Words

maanantai 11. maaliskuuta 2024 11.57

WORDS

DICE

KRUUNA

KLAAVA

= VAWE

TAIL

HEAD

Examples

maanantai 11. maaliskuuta 2024 12.14

EXAMPLE. YOU FLIP A COIN.

WITH WHAT $\underbrace{\text{PROBABILITY}}_{P}$ YOU GET A HEAD?

SOLUTION. $50\% = \frac{50}{100} = \frac{1}{2} = \frac{\text{1 DESIRED OPTION}}{\text{2 OPTIONS IN TOTAL}}$

EXAMPLE. YOU ROLL A DICE.

$$P(\text{WE GET 2 OR 3}) = ?$$

SOLUTION. $\frac{2}{6} = \frac{\text{DESIRED}}{\text{ALL}} = \frac{1}{3} = 33\%$

EXAMPLE. YOU ~~THROW~~ FLIP A COIN 10 TIMES.

WHAT IS THE EXPECTED NUMBER OF HEADS?

SOLUTION. $5 = \underbrace{10}_{\text{NUMBER OF TRIALS}} \cdot \underbrace{\frac{1}{2}}_{P}$

EXPERIMENT.

WE GOT 5 HEADS / 10 THROWS

IF WE CANNOT CALCULATE, WE CAN ALSO DO A SIMULATION ON COMPUTER.

SOMETIMES VISUALIZATION HELPS

4. Let's throw two dice. With what probability the sum is

(a) 1

(b) 5

(c) 11

(d) larger than 7?

Solution 0

Solution 0.11

Solution 0.056

Solution 0.42

$P = \frac{0}{36} = 0$

CALCULATOR

$\frac{4}{36} = \frac{1}{9} = 0{,}11$

$\frac{2}{36} = \frac{1}{18} = 0{,}056$

$\frac{15}{36} = \frac{5}{12} = 0{,}42$

DICE 1

SUM

DICE 2

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

IN TOTAL >7

36 IN TOTAL

SOMETIMES
YOU CAN LIST ALL THE OPTIONS

5. Let's throw a coin three times. With what probability

(a) 4

(b) 3

(c) 2

(d) 1

(e) 0

heads are obtained?

1 = HEADS
0 = TAILS

0 0 0
0 0 1
0 1 0
1 0 0
0 1 1
1 0 1
1 1 0
1 1 1

Exercises

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1.1 Mostly single events

1. The height distribution of IT students is as follows.

length [m]	number of students	cumulative
1.51 ... 1.53	2	2
1.54 ... 1.56	2	4
1.57 ... 1.59	5	9
1.60 ... 1.62	38	47
1.63 ... 1.65	62	109
1.66 ... 1.68	110	219
1.69 ... 1.71	126	345
1.72 ... 1.74	130	475
1.75 ... 1.77	126	601
1.78 ... 1.80	72	673
1.81 ... 1.83	42	715
1.84 ... 1.86	23	738
1.87 ... 1.89	7	745
1.90 ... 1.92	1	746

With what probability a student has length

(a) greater than 180 cm Solution 0.098

(b) 163 ... 174 cm Solution 0.57

(c) less than 160 cm? Solution 0.012

2. Students estimated the length of a given segment. Their errors were [cm]

2	3	0	5	6	1	2	4	3	1	3	2
1	0	1	1	0	2	1	1	0	5	0	2
5	3	1	1	2	0	4	3	0	0	2	1
0	3	5	4	2	0	5	3	1	6	2	4
1	1	4	7	2	0	2	1	0	4	4	3

With what probability a random student estimated the length with at most 1 cm error? Solution 0.43

3. A car factory collected data; when a car had its first repair done

km	number of first repair cars	cumulative
0 ... 10 000	50	50
10 001 ... 20 000	93	143
20 001 ... 30 000	293	436
30 001 ... 40 000	391	827
40 001 ... 50 000	183	1010
50 001 ...	40	1050

With what probability a car from this factory needs its first repair when it has been driven

(a) at most 20 000km Solution 0.14

(b) 20 001 ... 30 000km Solution 0.28

(c) 30 001 ... 40 000km Solution 0.37

(d) over 40 000km Solution 0.21

(e) find the sum of probabilities a-d. Solution 1

(f) With what probability, a car which did not have repair during kilometers 0-30 000km has to have repair during next 10 000km? Solution 0.64

4. Let's throw two dice. With what probability the sum is

(a) 1 Solution 0

(b) 5 Solution 0.11

(c) 11 Solution 0.056

(d) larger than 7? Solution 0.42

5. Let's throw a coin three times. With what probability

(a) 4 Solution 0

(b) 3 Solution 1/8

(c) 2 Solution 3/8

(d) 1 Solution 3/8

(e) 0 Solution 1/8

heads are obtained?

6. Let's make a two digit number by choosing its digits randomly from 1, 2, 3, 4, and 5. The same digit can appear twice. With what probability the number is divisible by 2 or 5? Solution 0.6

7. Let's throw two dice. With what probability the sum is 10, 11 or 12? Solution 0.17

8. Grade 5 was obtained as follows. In the mathematics exam 15% of students, in the physics exam 12% of students and in both 7% of students. With what probability a random student gets grade 5 in at least one of these exams? Solution 0.2

9. In the pedestrian crossing, the lights are adjusted so that red light is on for 40 s and the green light is on for 20 s. With what probability a pedestrian has to wait max 30 s? Solution 0.83

Exercise 9

keskiviikko 13. maaliskuuta 2024 12.34

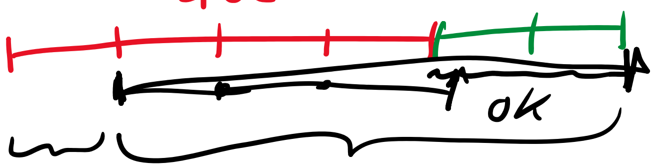
9. In the pedestrian crossing, the lights are adjusted so that red light is on for 40 s and the green light is on for 20 s. With what probability a pedestrian has to wait max 30 s? Solution 0.83

PICTURE

RED LIGHT 40S

GREEN 20S

60 TOTAL



10S NOT OK, NEED TO WAIT MORE THAN 30S

50S, OK

$P(\text{MAX } 30S)$

$= \frac{50}{60} = \frac{5}{6} = 0.8\overline{3}$

Exercise 1

keskiviikko 13. maaliskuuta 2024 12.52

1. The height distribution of IT students is as follows.

length [m]	number of students	cumulative
1.51 ... 1.53	2	2
1.54 ... 1.56	2	4
1.57 ... 1.59	5	9
1.60 ... 1.62	38	47
1.63 ... 1.65	62	109
1.66 ... 1.68	110	219
1.69 ... 1.71	126	345
1.72 ... 1.74	130	475
1.75 ... 1.77	126	601
1.78 ... 1.80	72	673
1.81 ... 1.83	42	715
1.84 ... 1.86	23	738
1.87 ... 1.89	7	745
1.90 ... 1.92	1	746

With what probability a student has length

(a) greater than 180 cm Solution 0.098

(b) 163 ... 174 cm Solution 0.57

(c) less than 160 cm? Solution 0.012

160 cm 161 cm 162 cm

38 IN TOTAL

~~626~~ 428

OK 73

TOTAL NUMBER OF STUDENTS

$P = \frac{73}{746}$

$P = \frac{428}{746} = \frac{475-47}{746}$

$P = \frac{9}{746}$

2. Students estimated the length of a given segment. Their errors were [cm]

2	3	0	5	6	1	2	4	3	1	3	2
1	0	1	1	0	2	0	1	0	5	0	2
5	3	1	1	2	0	4	3	0	0	2	1
0	3	5	4	2	0	5	3	1	6	2	4
1	1	4	7	2	0	2	1	0	4	4	3

With what probability a random student estimated the length with at most 1 cm error? Solution 0.43

TOTAL 60

ERROR 0 cm OK

ERROR 1 cm OK

OK FOR 26 STUDENTS

CALCULATOR

$\rightarrow P = \frac{26}{60} = 0.43$

3. A car factory collected data; when a car had its first repair done

km	number of first repair cars	cumulative
0 ... 10 000	50	50
10 001 ... 20 000	93	$\boxed{143}$
20 001 ... 30 000	293	436
30 001 ... 40 000	391	827
40 001 ... 50 000	183	1010
50 001 ...	40	1050

AT MOST 20000

OVER 40010

223

TOTAL

1

With what probability a car from this factory needs its first repair when it has been driven

(a) at most 20 000km — Solution 0.14

(b) 20 001 ... 30 000km — Solution 0.28

(c) 30 001 ... 40 000km — Solution 0.37

(d) over 40 000km — Solution 0.21

(e) find the sum of probabilities a-d. — Solution 1

(f) With what probability, a car which did not have repair during kilometers 0-30 000km has to have repair during next 10 000km? — Solution 0.64

$P_1 = \frac{143}{1050}$ $P_2 = \frac{293}{1050}$ $P_3 = \frac{391}{1050}$ $P_4 = \frac{183+40}{1050} = \frac{223}{1050}$

$P_1 + P_2 + P_3 + P_4 = 1$

$P = \frac{391}{391+183+40}$

5. Let's throw a coin three times. With what probability

(a) 4 — Solution 0

(b) 3 — Solution 1/8

(c) 2 — Solution 3/8

(d) 1 — Solution 3/8

(e) 0 — Solution 1/8

heads are obtained?

