\int \tan(x) dx$ Hint. Sijoita $\tan(x) = \frac{\sin(x)}{\cos(x)}$. Solution $-\ln(\cos(x))$

(c) $\int \sin(x)\sqrt{\cos(x)} dx$ Hint. Muista $\sqrt{a} = a^{1/2}$. Solution $-\cos(x)^{3/2}$

(d) $\int 2\sin(7x)\sin(3x) dx$ Hint. Käytä kaavaa $2\sin(a)\sin(b) = \cos(a-b) - \cos(a+b)$ Solution $-\cos(4x)/4 + \cos(10x)/10$

14. (a) $\int (\cos(x) - \cos(x)\sin(x)^2)\sin(x)^7$

$$= \int \underbrace{\cos(x)}_{f'(x)} \underbrace{(\sin(x))}_{f(x)}^7 - \cos(x)\sin(x)^9 \, dx$$

$$= \int \underbrace{\cos(x)}_{f'(x)} \underbrace{(\sin(x))^9}_{f(x)} - \cos(x)\sin(x)^{10}\, dx$$

$$= \frac{\sin(x)^8}{8} - \frac{\sin(x)^{10}}{10}$$

14. (b) $\int \tan(x)\, dx$

$$\boxed{\int \sqrt{x}\, dx = \int x^{\frac{1}{2}}\, dx}$$

$$= -\int \frac{-\sin(x)}{\underbrace{\cos(x)}_{f(x)}}\, dx \qquad f'(x) = -\sin(x)$$

$$= -\ln(\cos(x))$$

TARKISTUS. $D\ -\ln(\cos(x)) = -\dfrac{-\sin(x)}{\cos(x)}$ ok

(c) $\int \sin(x)\sqrt{\cos(x)}\, dx$

$$= -\int -\sin(x)\cos(x)^{\frac{1}{2}}\, dx = -\cos(x)^{\frac{3}{2}}$$

$$= -\cos(x)\sqrt{\cos(x)}$$

(d) $\int 2\sin(7x)\sin(3x)\, dx$

$$= \int \cos(7x-3x) - \cos(7x+3x)\, dx$$

$$= \int \cos(4x) - \cos(10x)\, dx$$

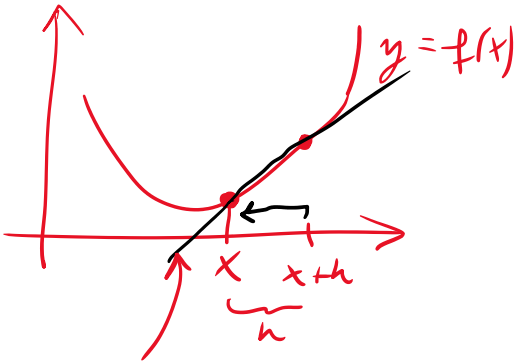
$$= \frac{\sin(4x)}{4} - \frac{\sin(10x)}{10}$$

6 numeerisia menetelmiä

tiistai 16. huhtikuuta 2024 19.16

DERIVAATAN MÄÄRITELMÄ

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$



$$k = \frac{y\text{:n muutos}}{x\text{:n muutos}} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

KURSSIN DERIVOINTI-KAAVAT VOI TODISTAA TÄMÄN AVULLA

TÄLLÄ VIIKOLLA ANNETAAN h:N OLLA JOKIN LUKU, ESIM. $h = 0{,}01$

$$f'(x) \approx \underbrace{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}_{\text{LIKIARVO DERIVAATALLE}}$$

ESIMERKKI. OLKOON $f(x) = xe^{2x}$.

