AJATUS FATOUN LEMMAN TODISTUKSEKSI

f_i FUNKTIOITA, $f_i : E \to [0,\infty]$, $E \subset \mathbb{R}^n$

(61)

$$\inf_{i \geq k} f_i(x) \leq f_i(x) \quad \forall i \geq k \quad \forall x \in E$$

$$\Rightarrow \underbrace{\int_E \inf_{i \geq k} f_i(x)\, dm}_{\text{ERÄS ALARAJA}} \leq \int_E f_i(x)\, dm \quad \forall i \geq k$$

$$\Rightarrow \int_E \inf_{i \geq k} f_i(x)\, dm \leq \inf_{i \geq k} \int_E f_i(x)\, dm \quad \forall k \in \mathbb{N}$$

$$\Rightarrow \int_E \inf_{i \geq k} f_i(x)\, dm \leq \underbrace{\sup_{k \in \mathbb{N}} \inf_{i \geq k}}\int_E f_i(x)\, dm$$

$$= \liminf_{i \to \infty} \int_E f_i(x)\, dm$$

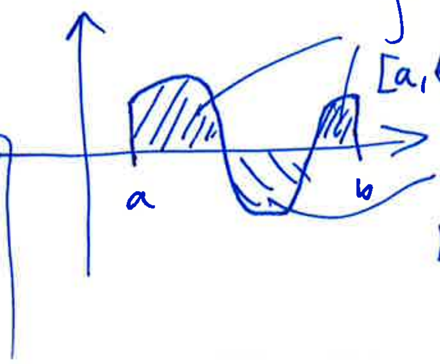
$$\Rightarrow \sup_{k \in \mathbb{N}} \overset{?}{\int_E} \inf_{i \geq k} f_i(x)\, dm \leq \liminf_{i \to \infty} \int_E f_i(x)\, dm$$

$$\overset{?}{\Rightarrow} \int_E \liminf_{i \to \infty} f_i\, dm \leq \liminf_{i \to \infty} \int_E f_i\, dm$$

$f^+ = \max(f, 0)$ $\qquad f = f^+ - f^-$

$f^- = \max(-f, 0)$

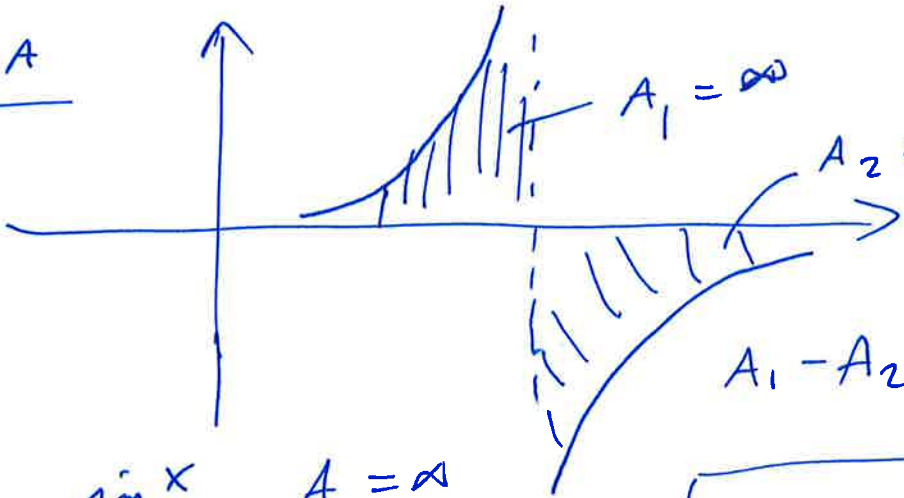
$$\int_{[a,b]} f\, dm = \int_{[a,b]} f^+\, dm - \int_{[a,b]} f^-\, dm$$