THIS NEVER HAPPENS, BECAUSE THERE ARE ONLY 3 COINS

$\boxed{\text{NEVER HAPPENS} \Rightarrow P=0}$

$P = 3/8$

1 { H H H

3 { H H T / H T H / T H H

"1 HEAD" 3 { H T T / T H T / T T H

"0 HEAD" 1 { T T T

$8 = 2^3$ TOTAL

H = HEAD

T = TAIL

Exercises 10-18

sunnuntai 17. maaliskuuta 2024 7.58

10. With what probability two picked playing cards are two aces? Solution 0.0045

$$P = \frac{4}{52} \cdot \frac{3}{51} \approx 0.0045$$

CALCULATOR

11. In a box, there are 6 red balls and 4 black balls. Let's pick up two balls without putting them back to the box. With what probability both of the balls are black? Solution 0.13

TOTAL 10 BALLS.

$$P = \frac{4}{10} \cdot \frac{3}{9} \approx 0.13$$

12. Let's throw a dice four times. With what probability

    (a) 2 number "2" Solution 0.12

    (b) 4 odd numbers Solution 0.0625

    (c) at least one "6" Solution 0.52

    are obtained?

DICE: 1, 2, 3, 4, 5, 6

(a) $P = 6 \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{5^2}{6^3} \approx 0.12$

EXAMPLE

1 3 1 5

2 6 5 4

X = NOT 2

6 OPTIONS

2 2 x x

2 x 2 x

2 x x 2

x 2 2 x

x 2 x 2

x x 2 2

2 NUMBER "2"

METHOD 2 BINOMIAL DISTRIBUTION

(DON'T WORRY WE WILL PRACTICE)

$\boxed{BIN(4 = n, \frac{1}{6} = p)}$

NUMBER OF TRIALS

$$\boxed{PMF(k) = \binom{n}{k} p^k (1-p)^{n-k}} = 6 \cdot \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \Rightarrow \text{SAME RESULT}$$

1 ← 0

1 1 ← 1

1 2 1 ← 2

1 3 3 1 ← 3

1 4 6 4 1 ← 4

$$\binom{n}{k} = \binom{4}{2} = 6$$

12. Let's throw a dice four times. With what probability

(a) 2 number "2" Solution 0.12

(b) 4 odd numbers Solution 0.0625

(c) at least one "6" Solution 0.52

are obtained?

1, 2, 3, 4, 5, 6

(b) $P = \left(\frac{3}{6}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16} = 0.0625$

(c) $\boxed{P(\bar{A}) = 1 - P(A)}$

$P(A) = P(\text{NO "6"}) = \left(\frac{5}{6}\right)^4$

$P(\bar{A}) = P(\text{AT LEAST ONE "6"}) = \underbrace{1 - \left(\frac{5}{6}\right)^4}_{} \approx 0.52$

METHOD 2 PMF OF $BIN(4, \frac{1}{6})$:

$P = P(\text{ONE } 6) + P(\text{TWO } 6) + P(\text{THREE } 6) + P(\text{FOUR } 6)$

$= \binom{4}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3 + \binom{4}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 + \binom{4}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1 + \binom{4}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0$

13. In a factory, there is a box of 100 circuit boards. Three of the boards are broken. Random two boards are chosen. With what probability at least one of the boards is intact? Solution 0.999

$\boxed{P(\bar{A}) = 1 - P(A)}$

$P(A) = P(\text{BOTH BROKEN}) = \underbrace{\frac{3}{100} \cdot \frac{2}{99}}_{}$

$P(\bar{A}) = P(\text{AT LEAST ONE}) = 1 - P(A)$

$$P(\bar{A}) = P(\text{AT LEAST ONE IN TACT}) = 1 - P(A)$$
$$= 1 - \frac{3}{100} \cdot \frac{2}{99} \approx 0.999$$

14. Two coins are thrown ~~twice~~. With what probability

(a) two heads — Solution 0.25

(b) at least one tail — Solution 0.75

are obtained?

(a) $P(\underbrace{\text{2 HEADS}}_{=A}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0.25$

(b) $P(\underbrace{\text{AT LEAST ONE TAIL}}_{\bar{A}}) = 1 - P(\text{2 HEADS}) = 1 - 0.25 = 0.75$

METHOD 2

4 TOTAL

HH 1

HT, TH, TT 3

$(2 \text{ OPTIONS})^{2 \text{ TIMES}} = 2^2 = 4$

(a) $P = \frac{1}{4}$

(b) $P = \frac{3}{4}$

THROW A COIN 10 TIMES

$2^{10} = 1024$ OPTIONS

HHH THTT HHT

1 2 3 4 5 6 7 8 9

15. Which probability is better: getting an odd number or at most 4 while throwing a dice? — Solution at most 4

ODD

$P = \frac{3}{}$

1, 2, 3, 4 AT MOST 4

$P = \frac{4}{}$

$P = \frac{3}{6}$ — 3, 4, 5, 6; 3, 4 } AT MOST 4 $\boxed{P = \frac{4}{6}}$

2 MEN
3 WOMEN
5 TOTAL

16. A plane was over booked. With 5 persons in the airport, random 2 are selected to the plane. Adam, Bella, Cecilia, Daniel and Emma are in the queue. With what probability

(a) With what probability Adam and Emma can board? Solution 0.1

(b) With what probability one man and one woman can board? Solution 0.6

(c) With what probability no man can board? Solution 0.3

(a) $P = \underbrace{\frac{1}{5}}_{A} \cdot \underbrace{\frac{1}{4}}_{E} + \underbrace{\frac{1}{5}}_{E} \cdot \underbrace{\frac{1}{4}}_{A}$

$= 2 \cdot \frac{1}{5} \cdot \frac{1}{4} = 2 \cdot \frac{1}{20} = \frac{1}{10} = \underline{0.1}$

$= \binom{5}{2} \cdot \frac{1}{5} \cdot \frac{1}{4}$

(b) $P = \frac{2}{5} \cdot \frac{3}{4} + \frac{3}{5} \cdot \frac{2}{4} = \cancel{2} \cdot \frac{\cancel{2}}{5} \cdot \frac{3}{\cancel{4}_2} = \frac{3}{5} = \underline{0.6}$

(c) $P = \frac{3}{5} \cdot \frac{2}{4} = \frac{3}{5} \cdot \frac{1}{2} = \frac{3}{10} = \underline{0.3}$

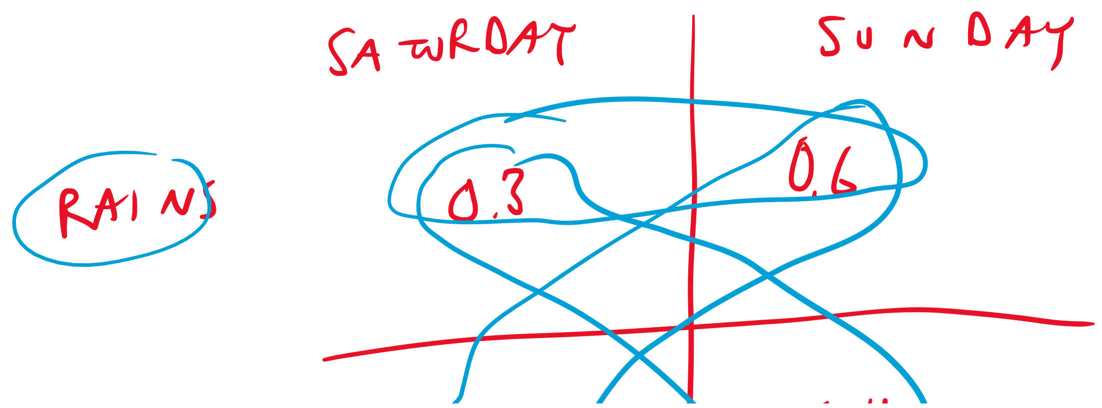
17. In a lottery, there are 4 tickets. The tickets have the numbers 1, 2, 3 and 4. One ticket is picked, put back to the box, and another ticked is picked. With what probability at least one "1" is obtained? Solution 0.4375

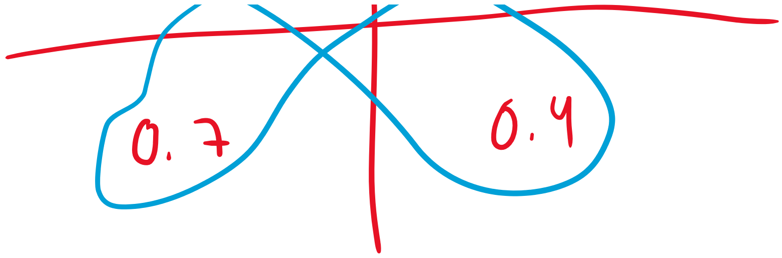
$\boxed{P(\bar{A}) = 1 - P(A)}$

$P(\underbrace{\text{NO "1"}}_{A}) = \frac{3}{4} \cdot \frac{3}{4}$

$P(\underbrace{\text{AT LEAST ONE "1"}}_{\bar{A}}) = 1 - \frac{3}{4} \cdot \frac{3}{4} \approx \underline{0.4375}$

18. Weather forecast announced that the chance of rain on Saturday is 30% and the chance of rain on Sunday is 60%. With what probability, it rains during the weekend? $\boxed{\text{Solution 0.72}}$





DOES NOT RAIN

$$P = 0.6 \cdot 0.7 + \underbrace{0.3 \cdot \overbrace{0.4} + 0.3 \cdot \overbrace{0.6}}_{0.3 \cdot 1}$$

$$= 0.42 + 0.3$$

$$= \underline{0.72}$$

18. Weather forecast announced that the chance of rain on Saturday is 30% and the chance of rain on Sunday is 60%. With what probability, it rains during the weekend? Solution 0.72

