Contents

1 Probability

1.1 Probability basics

1. The height distribution of IT students is as follows.

length [m]	number of students	cumulative
1.51 ... 1.53	2	2
1.54 ... 1.56	2	4
1.57 ... 1.59	5	9
1.60 ... 1.62	38	47
1.63 ... 1.65	62	109
1.66 ... 1.68	110	219
1.69 ... 1.71	126	345
1.72 ... 1.74	130	475
1.75 ... 1.77	126	601
1.78 ... 1.80	72	673
1.81 ... 1.83	42	715
1.84 ... 1.86	23	738
1.87 ... 1.89	7	745
1.90 ... 1.92	1	746

   With what probability a student has length

   (a) greater than 180 cm — Solution 0.098

   (b) 163 ... 174 cm — Solution 0.57

   (c) less than 160 cm? — Solution 0.012

2. Students estimated the length of a given segment. Their errors were [cm]

2	3	0	5	6	1	2	4	3	1	3	2
1	0	1	1	0	2	1	1	0	5	0	2
5	3	1	1	2	0	4	3	0	0	2	1
0	3	5	4	2	0	5	3	1	6	2	4
1	1	4	7	2	0	2	1	0	4	4	3

   With what probability a random student estimated the length with at most 1 cm error? — Solution 0.43

3. A car factory collected data; when a car had its first repair done

km	number of first repair cars	cumulative
0 ... 10 000	50	50
10 001 ... 20 000	93	143
20 001 ... 30 000	293	436
30 001 ... 40 000	391	827
40 001 ... 50 000	183	1010
50 001 ...	40	1050

With what probability a car from this factory needs its first repair when it has been driven

(a) at most 20 000km Solution 0.14

(b) 20 001 ... 30 000km Solution 0.28

(c) 30 001 ... 40 000km Solution 0.37

(d) over 40 000km Solution 0.21

(e) find the sum of probabilities a-d. Solution 1

(f) With what probability, a car which did not have repair during kilometers 0-30 000km has to have repair during next 10 000km? Solution 0.64

4. Let's throw two dice. With what probability the sum is

(a) 1 Solution 0

(b) 5 Solution 0.11

(c) 11 Solution 0.056

(d) larger than 7? Solution 0.42

5. Let's throw a coin three times. With what probability

(a) 4 Solution 0

(b) 3 Solution 1/8

(c) 2 Solution 3/8

(d) 1 Solution 3/8

(e) 0 Solution 1/8

heads are obtained?

6. Let's make a two digit number by choosing its digits randomly from 1, 2, 3, 4, and 5. The same digit can appear twice. With what probability the number is divisible by 2 or 5? Solution 0.6

7. Let's throw two dice. With what probability the sum is 10, 11 or 12? Solution 0.17

8. Grade 5 was obtained as follows. In the mathematics exam 15% of students, in the physics exam 12% of students and in both 7% of students. With what probability a random student gets grade 5 in at least one of these exams? Solution 0.2

9. In the pedestrian crossing, the lights are adjusted so that red light is on for 40 s and the green light is on for 20 s. With what probability a pedestrian has to wait max 30 s? Solution 0.83

10. With what probability two picked playing cards are two aces? Solution 0.0045

11. In a box, there are 6 red balls and 4 black balls. Let's pick up two balls without putting them back to the box. With what probability both of the balls are black? Solution 0.13

12. Let's throw a dice four times. With what probability

(a) 2 number "2" Solution 0.12

(b) 4 odd numbers Solution 0.0625

(c) at least one "6" Solution 0.52

are obtained?

13. In a factory, there is a box of 100 circuit boards. Three of the boards are broken. Random two boards are chosen. With what probability at least one of the boards is intact? Solution 0.999

14. Two coins are thrown twice. With what probability

    (a) two heads Solution 0.25

    (b) at least one tail Solution 0.75

    are obtained?

