PAPER

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Spatial coherence effects in second-harmonic generation of scalar light fields

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Abstract

We consider the spectral spatial coherence characteristics of scalar light fields in second-harmonic generation in an optically non-linear medium. Specifically, we take the fundamental-frequency (incident) field to be a Gaussian Schell-model (GSM) beam with variable peak spectral density and different coherence properties. We show that with increasing intensity the overall degree of coherence of both the fundamental and the second-harmonic field in general decreases on passage through the non-linear medium. In addition, the spectral density distributions and the two-point degree of coherence may, for both beams, deviate significantly from those of the GSM, especially at high intensities. Propagation in the non-linear medium is numerically analyzed with the Runge–Kutta and the beam-propagation methods, of which the latter is found to be considerably faster. The results of this work provide means to synthesize, via non-linear material interaction, random optical beams with desired coherence characteristics.

Keywords: coherence theory, non-linear optics, second-harmonic generation

(Some figures may appear in colour only in the online journal)

1. Introduction

Optical coherence [1, 2] and non-linear optics [3, 4] are central research areas of modern optics. Both topics are extensive but the influence of partial optical coherence (temporal, spatial, or spectral) in non-linear light–matter interactions has been analyzed only in a few specific cases. For example, in the context of second-harmonic generation (SHG), the conversion efficiency with an incident spatially [5] and temporally [6] partially coherent beam as well as with astigmatic beams [7] has been considered. Further, the spectral properties of the second-harmonic field induced by a Gaussian Schell-model (GSM) beam [8] and an incoherent conical beam [9] have been investigated. Other researches cover, e.g. the effect of an incoherent pump beam in parametric amplification [10], the spatial coherence of local second-harmonic fields at rough metal surfaces [11], pulse propagation in Kerr medium [12], and supercontinuum coherence [13–15].

In this work, we assess, within the scalar-field formalism, the coherence properties of the second-harmonic field produced by a stationary GSM beam in a non-linear optical material. We use the depleted (incident) beam model that takes into account the effect of the non-linear interaction on the incident fundamental-frequency GSM beam. Hence, besides the second-harmonic field, we also consider the coherence changes in the fundamental-frequency beam. The random GSM beam in front of the crystal is represented by constructing an ensemble of monochromatic (random-shaped) realizations. Each realization is then propagated through the non-linear crystal one at a time using the Runge–Kutta (RK) algorithm and the beam-propagation (BP) method tailored for the present context. Both techniques provide highly similar results, but the latter technique is observed to be two orders of magnitude faster. In general, the degrees of coherence of both the fundamental and the second-harmonic wave are found to decrease on passage through the crystal. The origin of this effect is the second-harmonic creation at the strongest peaks of the fundamental-wave realizations which increases the structural complexity in the realizations of both fields. The effect is stronger for higher incident-field peak intensities leading to fundamental and second-harmonic fields whose coherence properties may deviate significantly from the GSM. The non-linear material response can therefore be used to control and tailor the coherence properties of random beams.

This work is organized as follows. In section 2 the GSM beam and its propagation in a non-linear medium supporting SHG are described. Section 3 is devoted to the numerical analysis of the coherence effects taking place in SHG and section 4 is a summary of the main results. Several theoretical aspects have been relegated to appendices A–D. In A the construction of an ensemble representing a GSM beam is described. In B and C the RK and BP propagation methods, respectively, are outlined and D evaluates the sufficient number of realizations.

2. SHG with a GSM beam

In this section, we introduce the relevant concepts concerning the GSM beams and their propagation in a non-linear medium exhibiting SHG.

2.1. GSM beams

The spatial coherence properties of a stationary scalar light beam at points x_1 and x_2 in a plane $z = z_0$ and at frequency ω are described by the cross-spectral density (CSD) function. The CSD can be defined as [1]

$$W(x_1,x_2,z_0;\omega) = \langle E^*(x_1,z_0;\omega)E(x_2,z_0;\omega)\rangle, \tag{1}$$

where the asterisk denotes complex conjugation and the angular brackets refer to ensemble averaging. In addition, $E(x,z_0;\omega)$ is a monochromatic field realization representing a random electric field which in this work is taken to be linearly polarized. Also, the field generated in non-linear interaction is similarly linearly polarized allowing a scalar treatment of all fields. In this section, we drop the explicit frequency and z dependencies for notational simplicity and assume that the formulas are given at the entrance facet of a non-linear crystal. By setting $x_1 = x_2 = x$ we obtain the (average) spectral density of the field as $S(x) = W(x,x)$. The normalized CSD, namely the complex (spectral) degree of spatial coherence, reads

$$\mu(x_1,x_2) = \frac{W(x_1,x_2)}{\sqrt{S(x_1)S(x_2)}}. \tag{2}$$

It is known that the CSD admits the coherent-mode representation [1, 2], i.e. an expansion in terms of the mutually uncorrelated, spatially fully coherent modes. This is explicitly given by

$$W(x_1,x_2) = \sum_{m=0}^{\infty} \alpha_m \psi_m^*(x_1)\psi_m(x_2), \tag{3}$$

where the weights α_m are real and non-negative and the mode functions $\psi_m(x)$ are orthonormal in the considered region. The eigenvalues and eigenfunctions are solutions of a Fredhold integral equation [1]. For a GSM beam the mode functions are of Hermite–Gaussian (HG) form, written as [1, 16]

$$\psi_m(x) = \frac{(2/\pi)^{1/4}}{\sqrt{2^m m!}w_0} H_m\left(\frac{\sqrt{2}x}{w_0}\right)\exp\left(-\frac{x^2}{w_0^2}\right), \tag{4}$$

while the modal weights are

$$\alpha_m = S_0\sqrt{\frac{2\pi}{\beta}}\frac{w_0}{1+1/\beta}\left(\frac{1-\beta}{1+\beta}\right)^m. \tag{5}$$

Above, S_0 is the (spatial) peak spectral density, w_0 represents the mode width, and $H_m(x)$ is a Hermite polynomial. Parameter β is a real constant ranging between $0 \leqslant \beta \leqslant 1$ and it connects the width w of the GSM beam and w_0 as $w = w_0/\sqrt{\beta}$.