RAINS $\boxed{\text{AT LEAST}}$ ONE
ONE DAY

$$\boxed{P(\bar{A}) = 1 - P(A)}$$

P(NO RAIN SAT & NO RAIN SUN)

$$= 0.7 \cdot 0.4 = 0.28$$

P(RAINS AT LEAST ONE DAY) $= 1 - 0.28 = \underline{0.72}$

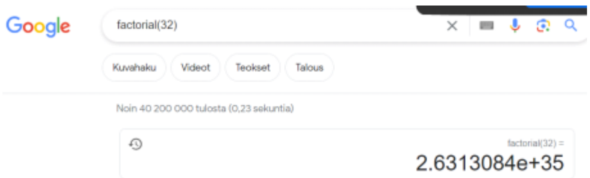
$n! = 32!$ $\qquad \frac{n!}{(n-k)!} = \frac{32!}{20!}$ $\qquad \frac{n!}{k!\,(n-k)!} = \frac{32!}{12!\;20!} = \binom{n}{k}$

19. In total 32 students sit in a class. In how many different ways they can sit? Solution $2.63 \cdot 10^{35}$

$32-12=20$

$\underbrace{32}_{\text{FIRST CHAIR, OPTIONS}} \cdot \underbrace{31}_{\text{SECOND CHAIR, OPTIONS}} \cdot 30 \cdot \ldots \cdot 3 \cdot 2 \cdot 1 = 32! =$ FACTORIAL(32)

$= 2.63 \cdot 10^{35}$



SOLUTION $1,082 \cdot 10^{17}$

$= n$

19. In total 32 students sit in a class. In how many different ways ~~they~~ can sit? ~~Solution $2.63 \cdot 10^{35}$~~

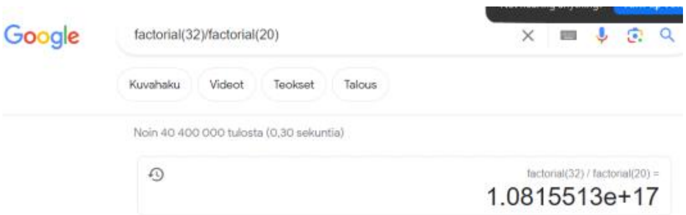
$k = 12$ OF THEM

$\overbrace{32 \cdot 31 \cdot \ldots \cdot 23 \cdot 22 \cdot 21}^{12 \text{ STUDENTS}}$ $= \frac{\overbrace{32 \cdot 31 \cdot \ldots \cdot 21 \cdot 19 \cdot 18 \cdot \ldots \cdot 3 \cdot 2 \cdot 1}^{n!}}{\underbrace{20 \cdot 19 \cdot 18 \cdot \ldots \cdot 3 \cdot 2 \cdot 1}_{(n-k)!}}$

1ST SEAT $\qquad$ 12th SEAT

$= \frac{32!}{20!}$

$= 1,082 \cdot 10^{17}$



19. In total 32 students sit in a class. In how many different ways ~~they can sit?~~ ~~Solution $2.63 \cdot 10^{35}$~~

12 STUDENTS CAN BE CHOSEN

IN HOW MANY WAYS

YOU CAN CHOOSE 12 FROM TOTAL 32 OPTIONS ?

BINOMIAL COEFFICIENT

$$\binom{32}{12} = \frac{32!}{12!\,(32-12)!} = \frac{32!}{12!\cdot 20!} = \underline{2{,}3\cdot 10^8}$$

PASCAL'S TRIANGLE

n \ k	0	1	2	3		
0	1					
1	1	1				
2	1	2	1			
3	1	3	3	1		
4	1	4	6	4	1	
5	1	5	10	10	5	1

$n = 11$

$$\binom{4}{2} = 6$$

$2^3 = 8$

TTT } 1
TTH, THT, HTT } 3
THH, HTH, HHT } 3
HHH } 1

5. Let's throw a coin three times. With what probability

(a) 4 — Solution 0
(b) 3 — Solution 1/8
(c) 2 $= k$ — Solution 3/8
(d) 1 — Solution 3/8
(e) 0 — Solution 1/8

heads are obtained?

$$P = \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \underbrace{\binom{3}{2}}_{\text{POSSIBLE ORDERS}} \underbrace{\left(\frac{1}{2}\right)^2}_{\text{HEADS}} \underbrace{\left(\frac{1}{2}\right)^1}_{\text{TAILS}} = \frac{3}{8}$$

EXCEL

0,375	=BINOM.DIST(2;3;1/2;FALSE)
0,375	=3/8

$= \frac{3}{8}$

3 OPTIONS

AB, AC, BC
BA, CA, CB

$\Rightarrow$ 6 OPTIONS $= 3 \cdot 2$

2 3 $\Rightarrow$ A B C PERMUTATIONS

$\overset{3}{\cancel{4}}$ → A B C

PERMUTATIONS

25. A password consists of $\overset{2}{\cancel{5}}$ different symbols. There are ~~115~~ symbols available.

   (a) How many different passwords exist, when the order matters and each symbol can be used only once? Solution
   $1.8 \cdot 10^{10}$

   (b) If you try to guess the password and each guess takes 10 s, how long it would take to make all the guesses? Solution
   5840 years

$\underbrace{115! \quad \text{TOO BIG}}$

(a) $115 \cdot 114 \cdot 113 \cdot 112 \cdot 111 = \underline{1.8 \cdot 10^{10}}$

(b) $\dfrac{1.8 \cdot 10^{10}}{60 \cdot 60 \cdot 24 \cdot 365} \cdot 10 = 5840$ YEARS

METHOD 2

$1.8 \cdot 10^{10}$ OPTIONS $\cdot \dfrac{10\ \text{S}}{\text{OPTION}}$

$= 1.8 \cdot 10^{11}$ SECONDS ARE NEEDED

NEEDED TIME IN YEARS $= \dfrac{1.8 \cdot 10^{11}}{60 \cdot 60 \cdot 24 \cdot 365} = 5840$ YEARS

26. Let's consider a deck of playing cards. → 52 CARDS

   (a) How many permutations does the deck have? Solution $8.1 \cdot 10^{67}$

   (b) With what probability 5 cards drawn contain 4 aces? Solution 0.000018 =1/54145

   (c) With what probability 5 cards drawn contain 3 clubs and 2 spades? Solution 0.00858

(a) $52! = 8.1 \cdot 10^{67}$

WITHOUT A CALCULATOR

$n! \approx \left(\frac{n}{2}\right)^n$

$52! \approx \left(\frac{52}{2}\right)^{52}$

(b) $\dfrac{4}{52} \cdot \dfrac{3}{51} \cdot \dfrac{2}{50} \cdot \dfrac{1}{49} \cdot \underbrace{\dfrac{48}{48}}_{\text{ANY CARD IS OK}}$

$P = 5 \cdot \boxed{\dfrac{4}{52}} \cdot \dfrac{3}{51} \cdot \dfrac{2}{50} \cdot \dfrac{1}{49} \cdot \dfrac{48}{48}$

$= 0.0000\ 18$

PATTERN
AAAAX
AAAXA
} 5 OPTIONS

TTH
THT
HTT
} COINS

(c) DECK : 52 CARDS

$P = \dfrac{\binom{13}{3}\binom{13}{2}}{\binom{52}{5}}$

SPADES 1, 2, ..., 13 ← NEED 2
HEARTS — " —
DIAMONDS — " —
CLUBS — " — 13 ← NEED 3

$\binom{52}{5}$ DIAMONDS — 11

CLUBS — 11 —13 ← NEED 3

binom(13,3)*binom(13,2)/binom(52,5)

NATURAL LANGUAGE   MATH INPUT

Input

$$\binom{13}{3} \times \frac{\binom{13}{2}}{\binom{52}{5}}$$

Exact result

$$\frac{143}{16660}$$

Decimal approximation

0.0085834333733493397358943577430972

$= 0.00858$

$$P = \frac{\binom{13}{3}\binom{13}{2}}{\binom{52}{5}}$$

IN TOTAL 35 COMBINATIONS $\binom{n}{k}$

27. For an entrance exam, 20 math questions and 15 physics questions are considered. The questions are chosen in random and their order does not matter. With what probability chosen 6 questions

   (a) are all math questions? Solution 0.024

   (b) contain 3 math questions and 3 physics questions? Solution 0.32

(a) $P = \dfrac{\binom{20}{6}}{\binom{35}{6}}$ ← 35 CHOOSE 6

(b) $P = \dfrac{\binom{20}{3}\binom{15}{3}}{\binom{35}{6}}$

28. A dice is rolled 10 times. With what probability exactly 3 results "6" are obtained? 0.15504

29. In football betting, the result of 13 matches is guessed at random. Each macth has 3 options (1,x,2) (This means (home wins, tie, guest wins).) With what probability the result of 12 matches is correct? Solution 0.000 016 3

30. English test contains 40 questions. Each question has 4 options. The student is guessing.

   (a) With what probability all answers are correct? Solution $8.28 \cdot 10^{-25}$

   (b) With what probability exactly 5 answers are correct? Solution 0.02723

   (c) With what probability at least 5 answers are correct? Solution 0.984

$\binom{13}{k} = \binom{13}{13-k}$

28. $\binom{10}{3}\underbrace{\left(\frac{1}{6}\right)^3}_{\text{GET "6" THREE TIMES}}\underbrace{\left(\frac{5}{6}\right)^7}_{\text{GET SOME OTHER NUMBER}} =$

666xxx xxxx

6xx6xx6xxxx

PATTERNS $\binom{10}{3}$

29. $\binom{13}{12}\underbrace{\left(\frac{1}{3}\right)^{12}}_{\text{RIGHT GUESS}}\underbrace{\left(\frac{2}{3}\right)^{1}}_{\text{WRONG GUESS}} = \ldots$

30. (a) $\left(\frac{1}{4}\right)^{40}$

(b) $\binom{40}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^{35}$

(c) $\sum^{40} \binom{40}{k}\left(\frac{1}{4}\right)^k\left(\frac{3}{4}\right)^{40-k}$

RIGHT GUESS

WRONG GUESS

(c) $\sum_{k=5}^{40} \binom{40}{k} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{40-k}$

Sum

$\sum_{k=5}^{40} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{40-k} \binom{40}{k}$

$\approx 0,98$

Decimal approxima

0.98395776018123033833287…

…

VIDEO ?