15. Which probability is better: getting an odd number or at most 4 while throwing a dice? Solution at most 4

16. A plane was over booked. With 5 persons in the airport, random 2 are selected to the plane. Adam, Bella, Cecilia, Daniel and Emma are in the queue. With what probability

    (a) With what probability Adam and Emma can board? Solution 0.1

    (b) With what probability one man and one woman can board? Solution 0.6

    (c) With what probability no man can board? Solution 0.3

17. In a lottery, there are 4 tickets. The tickets have the numbers 1, 2, 3 and 4. One ticket is picked, put back to the box, and another ticked is picked. With what probability at least one "1" is obtained? Solution 0.4375

18. Weather forecast announced that the chance of rain on Saturday is 30% and the chance of rain on Sunday is 60%. With what probability, it rains during the weekend? Solution 0.72

1.2 Permutaatiot, k-permutaatiot, kombinaatiot

19. In total 32 students sit in a class. In how many different ways they can sit? Solution $2.63 \cdot 10^{35}$

20. (a) 10 people shake each others hands. How many handshakes are performed? Solution 45

    (b) 8 teams play pairwise. How many matches are played? Solution 28

21. In a questionnaire, there were 6 questions each with 5 options to choose from. In how many ways the questionnaire was possible to answer (you had to answer to each question)? Solution 15 625

22. In a queue, there are 3 boys and 4 girls so that the girls are in the front. How many such queues can be made? Solution 144

23. A student answers 8 questions from 10 options.

    (a) How many ways the student can choose? Solution 45

    (b) How many ways the student can choose, if the first 3 questions are mandatory? Solution 21

24. There are 7 men and 5 women. How many groups with 3 men and 2 women exist? Solution 350

25. A password consists of 5 different symbols. There are 115 symbols available.

    (a) How many different passwords exist, when the order matters and each symbol can be used only once? Solution $1.8 \cdot 10^{10}$

    (b) If you try to guess the password and each guess takes 10 s, how long it would take to make all the guesses? Solution 5840 years

26. Let's consider a deck of playing cards.

    (a) How many permutations does the deck have? Solution $8.1 \cdot 10^{67}$

    (b) With what probability 5 cards drawn contain 4 aces? Solution 0.000018 =1/54145

    (c) With what probability 5 cards drawn contain 3 clubs and 2 spades? Solution 0.00858

27. For an entrance exam, 20 math questions and 15 physics questions are considered. The questions are chosen in random and their order does not matter. With what probability chosen 6 questions

    (a) are all math questions? Solution 0.024

    (b) contain 3 math questions and 3 physics questions? Solution 0.32

28. A dice is rolled 10 times. With what probability exactly 3 results "6" are obtained?

29. In football betting, the result of 13 matches is guessed at random. Each macth has 3 options (1,x,2) (This means (home wins, tie, guest wins).) With what probability the result of 12 matches is correct? Solution 0.000 016 3

30. English test contains 40 questions. Each question has 4 options. The student is guessing.

    (a) With what probability all answers are correct? Solution $8.28 \cdot 10^{-25}$

    (b) With what probability exactly 5 answers are correct? Solution 0.02723

    (c) With what probability at least 5 answers are correct? Solution 0.984

1.3 Probability distributions

31. A wheel of fortune has 8 equal sectors, one of which is a joker sector. Let X be a random variable which tells the number of jokers in 3 spins. Find the distribution and find the expected value. Solution $\mu = 0.4$

32. A basket ball player has probability 0.70 to score. He gets two throws. What is the expected value of scores? Solution 1.4

33. Let $X \sim Bin(n, p)$. Find n and p, when $\mu = 2$ and $\sigma^2 = \frac{4}{3}$. Solution $n = 6$ and $p = 1/3$

34. A dice is thrown 4 times. Find the distribution of getting a "6". Solution $Bin(4, 1/6), \mu = 1.6$ and $\sigma = 0.57$

35. A factory is doing quality inspection. A product is accepted with probability 0.8. Take two random products. Find the distribution, expected value and standard error. Solution $\mu =$

36. In a box, there are 3 black balls and 3 white balls. Three balls are drawed. Find the expected value of the number of white balls. Solution $\mu = 1.5$

37. There are 5 envelopes with 10€, 20€, 30€, 40€ and 50€ respectively. The winner chooses 2 envelopes. Random variable X tells the total amount won. Find the expected value. Solution $60e$

38. There are 1000 tickets. Price 100€ in 1 ticket, price 50€ in 10 tickets, price 20€ in 15 tickets. One ticket costs 1€. Find the expected value of the net win. Solution $\mu = -0.1e.$