The full CSD of a GSM beam can be expressed as

$$W(x_1,x_2) = S_0 \exp\left(-\frac{1+\beta^2}{2\beta}\frac{x_1^2+x_2^2}{w_0^2}\right) \times \exp\left(\frac{1-\beta^2}{\beta}\frac{x_1x_2}{w_0^2}\right), \tag{6}$$

and the related spectral density is

$$S(x) = S_0 \exp\left(-\frac{2x^2}{w^2}\right). \tag{7}$$

These enable us to write the complex degree of coherence of equation (2) in the form

$$\mu(x_1,x_2) = \exp\left[-\frac{(x_2-x_1)^2}{2\sigma^2}\right], \tag{8}$$

where $\sigma = \sqrt{\beta/(1-\beta)}w_0$ describes the coherence width of the GSM beam. We further introduce the overall (effective) degree of coherence [17, 18]

$$\bar{\mu}^2 = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|W(x_1,x_2)|^2 dx_1 dx_2}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}S(x_1)S(x_2)dx_1 dx_2}, \tag{9}$$

which characterizes the intensity-weighted degree of spatial coherence. For a GSM beam, $\bar{\mu} = \sqrt{\beta}$ holds.

2.2. Propagation in a non-linear medium

In this work, a GSM beam is propagated through a non-linear medium one realization $E(x,z;\omega)$ at a time. The construction of a statistical ensemble of realizations that represents a GSM beam is described in appendix A. We assume that the medium is nonmagnetic, source free, and its (dispersive) linear response is isotropic. The non-linear response of the medium is taken local and its strength is expressed by the non-linear susceptibility d (in contracted notation) under the Kleinman symmetry conditions. We from now on refer to field $E(x,z;\omega)$ as the fundamental (F) wave and invoke the notation $E_1(x,z) = E(x,z;\omega_1)$. The second-harmonic (SH) wave at frequency $\omega_2 = 2\omega_1$ generated in the medium is denoted by $E_2(x,z)$. The two waves are coupled in propagation and obey [3]

$$\nabla^2 E_1(x,z) + k_1^2 E_1(x,z) = -4\frac{d\omega_1^2}{c^2} E_1^*(x,z) E_2(x,z), \qquad (10)$$

$$\nabla^2 E_2(x,z) + k_2^2 E_2(x,z) = -2\frac{d\omega_2^2}{c^2} E_1^2(x,z), \qquad (11)$$

where $k_i = n(\omega_i)\omega_i/c$ with $n(\omega_i)$ being the refractive index, $i \in (1,2)$, and c is the vacuum speed of light.

The above coupled equations are numerically solved by employing the RK method outlined in appendix B and the BP method described in appendix C. We compared these methods in the context of field propagation in a non-linear medium. The implementation is with Matlab and the two methods rely on the available RK and fast Fourier transform (FFT) algorithms. As an example, with 600 and 20 000 sampling points in the transverse and longitudinal directions, respectively, the calculation times were 165.5 s with the RK and 0.7 s with the BP method. Thus, the BP method can here be regarded as two orders of magnitude faster than the RK technique. As an example, in appendix C the output spectral density distributions of the F and SH waves computed with the two methods are considered in a specific case. It is verified that with high accuracy the methods lead to identical results.

We notice that different ensembles can produce different SH-field coherence properties since the input F-field realizations themselves affect the generated SH field realizations via non-linear interaction. We assessed this possible effect by calculating the coherence properties of the F and SH fields at the output of the non-linear crystal for several ensembles of realizations. It turned out that for sufficiently large ensembles the SH-wave coherence properties were effectively the same for all sets. The same holds for the F waves. These are important justifications for the validity of the method. The influence of the number of members in an ensemble is considered in appendix D. It is found that the coherence properties reach convergence (are essentially unaltered if more realizations are included) when the number of realizations exceeds about 300.

3. Numerical results

In this section, we evaluate numerically the spatial coherence properties of the F and SH beams propagating in a non-linear medium. We employ the BP method for field propagation as it was confirmed to be significantly faster than the RK technique. The incident (F) field is a linearly (fully) polarized GSM beam at frequency ω_1. In the entrance facet of the crystal (chosen to be at $z = 0$) the field is represented by a set of realizations $\{E_{1n}(x,0)\}$ whose construction is explained in appendix A. Averaging over all realizations leads to the CSD of equation (6) as well as to the Gaussian spectral density of equation (7) and the degree of coherence described by equation (8). The width of the incident beam is in all considered cases fixed at $w = w_0/\sqrt{\beta} = 40\lambda_1$ with the (vacuum) wavelength of the F wave being $\lambda_1 = 0.8$ μm. The effective degree of coherence of the incident beam is varied by taking $\beta \in \{0.3, 0.5, 0.7, 0.85, 0.97\}$, where the extremes correspond to weakly and highly coherent beams, respectively. We do not consider lower β values since the beam width is $40\lambda_1$ and the transverse coherence length is in practice at least a few wavelengths. For each β value, we consider the spectral densities specified by $\sqrt{S_0} \in [0.001 \text{ GV m}^{-1},\ 0.8 \text{ GV m}^{-1}]$. This accordingly affects the amplitudes of the individual realizations in the ensemble and stronger amplitudes are expected to imply more notable non-linear effects. Furthermore, we choose the length L of the crystal as $L = 0.5$ mm and take the medium to be such that the F and SH waves have the same linear polarization state which is preserved on propagation. Hence, the non-linear susceptibility has only one nonzero element whose value is taken as $d = 2.0 \times 10^{-12}$ m V^{-1}. All the parameters are chosen such that measurements are possible, at least in principle, with a picosecond pulse laser, for instance. We remark that the strength of the SH wave can be enhanced by increasing either the incident spectral density or the crystal length [19]. However, in this work the crystal length is kept fixed.