31 – 41

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

① $n! = n \cdot (n-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

② $\frac{n!}{(n-k)!}$

③ $\binom{n}{k}$ = nchoosek(n,k) = binom(n,k) $= \frac{n!}{k!(n-k)!}$

④ $\binom{n}{k} p^k (1-p)^{n-k}$

PASCAL

1

1 1

1 2 1

1 3 3 1

1.3 Permutations, k-permutations, combinations

19. In total 32 students sit in a class. In how many different ways they can sit? $32!$ Solution $2.63 \cdot 10^{35}$

20. (a) 10 people shake each others hands. How many handshakes are performed? ⓐ $\binom{10}{2}$ Solution 45

    (b) 8 teams play pairwise. How many matches are played? ⓑ $\binom{8}{2}$ = Solution 28

21. In a questionnaire, there were 6 questions each with 5 options to choose from. In how many ways the questionnaire was possible to answer (you had to answer to each question)? $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^6$ Solution 15 625

22. In a queue, there are 3 boys and 4 girls so that the girls are in the front. How many such queues can be made? Solution 144

    PRODUCT OF $a!\, b!$

23. A student answers 8 questions from 10 options.

    (a) How many ways the student can choose? $\binom{10}{8}$ Solution 45

    (b) How many ways the student can choose, if the first 3 questions are mandatory? $\binom{7}{5}$ Solution 21

24. There are 7 men and 5 women. How many groups with 3 men and 2 women exist? Solution 350

25. A password consists of 5 different symbols. There are 115 symbols available.

    (a) How many different passwords exist, when the order matters and each symbol can be used only once? Solution $1.8 \cdot 10^{10}$

    (b) If you try to guess the password and each guess takes 10 s, how long it would take to make all the guesses? Solution 5840 years

Exercises 31-41

maanantai 25. maaliskuuta 2024 9.59

$\frac{1000}{6} = 166$.

IDEA

- LET'S CALCULATE MOST PROBLEMS ON VIDEO
- LET'S LEAVE 3 (?) PROBLEMS FOR EXERCISE (CALCULATE ON PAPER & SUBMIT)

1.4 Probability distributions

31. A wheel of fortune has 8 equal sectors, one of which is a joker sector. Let X be a random variable which tells the number of jokers in 3 spins. Find the distribution and find the expected value. Solution $\mu = 0.4$

$= 2.0, 7$

32. A basket ball player has probability 0.70 to score. He gets two throws. What is the expected value of scores? Solution 1.4

33. Let $X \sim Bin(n, p)$. Find n and p, when $\mu = 2$ and $\sigma^2 = \frac{4}{3}$. Solution $n = 6$ and $\boxed{p = 1/3}$

34. A dice is thrown 4 times. Find the distribution of getting a "6". Solution $Bin(4, 1/6), \mu = 1.6$ and $\sigma = 0.57$

35. A factory is doing quality inspection. A product is accepted with probability 0.8. Take two random products. Find the distribution, expected value and standard error. Solution $\mu =$

36. In a box, there are 3 black balls and 3 white balls. Three balls are drawed. Find the expected value of the number of white balls. Solution $\mu = 1.5$

ROLL $16^{=n}$ DICES. DENOTE BY X = NUMBER OF "6" YOU GET

$$\boxed{P(X = k) = \binom{n}{k} \underbrace{p}_{\frac{1}{6}}^{k} \underbrace{(1-p)}_{\frac{5}{6}}^{n-k}}$$

37. There are 5 envelopes with 10€, 20€, 30€, 40€ and 50€ respectively. The winner chooses 2 envelopes. Random variable X tells the total amount won. Find the expected value. Solution $60e$

38. There are 1000 tickets. Price 100€ in 1 ticket, price 50€ in 10 tickets, price 20€ in 15 tickets. One ticket costs 1€. Find the expected value of the net win. Solution $\mu = -0.1e$.

39. Two coins are tossed. If you get one head, you win 20€. If you get 2 heads, you win 40€. If you get 2 tails, you lose 100€. Find the expected value of the win. Solution $\mu = -5e$.

40. A wallet contains six 1€ coins, four 2€ coins, and two 0.50€ coins. One coin is picked up. Find the distribution.

41. Students threw six coins 90 times. The number of heads was counted.

0	2	3	2	4	3	4	3	3	4	1	4	3	4	5	3	4	4	4	4	3	2	1
4	2	3	4	2	5	3	4	2	4	2	1	2	4	2	1	4	3	2	2	1	5	2
3	1	5	3	3	3	1	3	4	2	3	4	1	3	5	5	3	2	5	3	2	3	4
2	3	3	3	2	3	4	5	5	0	5	2	3	3	4	3	0	4	1	2	4		

Find the distribution.

SOME THEORY

PROBABILITY DISTRIBUTION

5. Let's throw a coin three times. With what probability

(a) 4 Solution 0

(b) 3 Solution 1/8

(c) 2 Solution 3/8

(d) 1 Solution 3/8

(e) 0 Solution 1/8

heads are obtained?

8

TTT } 1

TTH, THT, HTT } 3

THH, HTH, HHT } 3

HHH } 1

X = NUMBER OF HEADS OBTAINED = RANDOM VARIABLE

k

$P(X = 4) = 0$

PROBABILITY

1

3

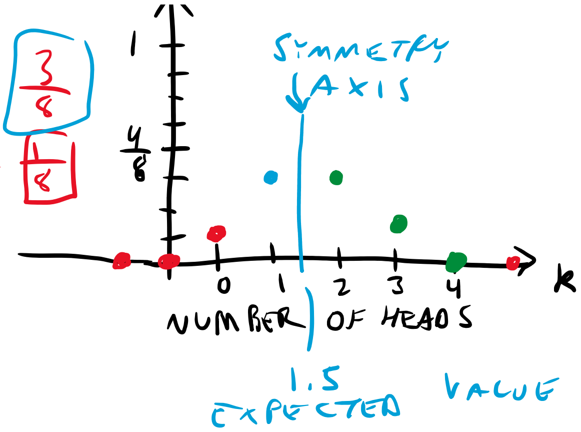
SYMMETRIC

SYMMETRY AXIS

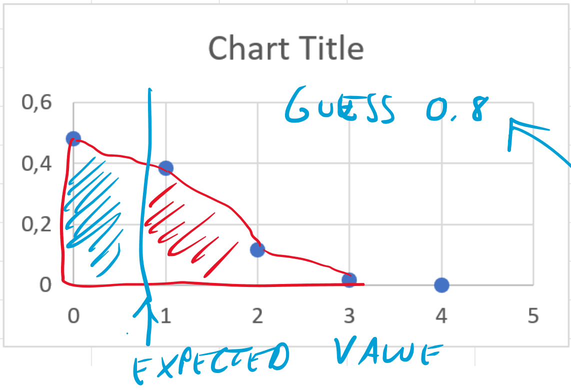
$P(X=4)=0$
$P(X=3)=\boxed{\frac{1}{8}}$
$P(X=2)=\boxed{\frac{3}{8}}$

$P(X=1)=\boxed{\frac{3}{8}}$
$P(X=0)=\boxed{\frac{1}{8}}$

$P(X=5)=0$
$P(X=-1)=0$
$P($



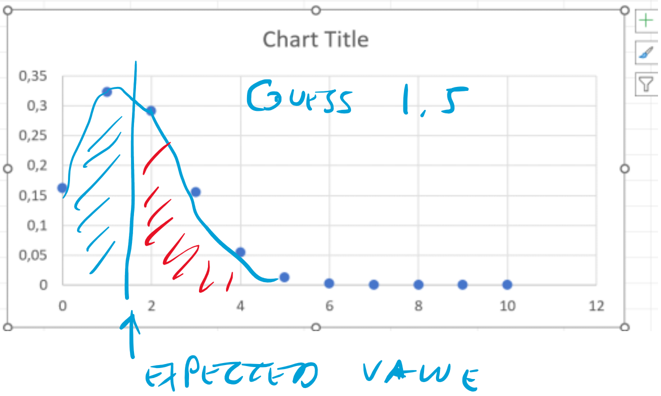
n=	4	p=	0,16667
		1-p=	0,83333
k=			
	0	0,48225	=BINOM.DIST(B4;B$1;E$1;FALSE)
	1	0,3858	
	2	0,11574	
	3	0,01543	
	4	0,00077	
		1	



x_i	p_i	x_i*p_i
0	0,48225	0
1	0,3858	0,3858
2	0,11574	0,23148
3	0,01543	0,0463
4	0,00077	0,00309
		0,66667