39. Two coins are tossed. If you get one head, you win 20€. If you get 2 heads, you win 40€. If you get 2 tails, you lose 100€. Find the expected value of the win. Solution $\mu = -5e.$

40. A wallet contains six 1€ coins, four 2€ coins, and two 0.50€ coins. One coin is picked up. Find the distribution.

41. Students threw six coins 90 times. The number of heads was counted.

0	2	3	2	4	3	4	3	3	4	1	4	3	4	5	3	4	4	4	4	3	2	1
4	2	3	4	2	5	3	4	2	4	2	1	2	4	2	1	4	3	2	2	1	5	2
3	1	5	3	3	3	1	3	4	2	3	4	1	3	5	5	3	2	5	3	2	3	4
2	3	3	3	2	3	4	5	5	0	5	2	3	3	4	3	0	4	1	2	4		

    Find the distribution.

1.4 Poisson distribution

42. A controller has a malfunction during a week with probability 0.007. A company maintains 900 controllers. Use the Poisson distribution to find the probability that

    (a) 7 Solution 0.14

    (b) more than 3 Solution 0.87

    controllers have a malfunction during a week.

43. On average, two cars arrive to a parking place during a minute. With what probability, during any given minute, 4 or more cars arrive to the parking place? Solution 0.14

44. A school has many computers. During one month, usually one computer breaks. With what probability less than two computers break? Solution 0.74

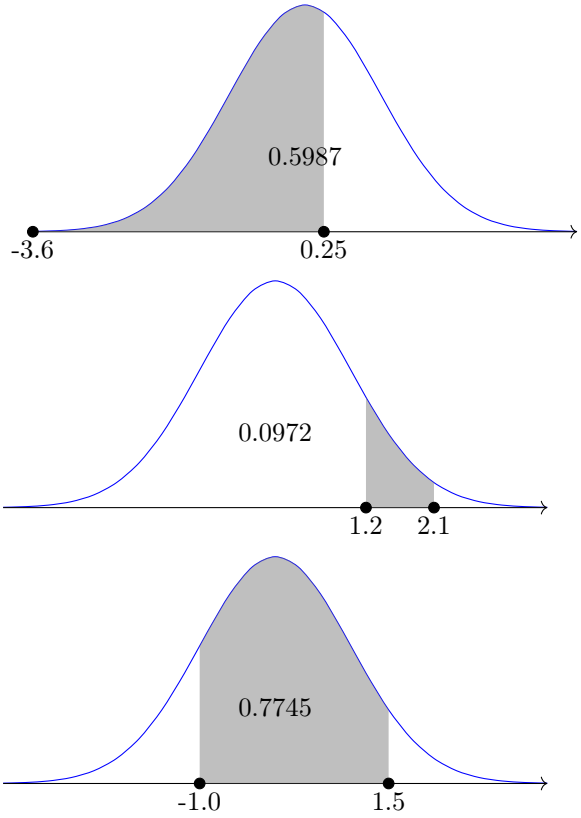
1.5 Normal distribution

45. A random variable X is normal distributed. Find the probabilities.

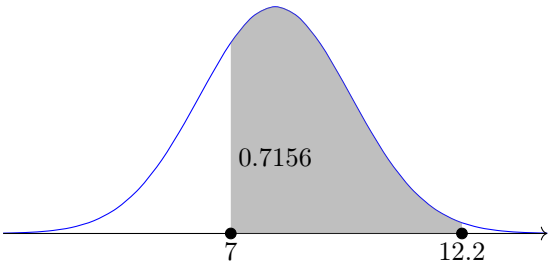
    (a) $P(z \leq 0.25)$ Solution 0.5987

    (b) $P(1.2 \leq z \leq 2.1)$ Solution 0.0972

    (c) $(P(-1 \leq z \leq 1.5)$ Solution 0.7745



46. A random variable X is normal distributed. Expected value is $\mu = 8$ and standard deviation is $\sigma = 1.7$. With what probability $7 \leq X \leq 12.2$? Solution 0.7156



47. A company produces mobile phone batteries. The life of a battery in years is normal distributed $N(\mu = 3; \sigma = 1.2)$. One batch is 7000 batteries. How many of them work after 5 years? Solution 330