LASKE $f'(1)$

ⓐ KÄSIN TARKASTI

ⓑ LIKIARVONA KAAVALLA

RATKAISU. ⓐ $f(x) = x e^{2x}$ $\quad \boxed{D(fg) = f'g + fg'}$

$$f'(x) = 1 \cdot e^{2x} + x \cdot e^{2x} \underbrace{D(2x)}_{=2}$$

$$f'(x) = e^{2x}(1 + 2x)$$

$$\boxed{f(x) = x e^{2x}}$$

$$f'(1) = e^{2 \cdot 1}(\underbrace{1 + 2 \cdot 1}_{3}) = 3e^2 \approx 22{,}167$$

TARKKA DERIVAATAN ARVO

ⓑ $$f'(x) \approx \frac{f(\overbrace{x+h}^{1{,}01}) - f(\overbrace{x}^{=1})}{h}$$

$h = 0{,}01$ (VALITAAN ESIM. NÄIN)

h

ESIM. NÄIN)

$$= \frac{1.01\,e^{2\cdot 1.01} - 1\cdot e^{2\cdot 1}}{0.01}$$

x = TARKASTELUPISTE $= 1$

$x+h = 1.01$

≈ 22.46

MELKO LÄHELLÄ ARVOA 22,167

(1.01*exp(2*1.01)-1*exp(2*1))/0.01

NATURAL LANGUAGE | MATH INPUT

Input

$$\frac{1.01\exp(2\times 1.01) - 1\exp(2\times 1)}{0.01}$$

Result

22.4652...

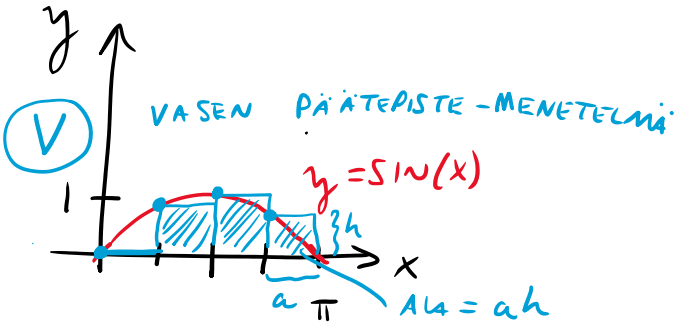
(SUHTEELLINEN VIRHE

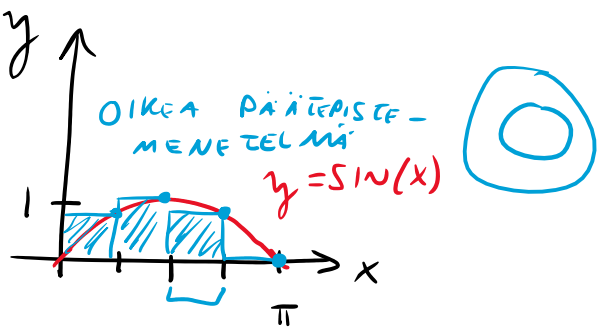
$$\frac{22.46 - 22.167}{22.167} = 0{,}013 \approx 1.3\%)$$

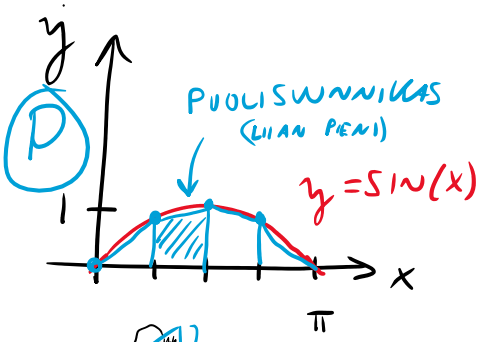
NUMEERISTA INTEGROINTIA

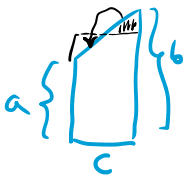
IDEA: INTEGRAALI ON JOKIN PINTA-ALA

→ ARVIOIDAAN PINTA-ALAA JOLLAKIN KUVIOLLA

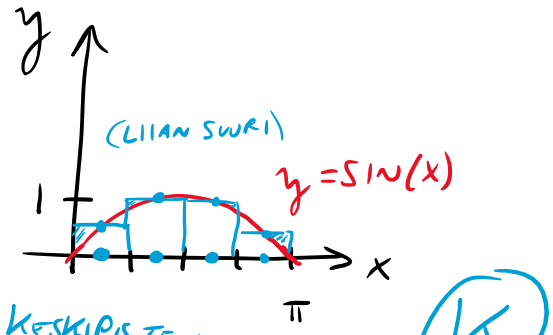








$$\text{ALA} = \frac{a+b}{2}\cdot c$$



KESKIPISTEMENETELMÄ K

NYKYÄÄN TIETOKONEET OVAT TEHOKKAITA => MIKÄ TAHANSA

V, O, P, K

TOIMII HYVIN.