In the following, we consider two cases. First, the refractive index of the crystal is the same at both frequencies, $n(\omega_1) = n(\omega_2) = 1.66$. Second, the indices are different, $n(\omega_1) = 1.660$, $n(\omega_2) = 1.661$. We refer to these situations as the 'phase-matched case' and the 'phase-mismatched case', respectively. We ignore the back reflections at the output facet as the related reflectance for the slab with chosen indices (surrounded by air) is a few percent at normal incidence.

3.1. Phase-matched case

Taking the refractive indices the same at the two frequencies, we calculate the squared overall degree of coherence for both the F and SH beams as a function of the incident beam's $\sqrt{S_0}$, which is a measure for the average peak amplitude. The computations are carried out for several β values and the results are shown in figure 1. The pink, yellow, blue, green, and orange curves refer to the β values of 0.3, 0.5, 0.7, 0.85, and 0.97, respectively. In addition, the solid lines indicate the F waves whereas the dashed lines denote the SH beam. We note that with small incident-field amplitudes, the degree of coherence

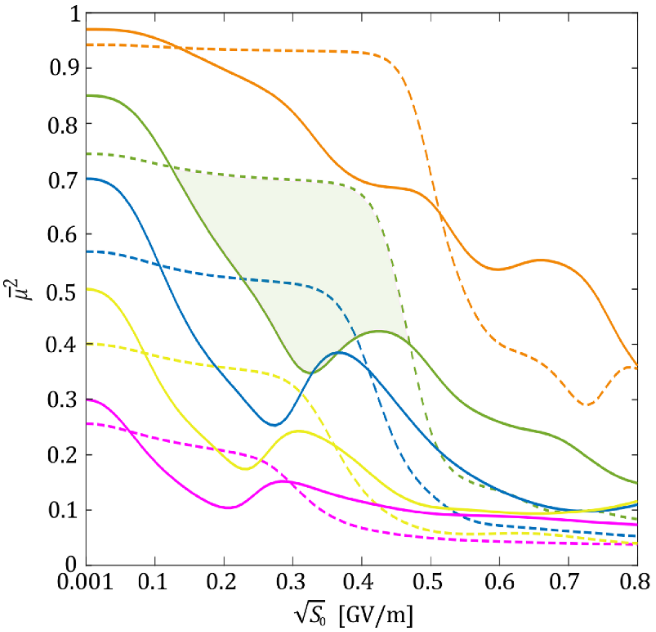

Figure 1. The squared effective degree of coherence of the F (solid lines) and SH (dashed lines) beams propagated through a non-linear crystal as a function of the incident beam's $\sqrt{S_0}$. The pink, yellow, blue, green, and orange curves correspond, respectively, to $\beta = \{0.3, 0.5, 0.7, 0.85, 0.97\}$ of the incident GSM beam. For the case of $\beta = 0.85$ the region where the SH beam $\bar{\mu}$ is larger than that of the F wave is shaded with green. The various beam and medium parameters are: $\lambda_1 = 0.8$ μm, $w = 40\lambda_1$, $L = 0.5$ mm, $d = 2.0 \times 10^{-12}$ m V^{-1}, and $n(\omega_1) = n(\omega_2) = 1.66$.

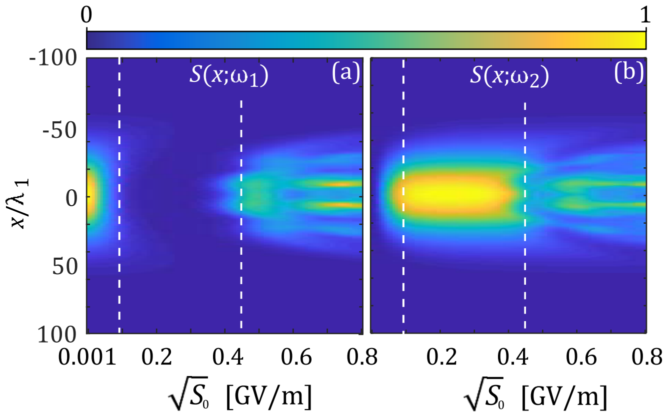

Figure 2. Normalized spectral density distributions of (a) the F field and (b) the SH beam, at the output facet of a non-linear crystal as a function of $\sqrt{S_0}$. The values at the vertical slices have been normalized by the corresponding S_0. The β value of the incident GSM beam is set to 0.85 and the white dashed lines correspond to the examples discussed in figure 4. Other materials and field parameters are as in figure 1.

of the F field does not change upon passage. This is explained by the fact that the propagation distance is short and the low amplitude does not induce significant SHG.

We further see from figure 1 that when $\sqrt{S_0}$ is increased, the effective degree of coherence of the F beam reduces rapidly for all β. At the same time, the SH beam's $\bar{\mu}$ remains nearly constant. Consequently, for all β values, the SH beam's degree of coherence at some point becomes larger than that of the F beam. This subsequently holds within a certain amplitude range which is the wider the higher is the incident F-wave β value. For $\beta = 0.85$ the range extends roughly from 0.12 to 0.5 GV m^{-1} and is marked in figure 1 with the green area. Near the end of the range the overall degree of coherence of the F beam increases slightly which is followed by a notable decrease in $\bar{\mu}$ of the SH beam. The above features can be explained by considering the behavior of the individual realizations as will be seen shortly. It is also observed that for high $\sqrt{S_0}$ values the output $\bar{\mu}$ of both beams is significantly smaller than that at the entrance facet. As an example, for the F wave with $\beta = 0.85$, the overall degree of coherence drops from $\sqrt{0.85} \approx 0.92$ to about $\sqrt{0.08} \approx 0.28$. Hence, interaction with a non-linear optical material provides a way to render a highly coherent beam into weakly coherent.

