ESMTB Distinguished Speaker
Odo Diekmann (*Utrecht University,* Netherlands)

Delay equations in biological context

Abstract. A delay equation is a rule for extending a function of time towards the future on the basis of the (assumed to be) known past. In population biology, the rule usually is a renewal equation specifying the population birth rate at the present time on the basis of the current population size and composition which, in turn, are determined from the birth rate in the past and models specifying development and survival of individuals. In other fields of application, like laser physics, the rule often takes the form of a differential equation incorporating delay.

By translation along the extended function (i.e., by updating the history) one defines a dynamical system. So it has technical advantages to choose a function space that makes translation a continuous operation. The rule concerns the value (or its derivative) in one point. So it has technical advantages to choose a function space such that the value in one point is well-defined and yet not constrained by values in nearby points. The theory of delay equations is nontrivial because one cannot have both technical advantages at the same time.

The aim of the lecture is to sketch some biological context, some aspects of the theory and some prospects for numerical bifurcation analysis.

Anne-Maria Ernvall-Hytönen (*Åbo Akademi,* Finland)

Measuring in transcendental number theory

Abstract. In transcendental number theory, it is possible to measure numbers in different ways. In this setting, however, the measuring does not correspond to how large a number is. One may use the irrationality measure which essentially tells how well a number can be approximated with rational numbers with a relatively small denominator. The transcendental number is more involved. It measures how far from zero we can bound the values of polynomials with integer coefficients evaluated at a given transcendental number.

In my talk, I will briefly tell about approximations by rational numbers and irrationality measures. I will then move to the transcendence measure of Napier's constant e, and I will briefly tell about the historical bounds, what is known now and what are the theoretical limits. At the end of the talk, I will tell how the transcendence measure can be generalized.

This is joint work with Kalle Leppälä, Tapani Matala-aho and Louna Seppälä.

Pauliina Ilmonen (*Aalto University*, Finland)

On typicality of functional observations

Abstract. With the rapid increase in measurement precision and storage capacity, we have seen a tremendous jump in the dimensionality of data. One of the common methodologies used when dealing with such high dimensional data is to assume that the observed units are random functions (from some generating process) instead of random vectors.

The concept of statistical depth was originally introduced as a way to provide a center-outward ordering from a depth-based multivariate median. Several different depth functions for functional observations have been presented in the literature. Most of these approaches, however, are solely interested in the -pointwise- centrality of the functions as a measure of (global) centrality. As a result, they are missing some important features inherent to functional data such as variation in shape, roughness or range. Thus, due to the richness of functional data, we opt to talk about typicality rather than centrality of an observation.

We discuss assessing typicality of functional observations. Moreover, we provide a new concept of depth for functional data. It is based on a new multivariate Pareto depth applied after mapping the functional observations to a vector of statistics of interest. These quantities allow incorporating the inherent features of the distribution, such as shape or roughness. In particular, in contrast to most existing functional depths, the method is not limited to centrality only. Properties of the new depth are explored and the benefits of a flexible choice of features are illustrated.

(S. Helander, G. Van Bever, S. Rantala, P. Ilmonen)

Jarkko Kari (*University of Turku*, Finland)

An Algebraic Geometric Approach to Multidimensional Symbolic Dynamics

Abstract. We study low complexity multidimensional words and subshifts using tools of algebraic geometry. The low complexity assumption is that, for some finite shape D, the word or the subshift has at most |D| distinct patterns of shape D. We express words as multivariate formal power series over integers and notice that the low complexity assumption implies that there is an annihilating polynomial: a polynomial whose formal product with the power series is zero. We prove that the word must then be a sum of periodic words over integers, possibly with unbounded values. As a specific application of the method we obtain an asymptotic version of the well-known Nivat's conjecture: we can show that a two-dimensional word that has low complexity with respect to arbitrarily large rectangles D must be periodic.

EMS Distinguished Speaker

Benoît Perthame (*Laboratoire Jacques-Louis Lions, Sorbonne Université*, France)

Tumor growth: from compressible models to free boundaries

Abstract. Tissue growth, as in solid tumors, can be described at a number of different scales from the cell to the organ. For a large number of cells, the 'fluid mechanical' approach has been advocated recently by many authors in mathematics or biophysics. Several levels of mathematical descriptions are commonly used, including possibly elasticity, visco-elastic laws, nutrients, active movement, surrounding tissue, vasculature remodeling and several other features.

We will focus on the links between two types of mathematical models. The 'microscopic' or 'compressible' description is at the cell population density level and a more macroscopic or 'incompressible' description is based on a free boundary problem close to the classical Hele-Shaw equation. In the stiff pressure limit, we are going to derive a weak formulation of the corresponding Hele-Shaw free boundary problem and we will make the connection with its geometric form.

Including additional features also opens other questions as circumstances in which singularities and instabilities may develop.

Brett Wick (*Washington University - St. Louis*, USA)

The Corona Theorem

Abstract. Carleson's Corona Theorem from the 1960's has served as a major motivation for many results in complex function theory, operator theory and harmonic analysis. In a simple form, the result states that for N bounded analytic functions $f_1, \ldots, f_N$ on the unit disc such that $\inf(|f_1| + \cdots + |f_N|) \geq \delta > 0$ it is possible to find N other bounded analytic functions $g_1, \ldots, g_N$ such that $f_1 g_1 + \cdots + f_N g_N = 1$. Moreover, the functions $g_1, \ldots, g_N$ can be chosen with some norm control.

In this talk we will discuss some generalizations of this result to certain vector valued functions and connections with geometry and to function spaces on the unit ball in several complex variables.