(KÄSIN LASKIESSA?)

SIMPSONIN MENETELMÄ

$$\frac{1 \cdot P + 2K}{1+2}$$

PAINOTETTU MENETELMISTÄ P & K.

KESKIARVO

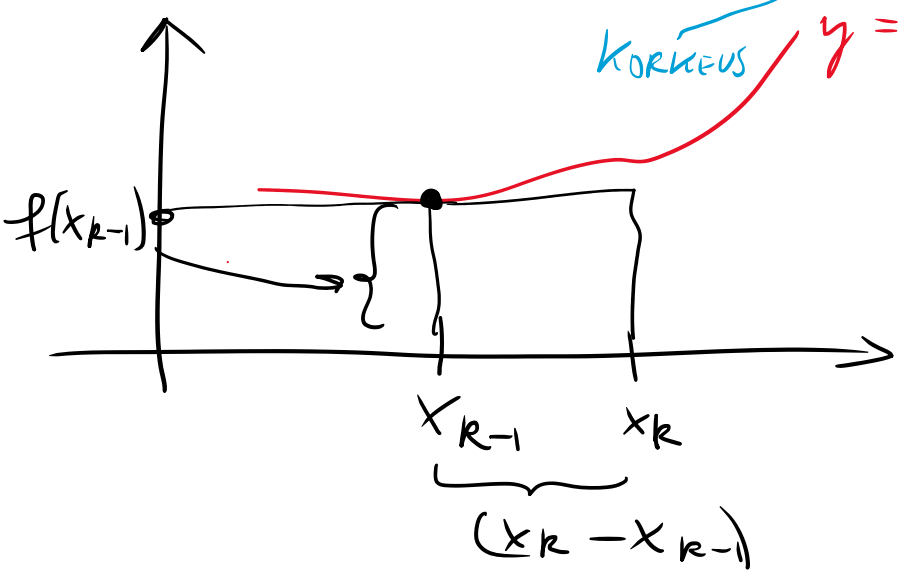
Integraalia voidaan arvioida vasemman päätepisteen menetelmällä

$$\int_a^b f(x)dx \approx V = \sum_{k=1}^{N} f(x_{k-1})(x_k - x_{k-1})$$

SUORAKULMIOIDEN PINTA-ALOJEN SUMMA

KORKEUS

KANTA



ESIM. $\int^1 x^2 dx$

ESIM. $\int_0^1 x^2\,dx$

$$= \left.\frac{x^3}{3}\right|_0^1 = \frac{1^3-0^3}{3} = \frac{1}{3} = 0{,}333\ldots$$

NUMEERISESTI

OK LIKIARVO

sum k=1 to k=100 x^2*w where x=a+(b-a)*k/100 and w=(b-a)/100 and a=0 and b=1

NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EXAMPLES

Input interpretation

$$\sum_{k=1}^{100} x^2 w \text{ where } x = a + (b-a)\times\frac{k}{100},\ w = \frac{b-a}{100},\ a = 0,\ b = 1$$

Result

$$\frac{6767}{20000} \approx 0{,}33835$$

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POWERED BY T

```
octave:9> format long
a=0;
b=1;
N=100;
dx=(b-a)/N;
s=0;
for k=1:100
x=a+(b-a)*k/N;
s=s+x^2*dx;
end
s
s = 0.338350000000000
```