$\sum x_i p_i$ EXPECTED VALUE

n=	10	p=	0,16667
		1-p=	0,83333
k=			
	0	0,16151	=BINOM.DIST(B4;B$1;E$1;FALSE)
	1	0,32301	
	2	0,29071	
	3	0,15505	
	4	0,05427	
	5	0,01302	
	6	0,00217	
	7	0,00025	
	8	1,9E-05	
	9	8,3E-07	
	10	1,7E-08	



x_i	p_i	x_i*p_i
0	0,16151	0
1	0,32301	0,32301
2	0,29071	0,58142
3	0,15505	0,46514
4	0,05427	0,21706
5	0,01302	0,06512
6	0,00217	0,01302
7	0,00025	0,00174
8	1,9E-05	0,00015
9	8,3E-07	7,4E-06
10	1,7E-08	1,7E-07
		1,66667

$= \sum_{i=0}^{10} x_i p_i$

10	1,7E-08	1,7E-07
		1,66667

$= \sum_{i=0} x_i p_i$

$X \sim BIN(n, \frac{1}{p})$

$1000 \cdot \frac{1}{6} = 166$ ✓

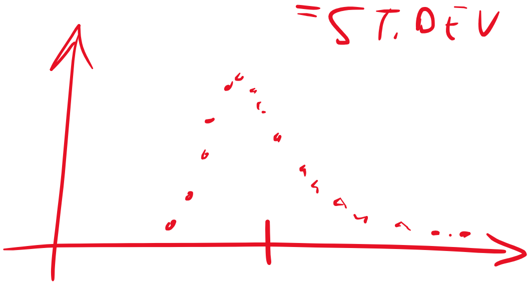
PMF $\binom{n}{k} p^k (1-p)^{n-k}$

EXPECTED VALUE = MEAN $= \mu = np$

VARIANCE $= \sigma^2 = np(1-p)$

STANDARD DEVIATION $= \sigma = \sqrt{\text{VARIANCE}}$

= ST.DEV

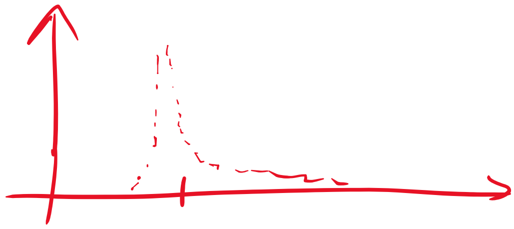


μ

σ = SMALL

μ

σ LARGE



μ

σ VERY SMALL

NORMAL DISTRIBUTION

RELATIVE AREA 68.2%

EXAMPLE.

LENGTH OF A HUMAN IS NORMAL DISTRIBUTED

$\mu = 173$ cm

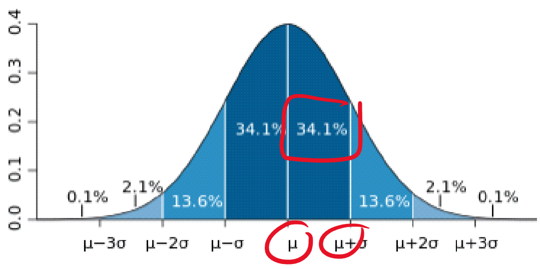
$\sigma = 7$ cm

$\mu + \sigma$

$P(173..., 180 \text{ cm}) = 34.1\%$

Part of a series on statistics

Probability theory



LET'S SOLVE THE EXERCISES

$Bin(n, p)$

$\mu = np$

$1000 \cdot \frac{1}{6}$

SOLVE YOURSELF

1.4 Probability distributions

31. A wheel of fortune has 8 equal sectors, one of which is a joker sector. Let X be a random variable which tells the number of jokers in 3 spins. Find the distribution and find the expected value. Solution $\mu = 0.4$

32. A basket ball player has probability 0.70 to score. He gets two throws. What is the expected value of scores? Solution 1.4 $= 2 \cdot 0.7$

33. Let $X \sim Bin(n, p)$. Find n and p, when $\mu = 2$ and $\sigma^2 = \frac{4}{3}$. Solution $n = 6$ and $p = 1/3$

34. A dice is thrown 4 times. Find the distribution of getting a "6". Solution $Bin(4, 1/6), \mu = \cancel{1.6}\ 0.66$ and $\sigma = 0.57$

~~35. A factory is doing quality inspection. A product is accepted with probability 0.8. Take two random products. Find the distribution, expected value and standard error. Solution $\mu =$~~

36. In a box, there are 3 black balls and 3 white balls. Three balls are drawed. Find the expected value of the number of white balls. Solution $\mu = 1.5$

x_i	p_i	$x_i p_i$
0	$\frac{3}{6} \cdot \frac{2}{5} \cdot \frac{1}{4}$	~
1	$3 \cdot \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{3}{4}$	~
2	$3 \cdot \frac{3}{6} \cdot \frac{3}{5} \cdot \frac{2}{4}$	~
3	$\frac{3}{6} \cdot \frac{2}{5} \cdot \frac{1}{4}$	~

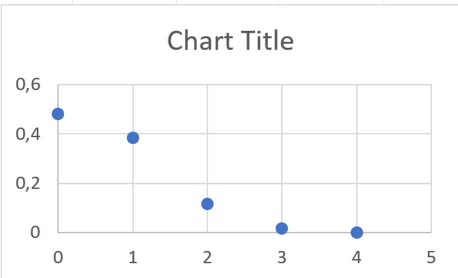
$\sum x_i p_i = 1,5$

SIMPLE.

HALF OF THE BALLS ARE WHITE (ON AVERAGE) $\Rightarrow \boxed{\mu = 1.5}$

34.

n=	4	p=	0,16667
		1-p=	0,83333
k=			
0	0,48225	=BINOM.DIST(B4;B$1;E$1;FALSE)	
1	0,3858		
2	0,11574		
3	0,01543		
4	0,00077		
	1		



$\mu = 4 \cdot \frac{1}{6} = \frac{2}{3} = 0,66$

$$\boxed{\begin{aligned} \mu &= 0,66 \\ \sigma &= 0,57 \end{aligned}}$$

33.

$\mu = 2$

$\sigma^2 = \frac{4}{3}$

$n = ?$

$p = ?$

SOLUTION

$\mu = np$

$\sigma^2 = np(1-p)$

$p = 1 - \frac{\sigma^2}{\mu}$

$= 1 - \frac{\frac{4}{3}}{2}$

$= 1 - \frac{2}{3} = \underline{\frac{1}{3}}$

$P(WIN\ 100€)$

$n = \frac{\mu}{p} = \frac{2}{\frac{1}{3}} = 2 \cdot 3 = \underline{6}$

SOLVE

SOLVE YOURSELF

$P(\text{WIN } 100€) = \frac{1}{1000} =$

$n = \frac{m}{p} = \frac{2}{\frac{1}{3}} = 2 \cdot 3 = 6$

37. There are 5 envelopes with 10€, 20€, 30€, 40€ and 50€ respectively. The winner chooses 2 envelopes. Random variable X tells the total amount won. Find the expected value. Solution 60e

38. There are 1000 tickets. Price 100€ in 1 ticket, price 50€ in 10 tickets, price 20€ in 15 tickets. One ticket costs 1€. Find the expected value of the net win. Solution $\mu = -0.1e$.

39. Two coins are tossed. If you get one head, you win 20€. If you get 2 heads, you win 40€. If you get 2 tails, you lose 100€. Find the expected value of the win. Solution $\mu = -5e$.

40. A wallet contains six 1€ coins, four 2€ coins, and two 0.50€ coins. One coin is picked up. Find the distribution. ?

$\mu = 1{,}25$

41. Students threw six coins 90 times. The number of heads was counted.

0	2	3	2	4	3	4	3	3	4	1	4	3	4	5	3	4	4	4	4	3	2	1
4	2	3	4	2	5	3	4	2	4	2	1	2	4	2	1	4	3	2	2	1	5	2
3	1	5	3	3	3	1	3	4	2	3	4	1	3	5	5	3	2	5	3	2	3	4
2	3	3	3	2	3	4	5	5	0	5	2	3	3	4	3	0	4	1	2	4		

Find the distribution.

x_i	n_i	p_i	$x_i p_i$
1	6	$\frac{6}{12}$	$1 \cdot \frac{6}{12}$
2	4	$\frac{4}{12}$	$2 \cdot \frac{4}{12}$
0.5	2	$\frac{2}{12}$	$0{,}5 \cdot \frac{2}{12}$
	TOTAL = 12		

$\Sigma = \frac{6+8+1}{12} = \frac{15}{12} = \frac{5}{4} = 1{,}25$

THIS WEEK,

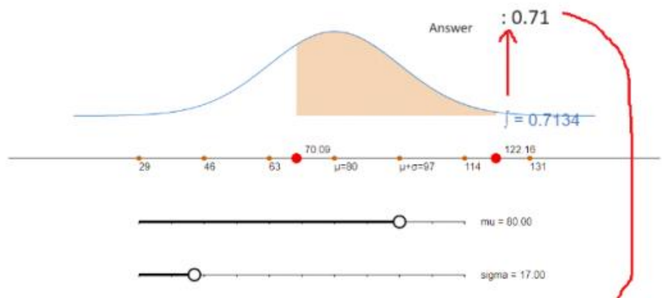
SUBMIT JUST 31, 38, 39

Exercises 42-51

torstai 28. maaliskuuta 2024 14.33

FOR NORMAL DISTRIBUTION PROBLEMS, YOU CAN USE THE SIMULATOR

46. A random variable X is normal distributed. Expected value is $\mu = 8$ and standard deviation is $\sigma = 1.7$. With what probability $7 \leq X \leq 12.2$? Solution 0.7156



45. a) 0,5987 b) 0,0972 c) 0,7745

46. 0,7156 ← correct answer

47. 330

PAGE

Normal distribution simulator

= FISH (IN FRENCH)

1.5 Poisson distribution (a limit of the binomial distribution)

42. A controller has a malfunction during a week with probability 0.007. A company maintains 900 controllers. Use the Poisson distribution to find the probability that

    (a) 7 Solution 0.14

    (b) more than 3 Solution 0.87

    controllers have a malfunction during a week.