48. The height of men is normal distributed with $\mu = 175cm$ and $\sigma = 8cm$. With what probability a man is over 2 meters tall? Solution 0.001%

49. Weight of product is normal distributed $N(\mu = 80kg; \sigma = 4kg)$.

    (a) With what probability a random product has weight 77 ... 80kg? Solution 0.27

    (b) With what probability a random product has weight over 80kg? Solution 0.5

50. Let $X \sim N(\mu = 5; \sigma = 0.2)$. Find k with probabilities

    (a) $P(X \leq k) = 95\%$ Solution 5.328

    (b) $P(X > k) = 1\%$ Solution 5.466

51. The resistance of a resistor is normal distributed $N(\mu = 3\ \Omega; \sigma = 0.1\ \Omega)$. With what probability the resistance is

    (a) 2.9...3.05 Ω Solution 0.5328

    (b) more than 2.95 Ω Solution 0.6915

    (c) less than 2.83 Ω. Solution 0.0446

2 Statistics

2.1 Confidence interval

52. Find the 99% confidence interval for the mean when the distribution is normal and $\sigma = 2.5$ with the sample: 30,8; 30,0; 29,9; 30,1; 31,7; 34,0. Solution $28.45 \leq \mu \leq 33.71$

53. Find the 95% confidence interval for the mean when the distribution is normal. It is known that $\mu = 74.81$ and $\sigma = 4$ when $n = 200$. Solution $74.25 \leq \mu \leq 75.36$

54. Laptop should have average weight of at least 2,0kg. Sample contains the weights: 8 laptops 1,90kg, 10 laptops 1,95kg, 12 laptops 1,98kg and 4 laptops 2,05kg. How much underweight the products are if confidence of 95% is used? Solution at least 24g

55. Average weight of a cell phone was announced to be $0.700kg$. A sample of 10 phones was studied: 6 phones were 692g and 4 phones were 701g. Is the weight in the approved limits, if confidence of 99% is used? Solution $690.8 \leq \mu \leq 700.4g$

56. Waiting time in IT service hotline (min) was recorded and $X \sim N(\mu, \sigma = 25)$.

97,0	101,5	102,1	103,9	93,4	103,3	104,1	98,6	97,3
96,2	107,7	104,8	98,5	99,2	93,8	100,3	103,7	96,4

    Find the confidence intervals for X with confidences 95%, 99% and 99.9%. Solution We have
    $88.6 \leq \mu \leq 111.7$
    $84.9 \leq \mu \leq 115.3$
    $80.7 \leq \mu \leq 119.5$

2.2 Testing

57. The ultimate tensile strength X of rope was studied ($n = 16$). (The rope is pulled until it breaks.) The mean was $\mu = 4482kg$ and the standard deviation $\sigma = 115kg$. Assume that X is normal distributed. Test the hypothesis $\mu_0 = 4500kg$ compared to the hypothesis $\mu_1 = 4400kg$. Solution $\mu_0 = 4500kg$ accepted,$\mu_1 = 4400$ accepted if $\alpha = 5\%$ or $\alpha = 1\%$

58. Let $X \sim N(\mu, \sigma = 60)$. Test the hypothesis $\mu_0 = 120$ with the sample

115	125	102	129	121	119	120	120	126	120
124	118	116	132	114	108	127	131	130	181

Solution Accepted with all significance levels.

59. Let $X \sim N(\mu, \sigma^2)$. Test the hypothesis $\mu_0 = 55$ with the significance level 5% for the sample

53,08	56,02	57,32	51,76	57,07	59,08	59,00	52,31	54,10	55,78
54,91	60,50	56,81	56,72	58,13	58,31	58,85	54,92	60,69	58,70

Solution Not accepted.

60. Two voltage meters differ by (in Volts): 0,4; -0,6; 0,2; 0,0; 1,0; 1,4; 0,4; 1,6. With significance level 5% can it be assumed that the calibration of the meters do not differ? Solution Accepted. Calibration does not differ.