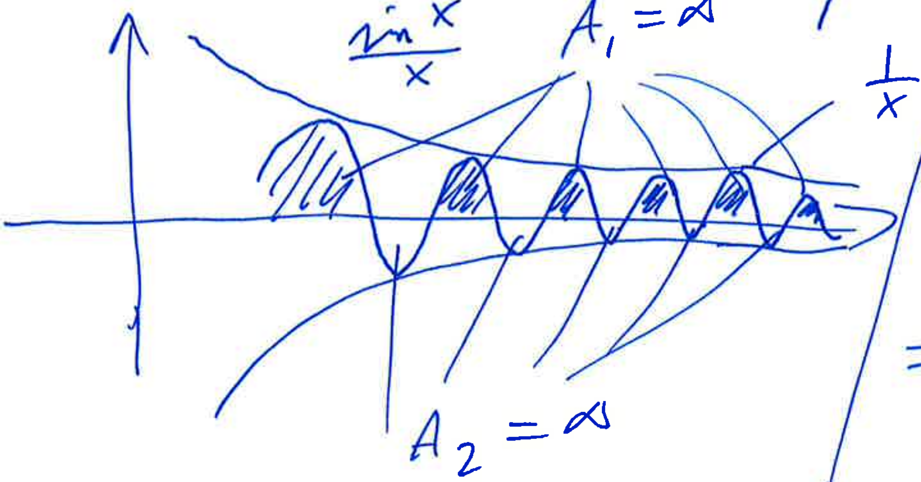
$\int_{[a,b]} f^+\, dm$

$\int_{[a,b]} f^-\, dm$

ONGELMIA



$A_1 - A_2$ EI MÄÄR.



$$\int_1^{\infty} \frac{\sin x}{x}\,dx$$

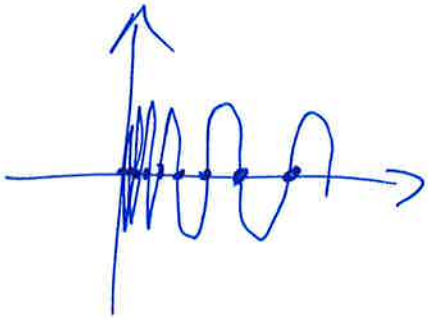
$$= \lim_{n\to\infty} \int_1^{n} \frac{\sin x}{x}\,dx \in \mathbb{R}$$

ON OLEMASSA

$$\int_0^1 \frac{dx}{x} = \infty$$

$$\int_0^1 \frac{dx}{\sqrt{x}} \in \mathbb{R}$$

$$\frac{1}{\sin \frac{1}{x}} \qquad [0,1]$$



$$f(x) = \frac{1}{\sqrt{|\sin \frac{1}{x}|}}$$

ON INTEGROITUVA VÄLILLÄ $[0,1]$

$f(x) = \infty$ MONILLA $x \in [0,1]$

(AINA KUN $\sin \frac{1}{x} = 0$).

$$g(x) = -\frac{1}{(|\sin \frac{1}{x}|)^{\frac{1}{3}}}$$

MONISSA PISTEISSÄ (NUMEROITUVAN MONTA)

$f(x) = \infty$, $g(x) = -\infty$

$h(x) = f(x) + g(x)$ EI HYVIN MÄÄR (MUOTOA $\infty - \infty$)

MERK. $F = \{x \in (0,1) : \sin \frac{1}{x} = 0\}$

ASETETAAN
$$h(x) = \begin{cases} f(x) + g(x), & x \in [0,1] \setminus F \\ 0, & x \in F \end{cases}$$

NUMEROITUVA, $m(F) = 0$

64

$$\int_{[0,1]} h \, dm = \int_{[0,1]\setminus F} h \, dm + \underbrace{\int_F h \, dm}_{=0}, \quad m(F) = 0$$

$$= \int_{[0,1]\setminus F} f + g \, dm + 0$$

$f(x) = x^{-\frac{1}{2}}$ $\int x^{-\frac{1}{2}} dx = 2x^{\frac{1}{2}} + c$ 65

$$\int_{\frac{1}{k}}^{1} f(x)\,dx = \Big/_{\frac{1}{k}}^{1} 2x^{\frac{1}{2}} = 2 \Big/_{\frac{1}{k}}^{1} x^{\frac{1}{2}}$$

$$= 2\left(1 - \left(\frac{1}{k}\right)^{\frac{1}{2}}\right) = 2 - 2\sqrt{\frac{1}{k}}$$