The normalized spectral densities of the F and SH beams at the crystal output are shown in figures 2(a) and (b) as a function of $\sqrt{S_0}$ in the case of $\beta = 0.85$. Each vertical slice corresponds to a fixed $\sqrt{S_0}$ value. The figures illustrate the coupling of the F-beam and SH-beam energies on transmission. The meaning of the white dashed lines will be explained later. By comparing the spectral density distributions with the overall degrees of coherence of figure 1, it is evident that a range of almost constant $\bar{\mu}$ of the SH beam takes place when most of the F-beam energy has been transferred to the SH beam. In contrast, at 0.4–0.5 GV m^{-1}, where the abrupt decrease of the degree of coherence of the SH field occurs, the F-wave total output energy is larger than that of the SH field. In general, for a fixed crystal length the output energy oscillates between the F and SH fields as a function of $\sqrt{S_0}$. This behavior is similar to what has been found as a function of the propagation distance in works assessing the efficiency of SHG [3, 19].

To explain the behavior of the overall degree of coherence of the SH beam we consider an ensemble of incident GSM-beam realizations. Averaging over all realizations produces a Gaussian spectral density distribution. However, individual realizations are not necessarily Gaussian. In particular, when the beam is partially coherent the realizations exhibit random spatial shapes possibly with several intensity peaks. Figure 3 shows the spectral density of a typical GSM-beam realization (black dashed lines) as well as the related F-wave (blue solid curves) and SH-wave (red solid curves) realizations in the case of $\beta = 0.5$ for $\sqrt{S_0}$ values of (a) 0.01 GV m^{-1}, (b) 0.1 GV m^{-1}, and (c) 0.3 GV m^{-1}. Note that the GSM-beam realization has the same shape in all three cases. The lower the overall degree of coherence, the more random structure the realizations in general show. For a single realization the SHG is strongest at the locations of high intensity. For a low $\sqrt{S_0}$ value, only the strongest intensity peak in a realization can contribute significantly to the SH-wave realization whose shape consequently exhibits a peak at this position [red curve in 3(a)]. The spatial locations of these peaks in the SH realizations are highly randomly distributed and represent a field that is less coherent than the F field. This explains

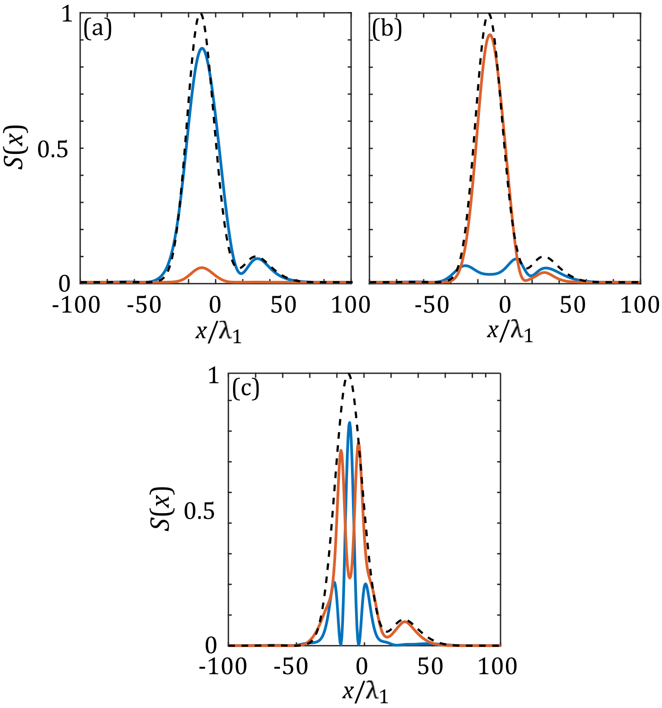

Figure 3. Spectral densities of a typical incident GSM-beam realization (black dashed curve) and the corresponding transmitted F (blue solid curve) and SH (orange solid curve) wave realizations. The incident GSM beam has $\beta = 0.5$ and $\sqrt{S_0}$ equals (a) 0.01 GV m^{-1}, (b) 0.1 GV m^{-1}, and (c) 0.3 GV m^{-1}. All plots have been normalized with S_0.

why for small $\sqrt{S_0}$ values $\bar{\mu}$ is smaller for the SH beam in figure 1.

When $\sqrt{S_0}$ increases more peaks in an F-wave realization can contribute to the SHG and randomness in the SH-field realizations increases [red curve in figure 3(b)]. This tends to decrease the overall degree of coherence of the SH field as a function of $\sqrt{S_0}$ as observed in figure 1. Simultaneously, the peaks in the F-field realizations contributing to the SHG are split into two as the energy in the middle of a peak is transferred to the SH-field realization [blue curve in 3(b)]. This effect increases the number of peaks in the F-wave realizations and likewise leads to a decreasing trend in $\bar{\mu}$. The abrupt decrease of the SH-field $\bar{\mu}$ occurs when $\sqrt{S_0}$ becomes sufficiently large to induce SHG also in the tail parts of the F-field realizations. Simultaneously, the high amplitude part of the SH realization converts back to the F-wave realization splitting the SH-wave realization. In this case, the SH (and F) realizations show highly peaked (random) structures [red and blue curves in figure 3(c)]. Similar effects also explain the filamentation of the spectral density distributions as observed in the right-hand sides of figures 2(a) and (b) in the case of $\beta = 0.85$. We also remark that multipeaked realizations, such as those found by increasing S_0, can likewise be obtained by increasing the crystal length.