43. On average, two cars arrive to a parking place during a minute. With what probability, during any given minute, 4 or more cars arrive to the parking place? Solution 0.14

44. A school has many computers. During one month, usually one computer breaks. With what probability less than two computers break? Solution 0.74

$X \sim BIN(n, p)$

n ↑ TRIALS

p ↑ PROBABILITY OF A SUCCESS

NUMBER OF SUCCESSFUL ROLLS

FOR EXAMPLE

$n = 10$

$p = \frac{1}{6}$

X = NUMBER OF "6" FROM ROLLING

SUCCESSFUL ROLLS

$$P(X = k) = \underbrace{\binom{n}{k} p^k (1-p)^{n-k}}$$

FROM ROLLING DICE

1 $\boxed{np = a}$

$\Rightarrow p = \frac{a}{n}$

$\xrightarrow{n \to \infty}$

2 $$\underbrace{\frac{a^k}{k!} e^{-a}} = P(X = k)$$

POISSON DISTRIBUTION

1.5 Poisson distribution (a limit of the binomial distribution)

42. A controller has a malfunction during a week with probability 0.007. A company maintains 900 controllers. Use the Poisson distribution to find the probability that

    (a) 7 — Solution 0.14

    (b) more than 3 — Solution 0.87

    controllers have a malfunction during a week.

43. On average, two cars arrive to a parking place during a minute. With what probability, during any given minute, 4 or more cars arrive to the parking place? — Solution 0.14

    a

44. A school has many computers. During one month, usually one computer breaks. With what probability less than two computers break? — Solution 0.74

SOLUTION.

$n = 100$, $p = 0{,}02$ | $n = 10000$, $p = 0{,}0002$

$a = np = 100 \cdot 0{,}02 = 2$

$$P(X \geq 4) = 1 - P(X < 4)$$
$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

	k	P(X=k)	
	0	0,1353353	
	1	0,2706706	
	2	0,2706706	
	3	0,180447	
		0,8571235	
a=2			
	P(X >=4)	1-0,857123	0,142877

INCLINATION POINTS

WITHOUT A SIMULATOR

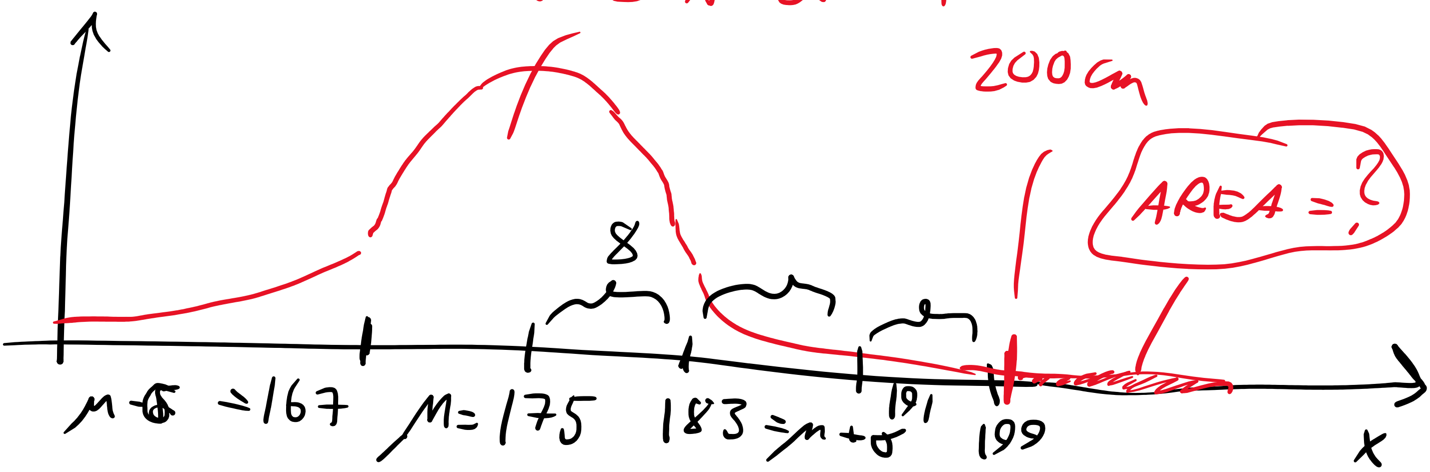
$f''(x_0)=0$ $y=f(x)$

$\mu+\sigma$

MOST STEEP $\mu-\sigma$

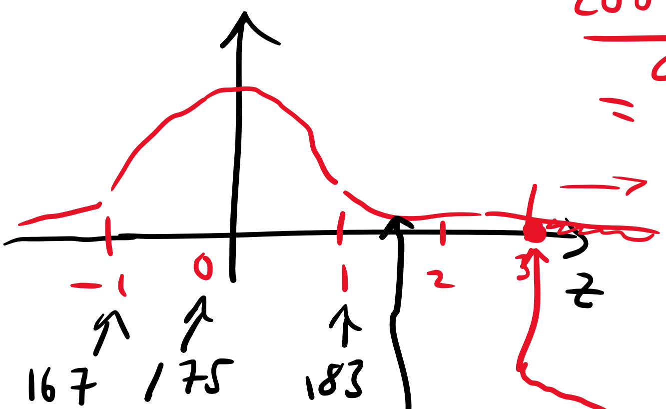
48. The height of men is normal distributed with $\mu = 175cm$ and $\sigma = 8cm$. With what probability a man is over 2 meters tall?
Solution 0.001

TOTAL AREA = 1



SOLUTION: (1) NORMALIZATION

$$z = \frac{x-\mu}{\sigma}$$



$$\frac{200-\mu}{\sigma} = \frac{200-175}{8} = \frac{25}{8} = 3{,}125$$

$$P(x \geq 200)$$

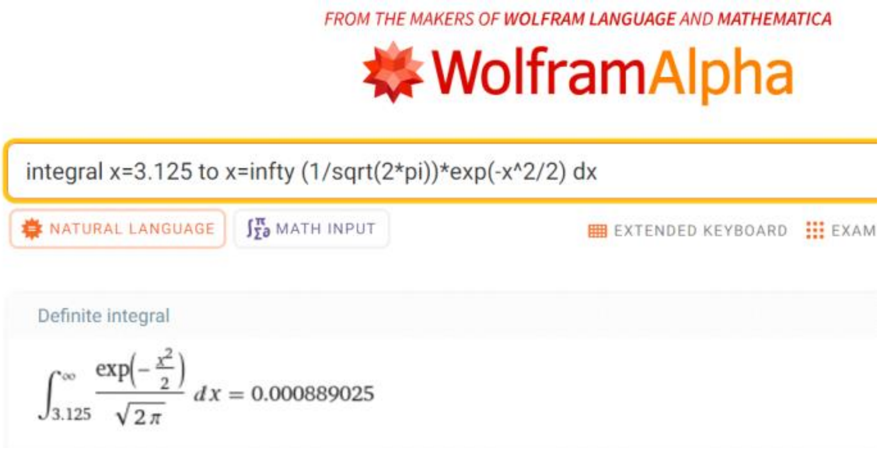
$$= P(z \geq 3{,}125)$$

$$= \int^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx$$

$$= \int_{3.125}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx = 0.0008890\ldots$$

$$\approx 0.001$$

$$= \underline{\underline{0.1\,\%}}$$



$$P(z \geq \underline{3.125})$$

$$= 1 - \underbrace{\Phi(3.125)}_{\text{CAN FIND FROM A TABLE}}$$

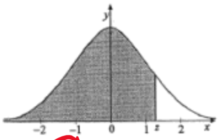
$$\approx 1 - 0.999$$

$$= \underline{\underline{0.001}}$$

BY USING A TABLE

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx$$

NORMED NORMAL DISTRIBUTION



$\Phi(z) = P(X \le z) = \int_{-\infty}^{z} f(x)\,dx$

Values of the cumulative function of the normed normal distribution.

Example

$P(X < -1{,}2) = 1 - P(X \le 1{,}2)$
$= 0{,}1151$

z	0	1	2	3	4	5	6	7	8	9
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,9970	0,9971	0,9972	0,9973	0,9974
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,9980	0,9981
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986
3,	0,9987	$\boxed{0{,}9990}$	0,9993	0,9995	0,9997	0,9998	0,9998	0,9999	[illegible]	1,0000

$\Rightarrow \Phi(3{,}1) = 0{,}999$

0,999

Exercises 52-61

torstai 4. huhtikuuta 2024 12.51

28. A dice is rolled 10 times. With what probability exactly 3 results "6" are obtained?

$a = np = \frac{10}{6}$

$$\boxed{\begin{gathered} X \sim BIN(n = 10,\ p = \tfrac{1}{6}) \\ P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \end{gathered}}$$

$k = 3$

ANSWER

$$P = \binom{10}{3}\left(\frac{1}{6}\right)^3\left(1-\frac{1}{6}\right)^7$$

$= \ldots$ 0.1550453 $\approx 15.5\%$

$np = a =$ CONSTANT

$n \to \infty$

$$\boxed{P(X = k) = \frac{a^k}{k!} e^{-a}}$$

SIMILAR

0.1457373 $\approx 14.5\%$

IF n WAS LARGER, FOR EXAMPLE $n = 100$, THE NUMBERS WOULD BE ALMOST SAME.

n = NUMBER OF TRIALS

p = PROBABILITY

$a = np$

$$\boxed{P(X = k) = \frac{a^k}{k!} e^{-a}}$$

42. A controller has a malfunction during a week with probability 0.007 ($= p$). A company maintains 900 ($= n$) controllers. Use the Poisson distribution to find the probability that

(a) 7 ($= k$) — Solution 0.14

(b) more than 3 — Solution 0.87

controllers have a malfunction during a week.