HYVÄ TAPA LASKEA ARDUINOSSA

integrate from x=0 to x=1 x^2

NATURAL LANGUAGE | MATH INPUT

Definite integral

$$\int_0^1 x^2\,dx = \frac{1}{3} \approx 0.33333$$

DERIVAATAN SOVELLUS: TAYLORIN SARJA / TAYLORIN POLYNOMI

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

SILEÄ / DERIVOITUVA

$\boxed{a=0}$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$3! = 1\cdot 2\cdot 3 = 6$

$4! = 1\cdot 2\cdot 3\cdot 4 = 24$

$1! = 1$

$0! = 1$

SARJOJA LÖYTYY TAULUKKOKIRJASTA / WIKIPEDIASTA

ESIM.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

SAMAT

$$D\, e^x = 0 + 1 + \frac{\cancel{2}x}{1\cdot\cancel{2}} + \frac{\cancel{3}x^2}{1\cdot 2\cdot\cancel{3}} + \frac{\cancel{4}x^3}{1\cdot 2\cdot 3\cdot\cancel{4}} + \dots$$

SAMAT

$1 \quad x \quad \frac{x^2}{2} \quad \frac{x^3}{3!}$

$\boxed{D\, e^x = e^x}$

HYÖTY?

KATKAISTAAN $e^x \approx \underbrace{1 + x + \frac{x^2}{2} + \frac{x^3}{6}}_{\text{TAYLORIN POLYNOMI}}$, KUN $x \approx 0$

ESIM. $e^2 = 1 + 2 + \frac{2^2}{2} + \frac{2^3}{6}$

$$= 1 + 2 + 2 + \underbrace{\frac{8}{6}}_{1 + \frac{2}{6}}$$

$$= 6 + \frac{1}{3}$$

$$\approx \underline{\underline{6.33}}$$

$\frac{1}{3}$

LASKIN

$e^2 \approx \underline{\underline{7.38}}$

EI IHAN HUONO

ESIM. INTEGRAALI, JOTA EI VOIDA LASKEA TARKASTI KÄSIN

← PUUTTUU

$$\int_0^1 e^{x^2}\,dx$$

$$\int \underbrace{2x}_{g'(x)}\, \underbrace{e^{x^2}}_{f(g(x))}\,dx = e^{x^2}$$

$$e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}$$

$$\approx \int_0^1 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}\,dx$$

$$= \Big/_0^1 x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42}$$

$$= 1 + \frac{1}{3} + \frac{1}{10} + \frac{1}{42}$$

$$\approx \underline{\underline{1.457}}$$

LASKIN

$\underline{\underline{1.462}}$

OK

VEKTORIT

JA

KOMPLEKSILUVUT

$i = \sqrt{-1}$

$i^2 = -1$

KOMPLEKSILUVUT

$i^2 = -1$

$$e^{ix} = \cos(x) + i\sin(x)$$

PERUSTELU TAYLORIN SARJALLA (ESIM.)

1. (Viikolta 2) Olkoon $f(x) = x^2 - 1$. Laske numeerisesti

(a) (ohi $f'(x)$)	Solution $2x$
(b) $f'(-2)$	Solution -4
(c) $f'(-1)$	Solution -2
(d) $f'(0)$	Solution 0
(e) $f'(1)$	Solution 2
(f) $f'(2)$	Solution 4

(b) $f'(-2) = ?$

OHJE

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

x on tarkastelupiste ja $h = 0.01$.

$$\begin{cases} x = -2 \\ h = 0{,}01 \\ x + h = -2 + 0{,}01 = -1{,}99 \end{cases}$$

(MITÄ PIENEMPI, SITÄ TARKEMPI ARVIO)

$f(x) = (x)^2 - 1$

$$f'(-2) \approx \frac{f(-1{,}99) - f(-2)}{0{,}01} = \frac{((-1{,}99)^2 - 1) - ((-2)^2 - 1)}{0{,}01}$$