61. Assume $X \sim N(\mu, \sigma = 3)$. Compare the hypothesis $\mu_0 = 60.0$ and $\mu_1 = 57.0$ with sample size $n = 20$, mean $\mu = 58.05$ and by choosing significance level $\alpha = 5\%$. Solution $\mu_0 = 60.0$ rejected, $\mu_1 = 57.0$ accepted

3 Formulas and tables

3.1 Formulas

permutations of n: in how many ways n elements can be arranged

$$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1)n$$

k-permutations of n: k-element subset of n element set can be chosen and arranged

$$\frac{n!}{(n-k)!} = (n-k+1)(n-k+2)\ldots(n-1)n$$

combinations: k-element subset of n element set can chosen

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

3.1.1 Some distributions

PMF = probability mass function
μ mean
σ^2 variance
σ standard deviation

$$\mu = \sum_{k=0}^{\infty} k \cdot p_k$$

$$\sigma^2 = \sum_{k=0}^{\infty} (k-\mu)^2 \cdot p_k$$

Binomial distribution

$$pmf = \binom{n}{k} p^k (1-p)^k, \quad \mu = np, \quad \sigma^2 = np(1-p)$$

Poisson distribution (For $Bin(n,p)$ let $\lambda = np$ and let $n \to \infty$.)

$$pmf = \frac{\lambda^k}{k!} e^{-\lambda}, \quad \mu = \lambda \quad \sigma^2 \lambda$$

Normal distribution

$$pmf = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad \mu = \mu, \quad \sigma = \sigma$$

Normalized normal distribution

$$pmf = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, \quad \mu = 1, \quad \sigma = 1$$

$$P(A) = \frac{\text{suotuisten alkeistapausten lukumäärä}}{\text{kaikkien alkeistapausten lukumäärä}}$$

Complementary event $\bar{A}$ satisfies $P(\bar{A}) = 1 - P(A)$
Always $P(A \; or \; B) = P(A) + P(B) - P(AjaB)$
Independent events $P(A \; or \; B) = P(A) + P(B)$
Always $P(A \; and \; B) = P(A) \cdot P(B|A)$
Independent events $P(A \; and \; B) = P(A) \cdot P(B)$
Expected value

$$E(X) = \sum_{i=1}^{n} x_i p_i$$

Standard deviation σ Variance

$$\sigma^2 = Var(x) = E((X-\mu)^2)$$

Uniform distribution

$$E(X) = \frac{b-a}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}$$

Normal distribution $P(a \leq X \leq b) = \Phi(b) - \Phi(a)$ $P(X \leq b) = \Phi(b)$ $P(a \leq X) = \Phi(\infty) - \Phi(a) = 1 - \Phi(a) = \Phi(-a)$

3.2 Pascal triangle

At row n, value k is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

1

1 + 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

1 11 55 165 330 462 462 330 165 55 11 1

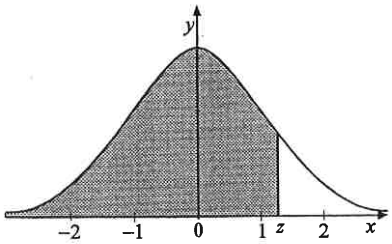
1 12 66 220 495 792 924 792 495 220 66 12 1

1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1

1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1

1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1

NORMED NORMAL DISTRIBUTION



$$\Phi(z) = P(X \le z) = \int_{-\infty}^{z} f(x)\,dx$$

Values of the cumulative function of the normed normal distribution.

Example

$P(X < -1{,}2) = 1 - P(X \le 1{,}2)$
$= 0{,}1151$

z	0	1	2	3	4	5	6	7	8	9
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,9970	0,9971	0,9972	0,9973	0,9974
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,9980	0,9981
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986
3,	0,9987	0,9990	0,9993	0,9995	0,9997	0,9998	0,9998	0,9999	0,9999	1,0000

Critical values of *t* for two-tailed tests

Significance level (α)