Figures 4(a) and (b) show the spectral density distributions of the F field (blue solid curve) and the SH field (orange solid curve) along the white dashed lines located at 0.09 and 0.45 GV m^{-1}, respectively, in figure 2. Black dashed lines depict the incident-wave Gaussian spectral density whose peak value is used to normalize all the curves. In figure 4(a) the dip in the middle part of the F-beam curve demonstrates the energy transfer to the SH beam which displays a nearly Gaussian shape whereas the filamentation of both beams is visible in (b). Hence, due to the non-linear optical response of the medium, both the F and SH output beams may, at large incident intensities, deviate significantly from a Gaussian shape. Notice also that in the cases of (a) and (b) the overall SH-field degree of coherence is in the flat and rapidly decreasing regions in figure 2 (green dashed curve). By comparing the spectral density distributions of the incident and transmitted beams we further observe that the widths are similar for all beams in (a) but in (b) the width of the transmitted F beam is half of that of the SH beam. This suggests that a non-linear light–matter interaction could be exploited to synthesize a beam with adjustable (narrower) width.

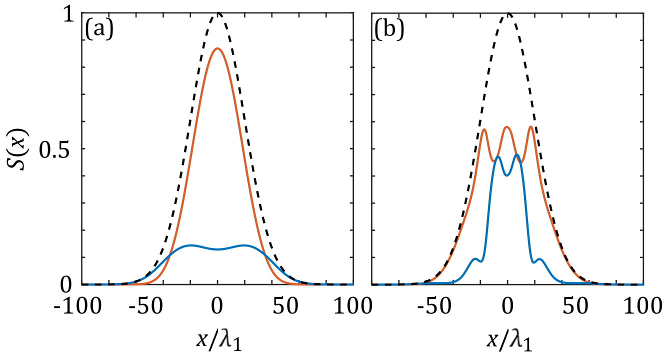

Figure 4. Distributions of the transmitted spectral densities of the F (blue solid curve) and SH (orange solid curve) beams in the cases of (a) $\sqrt{S_0} = 0.09$ GV m^{-1} and, (b) $\sqrt{S_0} = 0.45$ GV m^{-1}. Black dashed curves show the incident Gaussian spectral density. The situations in (a) and (b), respectively, correspond to the left and right white dashed lines in figures 2(a) and (b). All curves have been normalized with S_0.

Figure 5 presents the magnitude of the degree of coherence at the output of a non-linear crystal for the F beam (left column) and the SH beam (right column). The upper row corresponds to the low amplitude case of $\sqrt{S_0} = 0.09$ GV m^{-1} whereas the lower row represents the high amplitude situation with $\sqrt{S_0} = 0.45$ GV m^{-1}; both are also depicted with vertical white dashed lines in figure 2. As seen from 5(a), even with low incident-field amplitudes the degree of spatial coherence of the F beam deviates notably from that of the original GSM beam (which would be a straight diagonal bar but not shown). The origin of this modulation can be traced to the splitting of the F-field realizations as discussed earlier. In contrast, as seen from (b) the SH beam degree of coherence resembles the GSM beam coherence. At high intensities and for both beams the degree of coherence becomes strongly modulated as is visible in (c) and (d). The modulation is particularly strong in the region of high spectral density extending roughly from $-40\lambda_1$ to $40\lambda_1$ [see figures 4(a) and (b)]. The above observations suggest that SHG can be employed to alter and control the two-point spatial coherence properties of light beams.

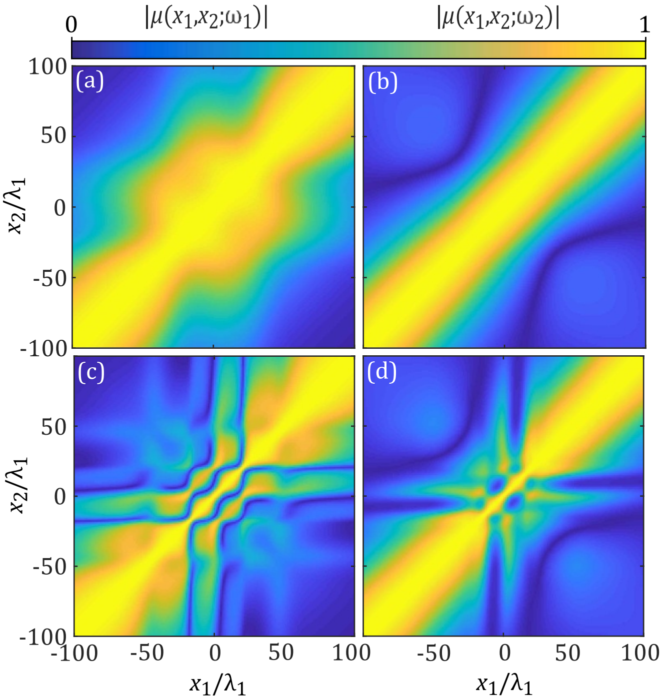

Figure 5. Magnitudes of the degree of coherence of the F beam (left column) and SH beam (right column) at the output facet of a non-linear crystal for $\beta = 0.85$ of the incident GSM beam. The upper and lower rows correspond to the cases of $\sqrt{S_0} = 0.09$ GV m^{-1} and $\sqrt{S_0} = 0.45$ GV m^{-1}, respectively, marked with white dashed lines in figure 2.

3.2. The phase-mismatched case

Next, we assess the situation in which the refractive index is different at different frequencies. We choose $n(\omega_1) = 1.660$ and $n(\omega_2) = 1.661$ which amount to a (dispersive) phase mismatch of $\Delta k = 2k_1 - k_2 = -1.5708 \times 10^4$ rad m^{-1} (for collinear components). The other parameters are as in section 3.1. The phase mismatch is expected to reduce the efficiency of the SHG which is clearly visible in figures 6(a) and (b) showing the output spectral density distributions for the F and SH beams, respectively, as a function of $\sqrt{S_0}$. In addition, the SHG occurs periodically as a function of $\sqrt{S_0}$. Analogous phase-mismatch-induced periodicity of the F- and SH-wave energies but as a function of the propagation distance has been discussed, e.g. in [3, 19].