(((-1.99)^2-1) -((-2)^2-1))/0.01

NATURAL LANGUAGE | MATH INPUT

Input

$$\frac{((-1.99)^2-1)-((-2)^2-1)}{0.01}$$

Result

-3.99

$$= \underline{\underline{-3{,}99}}$$

$h = 0$

$$f'(x) \approx \frac{f(x+h)-f(x)}{h} = \frac{f(x+0)-f(x)}{0}$$

x on tarkastelupiste ja $h = 0.01$.

$$= \frac{f(x)-f(x)}{0} = \frac{0}{0}$$

MÄÄRITTELEMÄTÖN

(c)

$f'(-1) = ?$

$$\begin{cases} x = -1 \\ h = 0{,}01 \\ x+h = -0{,}99 \end{cases}$$

$$f'(-1) \approx \frac{f(x+h)-f(x)}{h}$$

$$= \frac{f(-0{,}99)-f(-1)}{0{,}01}$$

$$\boxed{f(x) = x^2 - 1}$$

$$= \frac{((-0{,}99)^2-1)-((-1)^2-1)}{0{,}01}$$

$$= \underline{\underline{-1{,}99}}$$

(d) $f'(0) \approx 0{,}01$

(e) $f'(1) \approx 2{,}01$

(f) $f'(2) \approx 4{,}01$

5. Laske integraalit.

(a) $\int_0^{\pi} \sin(x)dx$ Solution 2

(b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x)dx$ Solution 0

(c) $\int_0^{2\pi} \sin(x)dx$ Solution 0

① KERTAUS : LASKUT KÄSIN

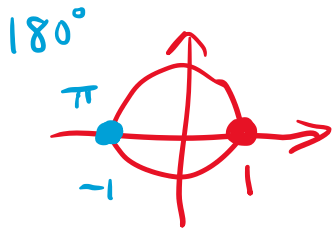
② TÄMÄN VIIKON VARSINAINEN ASIA : NUMEERINEN LASKU

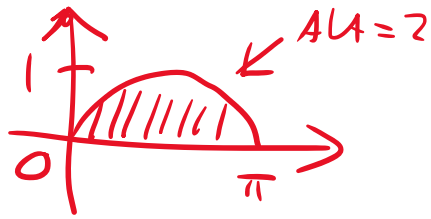
ⓐ $\int_0^{\pi} \sin(x)\,dx = \Big/_0^{\pi} -\cos(x) = -\underbrace{\cos(\pi)}_{=-1} - \left(-\underbrace{\cos(0)}_{=1}\right)$

$= -(-1) - (-1)$

$= 1 + 1 = \underline{\underline{2}}$

$\begin{cases} D\sin(x) = \cos(x) \\ D\cos(x) = -\sin(x) \end{cases}$





② NUMEERISESTI

ⓐ $\int_0^{\pi} \sin(x)\,dx = \underline{\underline{1.9998355\ldots}}$

OCTAVE

OK

TARKKA ARVO ON 2.

```
octave:1> format long
a=0;
b=pi;
N=100;
dx=(b-a)/N;
s=0;
for k=1:100
x=a+(b-a)*k/N;
s=s+sin(x)*dx;
end
s
s = 1.999835503887444
```

5. Laske integraalit.

(a) $\int_0^{\pi} \sin(x)dx$ — Solution 2

(b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x)dx$ $= 1,9998355...$ — Solution ~~0~~ 2

(c) $\int_0^{2\pi} \sin(x)dx$ $-2,49 \cdot 10^{-17} = 0$ — Solution 0

```
octave:18> format long
a=-pi/2;
b=pi/2;
N=100;
dx=(b-a)/N;
s=0;
for k=1:100
x=a+(b-a)*k/N;
s=s+cos(x)*dx;
end
s
s = 1.999835503887444
```

```
octave:28> format long
a=0;
b=2*pi;
N=100;
dx=(b-a)/N;
s=0;
for k=1:100
x=a+(b-a)*k/N;
s=s+sin(x)*dx;
end
s
s = -2.493034466764676e-17
```