43. On average, two cars arrive to a parking place during a minute. With what probability, during any given minute, 4 or more cars arrive to the parking place? Solution 0.14

44. A school has many computers. During one month, usually one computer breaks. With what probability less than two computers break? Solution 0.74

42.

(a) $a = np = 900 \cdot 0{,}007 = 6{,}3 \Rightarrow P(X=7) = \frac{6{,}3^7}{7!} e^{-6{,}3}$

(b) $P = P(X=4) + P(X=5) + P(X=6) + P(X=7) + \ldots$

$= 1 - \left(P(X=0) + P(X=1) + P(X=2) + P(X=3)\right)$

$e^{-6{,}3}$ $\qquad 6{,}3\, e^{-6{,}3}$ $\qquad \frac{6{,}3^2 e^{-6{,}3}}{2}$ $\qquad \frac{6{,}3^3 e^{-6{,}3}}{6}$

$= 1 - 0{,}1263$

$= 0{,}873\ldots$

$\approx 0{,}87$

$\frac{a^k}{k!} e^{-a}$

$= 0.87\ldots$

≈ 0.87

$\frac{a^k}{k!} e^{-a}$

$k=3$ $3! = 3\cdot 2\cdot 1 = 6$

42. A controller has a malfunction during a week with probability 0.007. A company maintains 900 controllers. Use the Poisson distribution to find the probability that

    (a) 7 Solution 0.14

    (b) more than 3 Solution 0.87

    controllers have a malfunction during a week.

$a=2$

43. On average, two cars arrive to a parking place during a minute. With what probability, during any given minute, 4 or more cars arrive to the parking place? Solution 0.14

44. A school has many computers. During one month, usually one computer breaks. With what probability less than two computers break? Solution 0.74

43. $P(X \geq 4)$

$$= 1 - \left(P(X=3) + P(X=2) + P(X=1) + P(X=0) \right)$$

$$\frac{2^3}{3!} e^{-2} \qquad \frac{2^2}{2!} e^{-2} \qquad \frac{2^1}{1!} e^{-2} \qquad \frac{2^0 e^{-2}}{0!}$$

42. A controller has a malfunction during a week with probability 0.007. A company maintains 900 controllers. Use the Poisson distribution to find the probability that

    (a) 7 Solution 0.14

    (b) more than 3 Solution 0.87

    controllers have a malfunction during a week.

43. On average, two cars arrive to a parking place during a minute. With what probability, during any given minute, 4 or more cars arrive to the parking place? Solution 0.14

44. A school has many computers. During one month, usually one computer breaks. With what probability less than two computers break? Solution 0.74

"a"

$$P(X<2) = P(X=0) + P(X=1)$$

$$= \frac{1^0}{0!} e^{-1} + \frac{1^1}{1!} e^{-1} = e^{-1}(1+1)$$

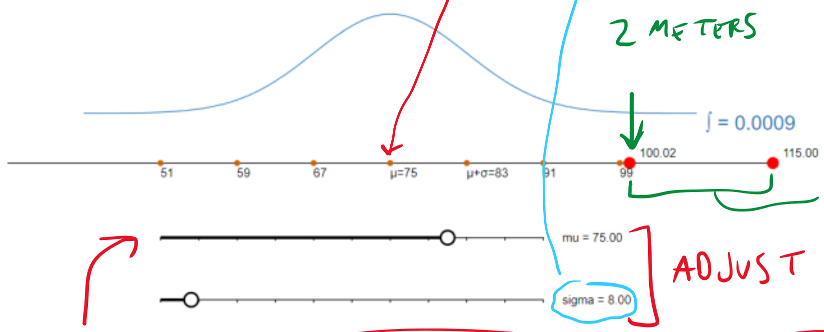
$$= 2e^{-1} = \frac{2}{e}$$

$$\approx 0.74$$

48. The height of men is normal distributed with $\mu = 175cm$ and $\sigma = 8cm$. With what probability a man is over 2 meters tall?

Solution ~~0.001%~~ 0,001

2 METERS



AREA = 0,0009...

≈ 0,001

ADJUST

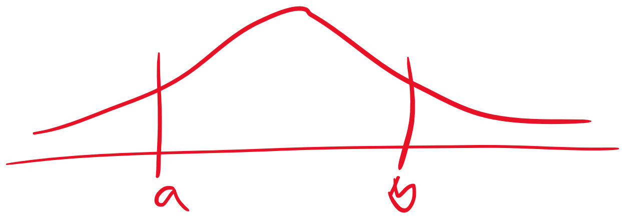
CAN BE ADJUSTED 0 .. 100, SO LET'S DENOTE M = 175 BY M = 75

Friday 5.4.2024

perjantai 5. huhtikuuta 2024 9.59

METHODS TO USE FOR NORMAL DISTRIBUTION EXERCISES

- SIMULATOR (IN MOODLE)
- TABLE (OLD FASHIONED METHOD)
- INTEGRAL :



P

$$= \text{AREA} = \int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

45. A random variable X is normal distributed. Find the probabilities.

(a) $P(z \leq 0.25)$ Solution 0.5987

SOLUTION. μ AND σ NOT MENTIONED $\Rightarrow \begin{cases} \mu = 0 \\ \sigma = 1 \end{cases}$

FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA

WolframAlpha

integral x=-infty to x=0.25 (1/(s*sqrt(2*pi)))*exp(-(1/2)*((x-m)/s)^2) dx where m=0 and s=1

NATURAL LANGUAGE MATH INPUT EXTENDED KEYBOARD EXAMPLES UPLC

Input interpretation

$$\int_{-\infty}^{0.25} \frac{1}{s\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-m}{s}\right)^2\right) dx \text{ where } m = 0, s = 1$$

Result

0.598706

45. A random variable X is normal distributed. Find the probabilities.

(a) $P(z \leq 0.25)$

(b) $P(1.2 \leq z \leq 2.1)$

(c) $(P(-1 \leq z \leq 1.5)$

Exercises 52-61, confidence interval and hypotheses

maanantai 8. huhtikuuta 2024 11.54

https://en.wikipedia.org/wiki/Confidence_interval

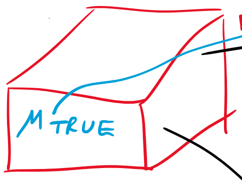
- This article **may be too technical** ... **understand**. *(March 2021)*

Informally, in frequentist statistics, a **confidence interval** (**CI**) is an interval which is expected to typically contain the parameter being estimated. More specifically, given a

WE CAN CHOOSE E.G.
95%
99%
99.9%

EXAMPLE.

RESISTORS, AROUND 5000 PIECES



WE CANNOT KNOW

→ SAMPLE 1 $n = 6$ RESISTORS

$R_1 = 1050\ \Omega$
$R_2 = 1027\ \Omega$
$\vdots$
$R_6 = \ldots$

SAMPLE GIVES SOME INFORMATION

FACTORY:

$\sigma_{RESISTANCE} = 3\ \Omega$

$$M_1 = \frac{R_1 + R_2 + \ldots + R_6}{6} = 1040\ \Omega$$

→ SECOND SAMPLE 2 $n = 6$

$M_2 = 1039\ \Omega$

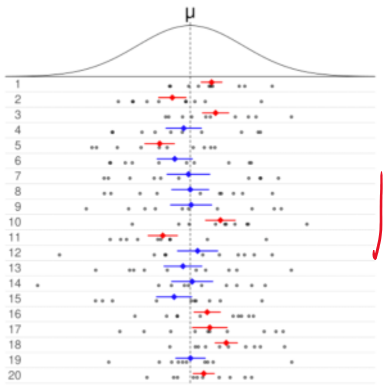
DIFFER BECAUSE OF THE SAMPLE

WE CAN CONTINUE TO TAKE MORE SAMPLES

→ M_3, M_4 ... MORE AVERAGES

CONFIDENCE INTERVAL (95%) OF SAMPLE $k = 1, 2, 3$ IS AN INTERVAL WHICH CONTAINS ~~OF M_1, M_2, M_3...~~

M TRUE 95% OF THE TIME



https://en.wikipedia.org/wiki/Confidence_interval

- This article **may be too technical** ... **understand**. *(March 2021)*

Informally, in frequentist statistics, a **confidence interval** (**CI**) is an interval which is expected to typically contain the parameter being estimated. More specifically, given a

FINDING THE CONFIDENCE INTERVAL.