The overall degree of coherence as a function of $\sqrt{S_0}$ is presented in figure 7 in the cases of $\beta \in \{0.3, 0.5, 0.7, 0.85, 0.97\}$ with pink, yellow, blue, green, and orange lines, respectively. The solid lines refer to the F beam while the dashed lines represent the SH field. For both beams the degree in general decreases with increasing $\sqrt{S_0}$. The mechanism behind this behavior is the same as in the phase-matched case of figure 1, i.e. more intensity peaks in the random realizations contribute to SHG while the intensity peaks of F-field realizations are split. Further, by comparing figures 6 and 7 we observe that the local maxima of the spectral density coincide with the maxima of the $\bar{\mu}$ curves for both beams. The periodic oscillations in the spectral density and

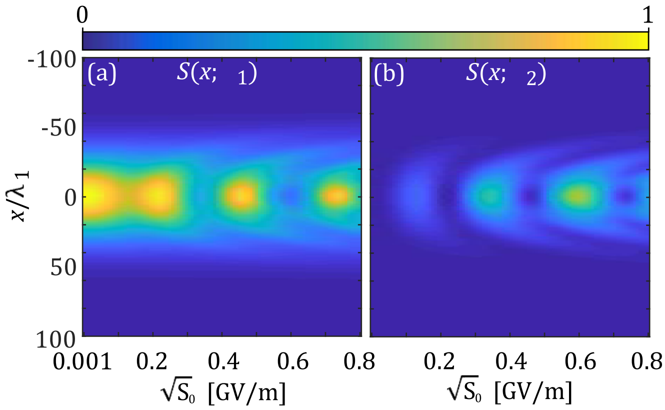

Figure 6. Same as in figure 2, but for $n(\omega_1) = 1.660$ and $n(\omega_2) = 1.661$.

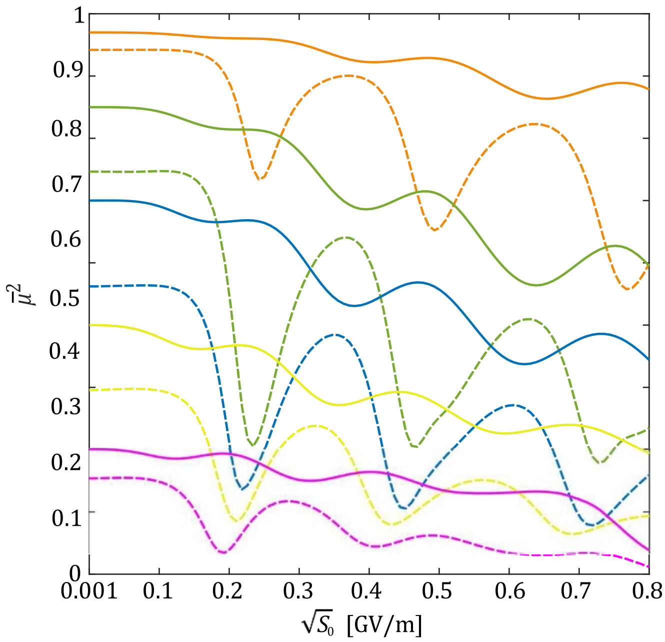

Figure 7. Same as in figure 1, but for $n(\omega_1) = 1.660$ and $n(\omega_2) = 1.661$.

the overall degree of coherence as a function of $\sqrt{S_0}$ originate from the phase mismatch. Similar effects are not present in figures 1 and 2.

4. Conclusions

We analyzed the spectral spatial coherence effects in SHG induced by a stationary GSM beam. The overall degrees of coherence of the F and SH fields at the exit facet of a non-linear crystal were found to decrease significantly with increasing peak intensity of the incident beam. In particular, at strong intensities, a highly coherent GSM beam may generate a weakly coherent SH beam and become highly incoherent itself on propagation. Hence, the SHG effect can be used to render a coherent beam into a weakly coherent one. In addition, the coherence properties of both fields may significantly differ from the GSM, especially at high F-wave intensities. In particular, the spectral density distributions of both F and SH waves may show filament structures or the F wave width may be significantly smaller than that of the incident GSM beam. Further, the two-point degree of coherence can be modified to deviate significantly from a Gaussian shape. Propagation in the non-linear medium was performed with the RK and BP methods, which led to identical results, but the latter was found to be two orders of magnitude faster. The results suggest that the coherence properties of light beams can be tailored and controlled by exploiting a non-linear material interaction.

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Appendix A. Construction of an ensemble of realizations

In this appendix we construct an ensemble of realizations describing a GSM beam. Specifically, we require that the CSD is of the form of equation (3) with the modes $\psi_m(x)$ given by the HG functions of equation (4). We write a single realization as

$$E_n(x) = \sum_{m=0}^{M-1} \alpha'_{nm}\psi_m(x), \tag{A1}$$

where M is a sufficiently large number and α'_{nm} are random complex numbers to be determined. Averaging over the ensemble as in equation (1) results in

$$W(x_1,x_2) = \sum_{m=0}^{M-1}\sum_{m'=0}^{M-1}\left(\frac{1}{N}\sum_{n=0}^{N-1}\alpha'^{*}_{nm}\alpha'_{nm'}\right) \times \psi^*_m(x_1)\psi_{m'}(x_2), \tag{A2}$$

where we assumed that the ensemble contains N realizations. Theoretically both N and M should approach infinity but in practise the summations have to be cut as we have done. For equation (A2) to coincide with the CSD of a GSM the following must hold

$$\frac{1}{N}\sum_{n=0}^{N-1}\alpha'^{*}_{nm}\alpha'_{nm'} = \alpha_m\delta_{mm'}, \tag{A3}$$

where α_m is the modal coefficient given in equation (5) and $\delta_{mm'}$ is the Kronecker delta function. We may set

$$\alpha'_{nm} = \sqrt{\alpha_m}\exp(i\phi_{nm}), \tag{A4}$$

where ϕ_{nm} is a random phase uniformly distributed within the interval $[0,2\pi]$. Equations (A3) and (A4) therefore imply

$$\frac{1}{N}\sum_{n=0}^{N-1}\exp[i(\phi_{nm'}-\phi_{nm})] = \delta_{mm'}. \tag{A5}$$