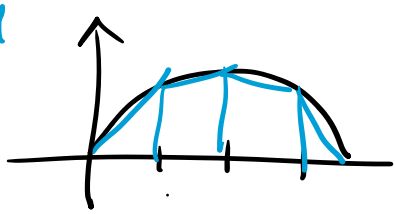
$= -2,49 \cdot 10^{-17} = 0$

HARJOITELLAAN OCTAVEN KÄYTTÖÄ:

① INTEGROINTIA

```
format long
a=0;
b=2*pi;
N=100;
dx=(b-a)/N;
s=0;
for k=1:100
x=a+(b-a)*k/N;
s=s+sin(x)*dx;
end
s
```

4



② MUUTA MEILLE TUTTUA

$$\begin{cases} ax + by = A \\ cx + dy = B \end{cases}$$

$$\begin{cases} x \\ y \end{cases}$$

$2^{x+3} = 5^{3x} \rightarrow \boxed{x = ?}$

$$\begin{cases} 6x + 5y &= 4 \\ x + 2y &= 2 \quad \| \cdot (-6) \end{cases}$$

MAT 1 – RATKAISU

$\begin{cases} 6x + 5y = 4 \end{cases}$

$$\begin{cases} 6x + 5y = 4 \\ -6x - 12y = -12 \end{cases} \quad +$$

$$0 \cdot x - 7y = -8$$

$$-7y = -8 \quad \| : (-7)$$

$$y = \frac{8}{7}$$

$$6x + 5y = 4$$

$$6x = 4 - 5y \quad \| : 6$$

$$x = \frac{2}{3} - \frac{5}{6} y$$

$$x = \frac{2}{3} - \frac{5}{\cancel{6}_{3}} \cdot \frac{\cancel{8}^{4}}{7}$$

$$= \frac{2}{3} - \frac{20}{21}$$

$$= \frac{14}{21} - \frac{20}{21} = \frac{-6}{21}$$

$$= -\frac{2}{7}$$

$$\begin{cases} x = -\frac{2}{7} \\ y = \frac{8}{7} \end{cases}$$

$$\begin{cases} 6x + 5y = 4 \\ 1 \cdot x + 2y = 2 \end{cases}$$

```
octave:69> A=[6,5; 1,2];
b=[4; 2];
x=A\b
x =

   -2/7
    8/7
```

$$\begin{cases} x = -\frac{2}{7} \\ y = \frac{8}{7} \end{cases}$$

$$\underbrace{\begin{pmatrix} 6 & 5 \\ 1 & 2 \end{pmatrix}}_{\text{MATRIISI } A} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{\text{TUNTEMATON VEKTORI } \bar{x}} = \underbrace{\begin{pmatrix} 4 \\ 2 \end{pmatrix}}_{\bar{b}}$$

$$A\bar{x} = \bar{b} \quad \| A \backslash$$

$$\underbrace{A \backslash A}_{\text{HÄVIÄÄ}} \bar{x} = A \backslash \bar{b}$$

$$\bar{x} = A \backslash \bar{b}$$

OCTAVELLA SIIS

$$A = \begin{pmatrix} 6 & 5 \\ 1 & 2 \end{pmatrix}$$

OCTAVELLA SIIS

$$A = \begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}$$

$$b = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$x = A \backslash b = \ldots$$

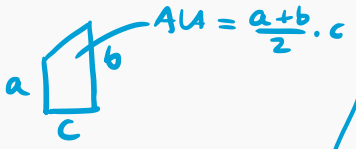
ESIM. LASKETAAN ARVIO INTEGRAALILLE

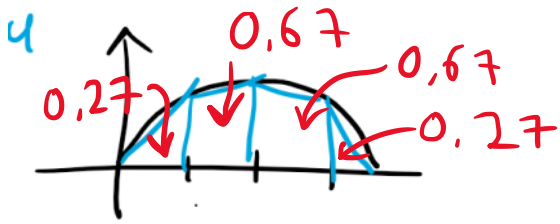
$$\int_0^{\pi} \sin(x)\, dx = 2$$

PUOLISUUNNIKASMENETELMÄLLÄ

NELJÄLLÄ OSAVÄLILLÄ

```
format long
a=0;
b=pi;
N=4;
dx=(b-a)/N;
s=0;
for k=1:N
x=a+(b-a)*k/N;
puoliSuunnikas=((sin(x)+sin(x-dx))/2)*dx
s=s+puoliSuunnikas
end
s
```





