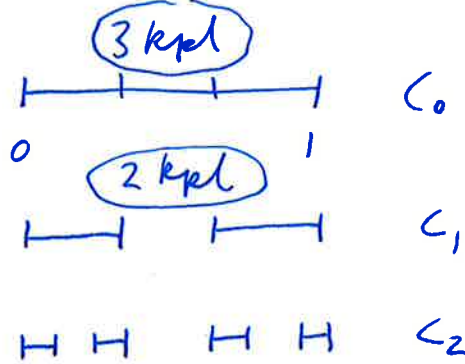
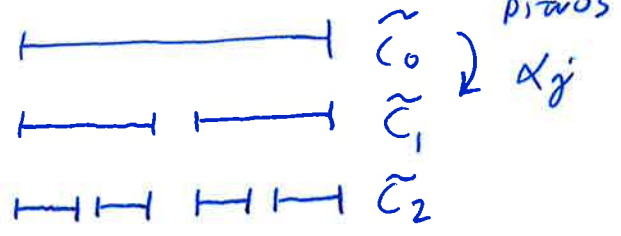


VII MEKSI



PITUUS  $\cdot \frac{2}{3}$



$$C = \bigcap_{i=1}^{\infty} C_i$$

$$\tilde{C} = \bigcap_{i=1}^{\infty} \tilde{C}_i$$

$$m(\tilde{C}) = \prod_{i=1}^{\infty} \alpha_j = a \in (0, 1)$$

- CANTORIN JOUKKO
- YLINUMEROITAVA

$$m(C) = 0$$

$$m(C) = \prod_{i=1}^{\infty} \frac{2}{3} = 0$$

MITEN?  

$$\sum_{i=1}^{\infty} \ln(\alpha_j) = \ln a$$
  
 ESIM. GEOMETRINEN SARJA

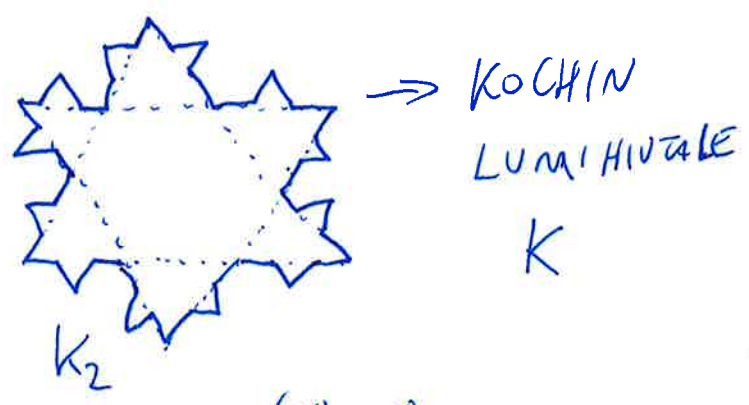
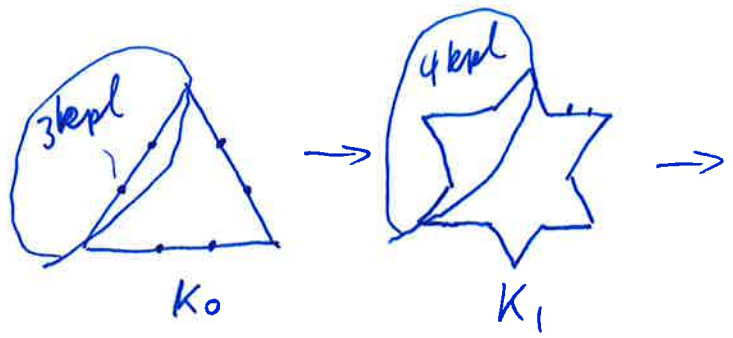
JOUKON C HAUSDORFFIN DIMENSIO

$$\dim_H(C) = \frac{\ln 2}{\ln 3} \approx 0.6309...$$

$$m_0(C) = \#C = \infty$$

$$m_1(C) = 0$$

$$m_{\dim_H(C)} = 1$$



$$\dim_H(K) = \frac{\ln 4}{\ln 3} \approx 1.2619$$

$$m_1(K) = \infty$$

$$m_2(K) = 0$$

$$m_{\dim_H(K)} = ?$$

LISÄÄ: MITRO MAKKONEN GRADU

$E_1$  MITALLINEN

$$\Rightarrow m^*(B) = m^*(B \cap E_1) + m^*(B \setminus E_1) \quad \forall B \subset \mathbb{R}^n$$

VALITTAAN  $B = A \cap E_1 \cup E_2$

$$\begin{aligned} \Rightarrow m^*(A \cap (E_1 \cup E_2)) &= m^*((A \cap E_1 \cup E_2) \cap E_1) \\ &\quad + m^*((A \cap E_1 \cup E_2) \cap E_1^c) \\ &= m^*(A \cap E_1) + m^*(A \cap E_2) \end{aligned}$$

$$\Rightarrow m^*(A \cap \bigcup_{i=1}^2 E_i) = \sum_{i=1}^2 m^*(A \cap E_i) \quad (*)$$

INDUKTIO - OLETUS

$$m^*(A \cap \bigcup_{i=1}^k E_i) = \sum_{i=1}^k m^*(A \cap E_i)$$

JOLLAKIN KEHU.

NYT

$$m^*(A \cap \bigcup_{i=1}^{k+1} E_i) = m^*(A \cap \left( E_{k+1} \cup \underbrace{\bigcup_{i=1}^k E_i}_{=C} \right))$$

$$= m^*(A \cap (E_{k+1} \cup C))$$

$$= (*) \quad m^*(A \cap E_{k+1}) + m^*(A \cap C)$$

$$= m^*(A \cap E_{k+1}) + m^*(A \cap \bigcup_{i=1}^k E_i)$$

$$\stackrel{1.0.0.}{=} m^*(A \cap E_{k+1}) + \sum_{i=1}^k m^*(A \cap E_i)$$

$$= \sum_{i=1}^{k+1} m^*(A \cap E_i)$$

INDUKTIOPERIAATE  $\rightarrow$

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LEMMA 2.4.5 PÄTEE KÄIKILLÄ KEN.  $\square$