Next we introduce the phase matrix

$$\phi = \begin{pmatrix} \exp(i\phi_{00}) & \cdots & \exp[i\phi_{0(M-1)}] \\ \exp(i\phi_{10}) & \ddots & \\ \vdots & & \\ \exp[i\phi_{(N-1)0}] & & \exp[i\phi_{(N-1)(M-1)}] \end{pmatrix}, \tag{A6}$$

whose rows represent individual realizations and columns are mutually orthogonal vectors if M is infinitely large. However, since M is finite we employ the Gram–Schmidt method to render them orthonormal. This then leads to a matrix

$$\mathbf{C} = \begin{pmatrix} c_{00} & \cdots & c_{0(M-1)} \\ c_{10} & \ddots & \\ \vdots & & \\ c_{(N-1)0} & & c_{(N-1)(M-1)} \end{pmatrix}, \tag{A7}$$

whose columns are orthonormal vectors and satisfy

$$\frac{1}{N}\sum_{n=0}^{N-1}c^*_{nm}c_{nm'} = \delta_{mm'}. \tag{A8}$$

We then set

$$\alpha'_{nm} = \sqrt{\alpha_m}c_{nm}, \tag{A9}$$

which via equation (A1) generates an ensemble of random realizations that represents a GSM beam. We point out that, e.g. in [8–10, 12] the above orthonormalization procedure was not employed and hence the resulting CSD may not accurately represent a GSM beam.

Appendix B. RK method

Below we describe the main points of the RK propagation method. We invoke the envelope representation of the field as $E_j(x,z) = A_j(x,z)\exp(ik_jz)$ and for expressing the required formulas we introduce a new variable $A'_j(x,z) = \partial_z A_j(x,z)$ with $\partial_z = \partial/\partial_z$, $j \in (1,2)$. This allows us to split the second-order z derivatives in equations (10) and (11) into four first-order derivatives leading to

$$\partial_z A_1(x,z) = A'_1(x,z), \tag{B1}$$

$$\partial_z A'_1(x,z) = -4\frac{d\omega_1^2}{c^2}A^*_1(x,z)A_2(x,z)\exp(-i\Delta kz) - i2k_1A'_1(x,z) - \partial_{xx}A_1(x,z), \tag{B2}$$

$$\partial_z A_2(x,z) = A'_2(x,z), \tag{B3}$$

$$\partial_z A_2'(x,z) = -2\frac{d\omega_2^2}{c^2}A_1^2(x,z)\exp(i\Delta kz) - i2k_1A_2'(x,z) - \partial_{xx}A_2(x,z), \quad \text{(B4)}$$

where $\partial_{xx} = \partial^2/\partial x^2$ and $\Delta k = 2k_1 - k_2$ are the spatial second-order derivative and the phase mismatch, respectively. The set of equations (B1)–(B4) can be integrated with respect to z by employing the RK algorithm [20]. When using an ordinary differential equation solver one needs to perform the spatial second order derivations numerically on every integration step. We also employ a non-linear coordinate transformation [21] to prevent the reflections from the calculation window boundaries.

Appendix C. Beam-propagation method

Next we outline the BP method [22] for field propagation in a non-linear medium. The solutions for the homogeneous versions of equations (10) and (11) can be written using the angular-spectrum representation as [23]

$$E_j(k_{xj},z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E_j(x,z)\exp(-ik_{xj}x)\mathrm{d}x, \quad \text{(C1)}$$

$$E_j(x,z) = \int_{-\infty}^{\infty} E_j(k_{xj},z_0) \times \exp(ik_{xj}x + ik_{zj}\Delta z)\mathrm{d}k_{xj}, \quad \text{(C2)}$$

where $j \in (1,2)$ and k_{xj} is the spatial frequency, i.e. the wave vector x component, and $\Delta z = z - z_0$ is the propagation step from an arbitrary reference plane at z_0 to z. The z component of the wave vector is $k_{zj} = (k_j^2 - k_{xj}^2)^{1/2}$ for $j \in (1,2)$. To find an expression when the source terms (right-hand sides) are included in equations (10) and (11), we assume that within a small propagation distance Δz the non-linear polarization changes linearly and its spatial spreading is negligible. Consequently, we may use the slowly-varying envelope approximation (SVEA) [3, 24] and write

$$\nabla^2 E_j(x,z) + k_j^2 E_j(x,z) \approx i2k_j\partial_z A_j(x,z), \quad \text{(C3)}$$

where we have denoted the slowly varying envelope by $A_j(x,z)$, $j \in (1,2)$. We neglected the phase terms $\exp(ik_{zj}z)$ in equation (C3) since the aim is to use the angular-spectrum representation for the field $E_j(x,z)$ that includes the phase. Equating the right-hand side of equation (C3) with those of equations (10) and (11), integrating from z_0 to z, rearranging and combining with equation (C2) yields