OCTAVEN TULOSTUS

```
puoliSuunnikas = 0.277680183634898
s = 0.277680183634898
puoliSuunnikas = 0.670379265333622
s = 0.948059448968520
puoliSuunnikas = 0.670379265333622
s = 1.618438714302142
puoliSuunnikas = 0.277680183634898
s = 1.896118897937040
s = 1.896118897937040
```

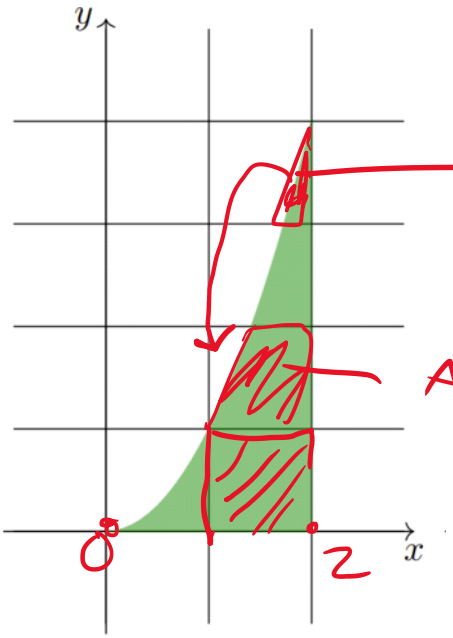
ARVIO = 1,89 ⟷ 2

ESIM. SAMA LASKU SIMPSONIN MENETELMÄLLÄ:

```
octave:1> format long
a=0;
b=pi;
N=4;
dx=(b-a)/N;
PS=0; %puolisuunnikasmenetelmä
KP=0; %keskipistemenetelmä
for k=1:N
x=a+(b-a)*k/N;
PS=PS+((sin(x)+sin(x-dx))/2)*dx;
keskipiste=x-dx/2; %puolikas siirtymä takaisinpäin
KP=KP+sin(keskipiste)*dx;
end
Simpson=(PS+2*KP)/3
Simpson = 2.000269169948388
```

TOSI LÄHELLÄ

TARKKA ARVO ON 2



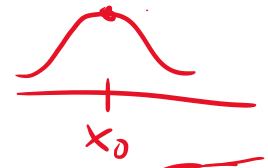
$D\,x^n = n x^{n-1}$

$\int x^2 dx = \frac{x^3}{3}$

$y = x^2$

$$\text{ALA} = \int_0^2 x^2 dx = \Big/_0^2 \frac{x^3}{3} = \frac{2^3 - 0^3}{3} = \frac{8}{3} = 2.33$$

$f'(x_0) = 0$



JYRKIN KOHTA

$f''(x_0) = 0$

$D\,x^7 = 7x^6$ $\qquad \int x^2 dx = \frac{x^3}{3}$

$D\,e^x = e^x$ $\qquad \int e^x dx = e^x$

~~$D\,2^x = 2^x \ln(2)$~~ $\qquad$ ~~$\int 2^x dx = \frac{2^x}{\ln(2)}$~~

$D\,e^{2x} = e^{2x} \underbrace{D(2x)}_{=2}$ $\qquad \int e^{2x} dx = \frac{e^{2x}}{}$

$$D\, e^{2x} = e^{2x}\, \underbrace{D(2x)}_{=2} = 2e^{2x} \qquad \int e^{2x}\,dx = \frac{e^{2x}}{2}$$

$$D \ln(x) = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \ln(x)$$

$$\cancel{D \log_7(x) = D\,\frac{\ln(x)}{\ln(7)} = \frac{1}{\ln(7)\,x}}$$

$$\int \frac{1}{2x}\,dx = \int \frac{1}{2}\cdot\frac{1}{x}\,dx = \frac{1}{2}\int \frac{1}{x}\,dx = \frac{1}{2}\ln(x)$$