2 CASES:

CASE 1: $\sigma_{POPULATION}$ = "σ_{TRUE}" IS KNOWN

CASE 2: s ~~σ~~$_{SAMPLE}$ IS USED

CASE 2: S_{SAMPLE} IS USED

1. TAKE A SAMPLE
2. FIND IS IT CASE 1 OR 2
3. CHOOSE CONFIDENCE LEVEL (95%, 99%, OR 99,9%)
4. CALCULATE μ (BASED ON THE SAMPLE)
5. FIND A RADIUS FROM t - DISTRIBUTION TABLE

CASE 1		CASE 2
95% ($\boxed{0,95}$) $p=0.05$	$R = \underbrace{1,96}\frac{S}{\sqrt{n}}$	$R = t_{n-1,\,p=0.05}\frac{S}{\sqrt{n}}$
99% $p=0,01$	$R = \underbrace{2,58}\frac{S}{\sqrt{n}}$	$R = t_{n-1,\,p=0,01}\frac{S}{\sqrt{n}}$
99,9% $p=0,001$	$R = \underbrace{3,29}\frac{S}{\sqrt{n}}$	$R = t_{n-1,\,p=0,001}\frac{S}{\sqrt{n}}$

6. $[\mu - R, \mu + R]$

52. Find the 99% confidence interval for the mean when the distribution is normal and $\boxed{\sigma = 2.5}$ with the sample: 30,8; 30,0; 29,9; 30,1; 31,7; 34,0. KNOWN
Solution $28.45 \le \mu \le 33.71$

SOLUTION.

1. SAMPLE ok
2. CASE 1 (GOOD, EASIER!)
3. 99% $\rightarrow R = 2,58\frac{S}{\sqrt{n}}$
4. $\mu = 31,0833$
5. $R = 2.58 \cdot \frac{2,5}{\sqrt{6}} = 2,6332$

$R = 2{,}00 \cdot \frac{\quad}{\sqrt{6}} = 2{,}0\ldots$

(6) INTERVAL

$[\mu - R, \mu + R]$

$\mu - R = 31{,}0833 - 2{,}6332$
$= \underbrace{28{,}\cancel{3}45} \approx 28{,}5$

$\mu + R = 31{,}0833 + 2{,}6332$
$= \underbrace{33{,}71}6 \approx 33{,}7$

INTERVAL

$[28{,}5 \ , \ 33{,}7]$

SIMILARLY, CALCULATE YOURSELF

CASE 1
KNOWN

53. Find the 95% confidence interval for the mean when the distribution is normal. It is known that $\mu = 74.81$ and $\sigma = 4$ when $n = 200$. Solution $74.25 \leq \mu \leq 75.36$

EXAMPLE

54. Laptop should have average weight of at least 2,0kg. Sample contains the weights: 8 laptops 1,90kg, 10 laptops 1,95kg, 12 laptops 1,98kg and 4 laptops 2,05kg. How much underweight the products are if confidence of 95% is used? Solution at least 24g

(1) SAMPLE

(2) σ NOT KNOWN $\Rightarrow$ CASE 2 (WE WILL USE THE t-DISTRIBUTION)

(3) $95\% \Rightarrow p = 1 - 95\% = 1 - 0{,}95 = 0.05$

$\boxed{p = 0{,}05}$

WE CAN FIND FROM A TABLE

(4) $\mu = 1{,}96059$

$n = 34$

$\Rightarrow \boxed{n - 1 = 33}$

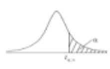
$t_{n-1,\, p} = 1{,}697$?

$R = 1{,}697 \cdot \frac{s}{\sqrt{34}}$

$p = 0{,}05$

Table of the Student's *t*-distribution

The table gives the values of $t_{\alpha;\nu}$ where $\Pr(T_\nu > t_{\alpha;\nu}) = \alpha$, with ν degrees of freedom



ν \ α	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

$n = 33$ NOT FOUND

USED THE ROW 30

HAD TO FIND s BASED ON SAMPLE

$s = 0{,}04505$

$$R = 1.697 \cdot \frac{s}{\sqrt{n}} = \underbrace{1.697}_{\text{TOO BIG ?}} \cdot \frac{0.04505}{\sqrt{34}} = 0.01311$$

INTERVAL $[\mu - R, \mu + R]$ $\mu = 1.96059$

$= [\, 1.947 \,,\, 1.974 \,]$

QUESTION: UNDERWEIGHT ?

TARGET WEIGHT $2.0 - 1.974$ kg

$= 0.026{,}299$ kg

$\approx \underline{26 \text{ g}}$

ANSWER: IT SEEMS THAT THE LAPTOPS ARE $\boxed{26 \text{ g}}$ UNDERWEIGHT

SIMILARLY, CALCULATE YOURSELF

σ NOT KNOWN $\Rightarrow$ CASE 2

55. Average weight of a cell phone was announced to be $0.700kg$. A sample of 10 phones was studied: 6 phones were 692g and 4 phones were 701g. Is the weight in the approved limits, if confidence of 99% is used? Solution $690.8 \leq \mu \leq 700.4g$

$p = 1 - 99\% = 0.01$

HYPOTHESES TESTING

IDEA ① FIND THE CONFIDENCE INTERVAL (OR MANY INTERVALS 95%, 99%, 99.9%)

② IS THE GUESS / HYPOTHESIS IN THE INTERVAL ? $\xrightarrow{\text{YES}}$ ACCEPT THE HYPOTHESIS

$\xrightarrow{\text{NO}}$ REJECT THE HYPOTHESIS

EXAMPLE. (REFORMULATION)

54. ~~Laptop should have average weight of at least 2,0kg.~~ Sample contains the weights: 8 laptops 1,90kg, 10 laptops 1,95kg, 12 laptops 1,98kg and 4 laptops 2,05kg. ~~How much underweight the products are if confidence of 95% is used?~~ ~~Solution at least 24g~~

TEST THE HYPOTHESES $\mu_0 = 2{,}0$ kg, $\mu_1 = 1{,}95$ kg WITH CONFIDENCE LEVEL 95%.

SOLUTION AS ABOVE, WE CAN FIND THE 95% - CONFIDENCE INTERVAL

$[1.947, 1.974]$ $\quad 1.974 < \mu_0$ (NOT OK)

BECAUSE $\mu_0 = 2$ IS EXCLUDED, μ_0 IS REJECTED.

BECAUSE $\mu_1 = 1.95$ IS INCLUDED, μ_1 IS ACCEPTED.

$1.947 < \mu_1 < 1.974$ (ok)

2.1 Confidence interval

PROBABLY THIS KIND OF QUESTION IN THE EXAM

52. Find the 99% confidence interval for the mean when the distribution is normal and $\sigma = 2.5$ with the sample: 30,8; 30,0; 29,9; 30,1; 31,7; 34,0. Solution $28.45 \leq \mu \leq 33.71$

53. Find the 95% confidence interval for the mean when the distribution is normal. It is known that $\mu = 74.81$ and $\sigma = 4$ when $n = 200$. Solution $74.25 \leq \mu \leq 75.36$

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56. Waiting time in IT service hotline (min) was recorded and $X \sim N(\mu, \sigma = 25)$.

97,0	101,5	102,1	103,9	93,4	103,3	104,1	98,6	97,3
96,2	107,7	104,8	98,5	99,2	93,8	100,3	103,7	96,4

Find the confidence intervals for X with confidences 95%, 99% and 99.9%. Solution We have
$88.6 \leq \mu \leq 111.7$
$84.9 \leq \mu \leq 115.3$
$80.7 \leq \mu \leq 119.5$

CALCULATE YOURSELF

2.2 Testing

VOLUNTARY

57. The ultimate tensile strength X of rope was studied ($n = 16$). (The rope is pulled until it breaks.) The mean was $\mu = 4482kg$ and the standard deviation $\sigma = 115kg$. Assume that X is normal distributed. Test the hypothesis $\mu_0 = 4500kg$ compared to the hypothesis $\mu_1 = 4400kg$. Solution $\mu_0 = 4500kg$ accepted, $\mu_1 = 4400$ accepted if $\alpha = 5\%$ or $\alpha = 1\%$

58. Let $X \sim N(\mu, \sigma = 60)$. Test the hypothesis $\mu_0 = 120$ with the sample

115	125	102	129	121	119	120	120	126	120
124	118	116	132	114	108	127	131	130	181

Solution Accepted with all significance levels.

59. Let $X \sim N(\mu, \sigma^2)$. Test the hypothesis $\mu_0 = 55$ with the significance level 5% for the sample

53,08	56,02	57,32	51,76	57,07	59,08	59,00	52,31	54,10	55,78
54,91	60,50	56,81	56,72	58,13	58,31	58,85	54,92	60,69	58,70

Solution Not accepted.

60. Two voltage meters differ by (in Volts): 0,4; -0,6; 0,2; 0,0; 1,0; 1,4; 0,4; 1,6. With significance level 5% can it be assumed that the calibration of the meters do not differ. Solution Accepted. Calibration does not differ.

53,08	56,02	57,32	51,76	57,07	59,08	59,00	52,31	54,10	55,78
54,91	60,50	56,81	56,72	58,13	58,31	58,85	54,92	60,69	58,70

Solution Not accepted.

60. Two voltage meters differ by (in Volts): 0,4; -0,6; 0,2; 0,0; 1,0; 1,4; 0,4; 1,6. With significance level 5% can it be assumed that the calibration of the meters do not differ? Solution Accepted. Calibration does not differ.

61. Assume $X \sim N(\mu, \sigma = 3)$. Compare the hypothesis $\mu_0 = 60.0$ and $\mu_1 = 57.0$ with sample size $n = 20$, mean $\mu = 58.05$ and by choosing significance level $\alpha = 5\%$. Solution $\mu_0 = 60.0$ rejected, $\mu_1 = 57.0$ accepted