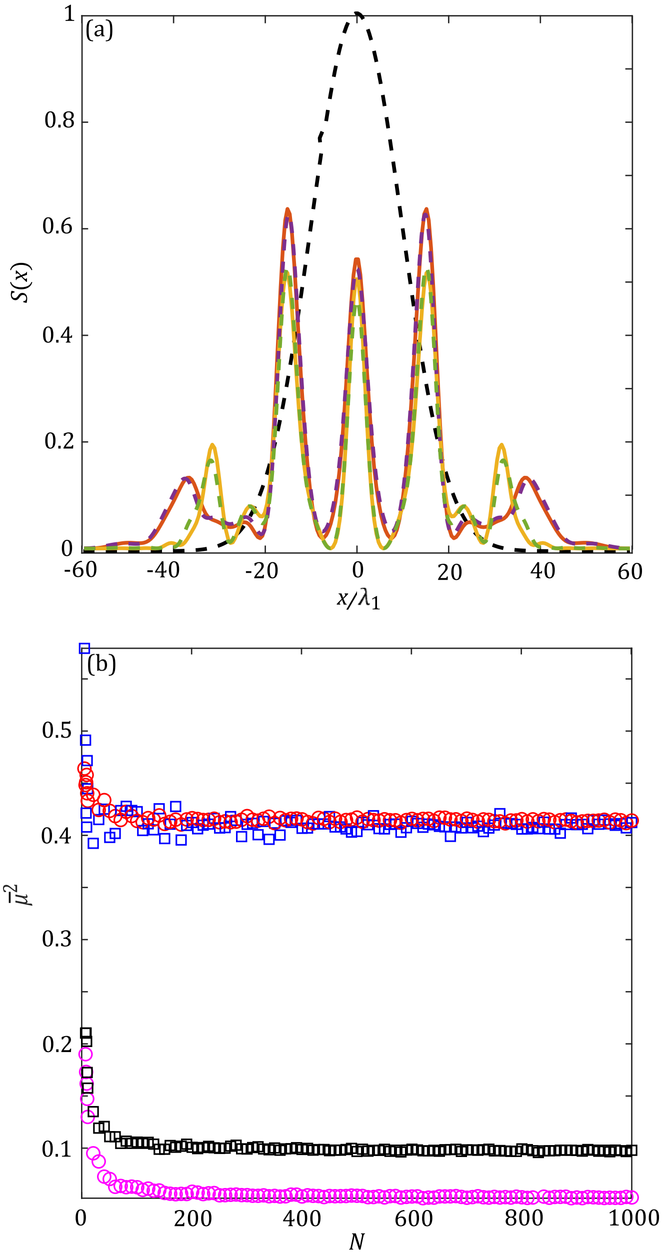

Figure C1. (a) Comparison of the RK and BP methods. Spectral density distributions are shown for the F and SH waves at the output facet of a non-linear crystal calculated with the RK method [red (F) and yellow (SH) solid lines] and the BP method [purple (F) and green (SH) dashed lines]. The black dashed curve illustrated the incident beam. (b) The squared overall degree of spatial coherence for the F (squares) and SH (circles) waves as a function of the number of realizations. The red and blue symbols denote the case when the initial GSM beam has $\beta = 0.85$, while black and magenta correspond to an incident beam of $\beta = 0.3$. In both cases, the incident beam's average amplitude is 0.45 GV m^{-1}.

$$\int_{-\infty}^{\infty} E_1(k_{x1},z_0)\exp[i(k_{x1}x + k_{z1}\Delta z)]\mathrm{d}k_{x1} = E_1(x,z_0) + \frac{2id\omega_1^2}{k_1c^2}E_1^*(x,z_0)E_2(x,z_0)\Delta z, \quad \text{(C4)}$$

$$\int_{-\infty}^{\infty} E_2(k_{x2},z_0)\exp[i(k_{x2}x + k_{z2}\Delta z)]\mathrm{d}k_{x2} = E_2(x,z_0) + \frac{id\omega_2^2}{k_2c^2}E_1^2(x,z_0)\Delta z. \quad \text{(C5)}$$

Above, the F and SH fields in the z_0 plane are known and using equation (C1), one can construct, by employing the FFT, an iterative method for the propagation of the coupled fields.

We tested the numerical methods presented in appendices B and C by propagating a deterministic Gaussian beam whose width and peak amplitude are $w_0 = 20\lambda_1$ and $A_1(0,0) = 1\times 10^9$ V m^{-1}, respectively, through a second-order non-linear crystal with the coupling constant $d = 2.0\times 10^{-13}$ m V^{-1} and thickness $L = 1$ mm. In a plane-wave model the chosen peak amplitude corresponds to the power of 100 GW cm^{-2} which is easily achieved, e.g. with picosecond pulsed lasers. Such long pulses can be regarded stationary conforming with the assumptions of this work. With 600 and 20 000 sampling points in the x and z directions, respectively, the calculation times were 165.5 s (RK) and 0.7 s (BP). Therefore, we may consider the BP method two orders of magnitude faster than the RK technique. Figure C1(a) shows the F and SH waves after propagation in the medium. The incident beam profile is shown with black dashed line whereas solid lines present, respectively, the F (red) and the generated SH (yellow) beams calculated with the RK method. The corresponding spectral densities obtained with the BP method are displayed with purple (F) and green (SH) dashed lines. The results are, to a good accuracy, identical.

Appendix D. Convergence arguments

In this appendix we assess the convergence of the results by considering the influence of the number of realizations in an ensemble. As a quantitative measure we employed the overall degree of coherence squared whose values at the non-linear crystals's exit facet are shown in figure C1(b) as a function of the number of realizations. The squares and circles refer to the F and SH waves, respectively, with blue and red colors associated with an incident beam of $\beta = 0.85$, whereas black and magenta represent the situation of $\beta = 0.3$. The incident average amplitude corresponds to the high-intensity case of 0.45 GV m^{-1}, which is numerically challenging since both field distributions decompose into narrow filaments. However, in all cases we observe that the overall degree of coherence does not significantly change if the number of realizations exceeds, say, 300.

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