ESIM.

$$D\, x^7 = 7x^6$$

$$D\, 7^x = 7^x \ln(7)$$

$$D\, x^x = D e^{\ln(x^x)} = D e^{x\ln(x)} = \underbrace{e^{x\ln(x)}}_{x^x} = x^x(\ln(x)+1)$$

$$D\, x\ln(x) = 1\cdot\ln(x) + x\cdot\frac{1}{x} = \ln(x)+1$$

$$a^b = e^{\ln(a^b)} = e^{b\ln(a)}$$

KANTALUVUKSI SAA AINA e:N

$$\boxed{\begin{aligned} D\sin(x) &= \cos(x) \\ D\cos(x) &= -\sin(x) \end{aligned}}$$

HARJOITELTIIN $(fg)' = f'g + fg'$

HARJOITELTIIN NYÖS

$$(fg)' = f'g + fg'$$

$$D\,\frac{f}{g} = \frac{f'g - fg'}{g^2}$$

$$D\,f(g(x)) = f'(g(x))\,g'(x)$$

ESIM. $D\,e^{\sin(x^3)} = e^{\sin(x^3)} \cdot \underbrace{D\sin(x^3)}_{= \cos(x^3)\cdot \underbrace{Dx^3}_{=3x^2}}$

$$= e^{\sin(x^3)}\cos(x^3)\cdot 3x^2$$

$$\boxed{\begin{aligned} \int \sin(3x)\,dx &= \frac{-\cos(3x)}{3} \\ \int \cos(5x)\,dx &= \frac{\sin(5x)}{5} \\ \int 3x^2 \sin(x^3)\,dx &= -\cos(x^3) \end{aligned}}$$

$$D\,f(g(x)) = f'(g(x))\,g'(x) \Rightarrow \boxed{\int f'(g(x))\,g'(x)\,dx = f(g(x))}$$

ok

$\rightarrow f'(x) = \cos(x)$

$$\boxed{\int \cos(x)\cdot \overbrace{\sin(x)}^{f(x)}{}^7\,dx = \frac{\sin(x)^8}{8}}$$

$$\boxed{\int f'(x)\,f(x)^n\,dx = \frac{f(x)^{n+1}}{n+1}, \quad n \neq -1}$$

$$\int \cos(x)^{\cancel{3}\,1+2}\,\sin(x)^7\,dx$$

$$\sin(x)^2 + \cos(x)^2 = 1$$

$$\cos^2(x) = 1 - \sin(x)^2$$

$$\int \cos(x)^3 \sin(x)^7 \, dx \qquad \cos^2(x) = 1 - \sin(x)^2$$

$$= \int \cos(x)\left(1 - \sin(x)^2\right)\sin(x)^7 \, dx$$

$$= \int \cos(x)\sin(x)^7 \, dx - \int \cos(x)\sin(x)^9 \, dx$$

$$= \frac{\sin(x)^8}{8} - \frac{\sin(x)^{10}}{10}$$

$$2\sin(a)\sin(b) = \cos(a-b) - \cos(a+b)$$

$$\frac{1}{2}\int 2\sin(7x)\sin(3x) \, dx$$

$$= \frac{1}{2}\int \cos(4x) - \cos(10x) \, dx$$

$$= \frac{\sin(4x)}{2 \cdot 4} - \frac{\sin(10x)}{2 \cdot 10}$$

$$-\int \frac{-\sin(x)}{\cos(x)} \, dx = -\int \frac{f'(x)}{f(x)} \, dx = -\ln(f(x))$$

$$= -\ln(\cos(x))$$

$$\int \frac{1}{x} \, dx = \ln(x)$$

$$D\ln(f(x)) = \frac{f'(x)}{f(x)}$$

$$\boxed{\int \frac{f'(x)}{f(x)} \, dx = \ln(f(x))}$$