Degrees of freedom *(df)*	.2	.15	.1	.05	.025	.01	.005	.001
1	3.078	4.165	6.314	12.706	25.452	63.657	127.321	636.619
2	1.886	2.282	2.920	4.303	6.205	9.925	14.089	31.599
3	1.638	1.924	2.353	3.182	4.177	5.841	7.453	12.924
4	1.533	1.778	2.132	2.776	3.495	4.604	5.598	8.610
5	1.476	1.699	2.015	2.571	3.163	4.032	4.773	6.869
6	1.440	1.650	1.943	2.447	2.969	3.707	4.317	5.959
7	1.415	1.617	1.895	2.365	2.841	3.499	4.029	5.408
8	1.397	1.592	1.860	2.306	2.752	3.355	3.833	5.041
9	1.383	1.574	1.833	2.262	2.685	3.250	3.690	4.781
10	1.372	1.559	1.812	2.228	2.634	3.169	3.581	4.587
11	1.363	1.548	1.796	2.201	2.593	3.106	3.497	4.437
12	1.356	1.538	1.782	2.179	2.560	3.055	3.428	4.318
13	1.350	1.530	1.771	2.160	2.533	3.012	3.372	4.221
14	1.345	1.523	1.761	2.145	2.510	2.977	3.326	4.140
15	1.341	1.517	1.753	2.131	2.490	2.947	3.286	4.073
16	1.337	1.512	1.746	2.120	2.473	2.921	3.252	4.015
17	1.333	1.508	1.740	2.110	2.458	2.898	3.222	3.965
18	1.330	1.504	1.734	2.101	2.445	2.878	3.197	3.922
19	1.328	1.500	1.729	2.093	2.433	2.861	3.174	3.883
20	1.325	1.497	1.725	2.086	2.423	2.845	3.153	3.850
21	1.323	1.494	1.721	2.080	2.414	2.831	3.135	3.819
22	1.321	1.492	1.717	2.074	2.405	2.819	3.119	3.792
23	1.319	1.489	1.714	2.069	2.398	2.807	3.104	3.768
24	1.318	1.487	1.711	2.064	2.391	2.797	3.091	3.745
25	1.316	1.485	1.708	2.060	2.385	2.787	3.078	3.725
26	1.315	1.483	1.706	2.056	2.379	2.779	3.067	3.707
27	1.314	1.482	1.703	2.052	2.373	2.771	3.057	3.690
28	1.313	1.480	1.701	2.048	2.368	2.763	3.047	3.674
29	1.311	1.479	1.699	2.045	2.364	2.756	3.038	3.659
30	1.310	1.477	1.697	2.042	2.360	2.750	3.030	3.646
40	1.303	1.468	1.684	2.021	2.329	2.704	2.971	3.551
50	1.299	1.462	1.676	2.009	2.311	2.678	2.937	3.496
60	1.296	1.458	1.671	2.000	2.299	2.660	2.915	3.460
70	1.294	1.456	1.667	1.994	2.291	2.648	2.899	3.435
80	1.292	1.453	1.664	1.990	2.284	2.639	2.887	3.416
100	1.290	1.451	1.660	1.984	2.276	2.626	2.871	3.390
1000	1.282	1.441	1.646	1.962	2.245	2.581	2.813	3.300
Infinite	1.282	1.440	1.645	1.960	2.241	2.576	2.807	3.291

Critical values of *t* for one-tailed tests

Significance level (α)

Degrees of freedom *(df)*	.2	.15	.1	.05	.025	.01	.005	.001
1	1.376	1.963	3.078	6.314	12.706	31.821	63.657	318.309
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173
5	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297
10	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144
11	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025
12	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930
13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787
15	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733
16	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686
17	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646
18	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610
19	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579
20	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552
21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527
22	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505
23	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485
24	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467
25	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450
26	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435
27	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421
28	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408
29	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396
30	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385
40	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307
50	0.849	1.047	1.299	1.676	2.009	2.403	2.678	3.261
60	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232
70	0.847	1.044	1.294	1.667	1.994	2.381	2.648	3.211
80	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195
100	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174
1000	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098
Infinite	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090

References

[1] A. Croft, R. Davison, M. Hargreaves, and J. Flint, Engineering Mathematics, 5th ed. (Pearson, Harlow, 2017)

[2] Calculus Volume 1, OpenStax (2016). https://openstax.org/details/books/calculus-volume-1

[3] Calculus Volume 2, OpenStax (2016). https://openstax.org/details/books/calculus-volume-2

[4] Stein, E.M., and R. Shakarchi Fourier Analysis: An Introduction. Princeton University Press (2003).