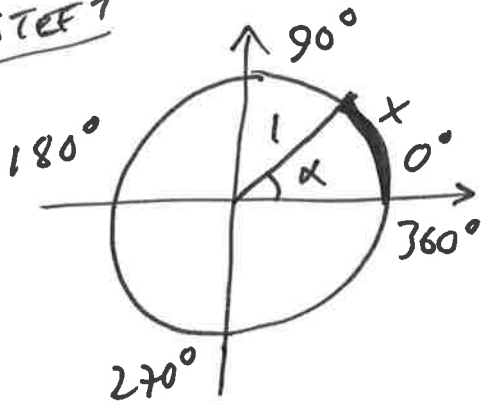


# KULMA -ASTEIKUT

## ASTEET

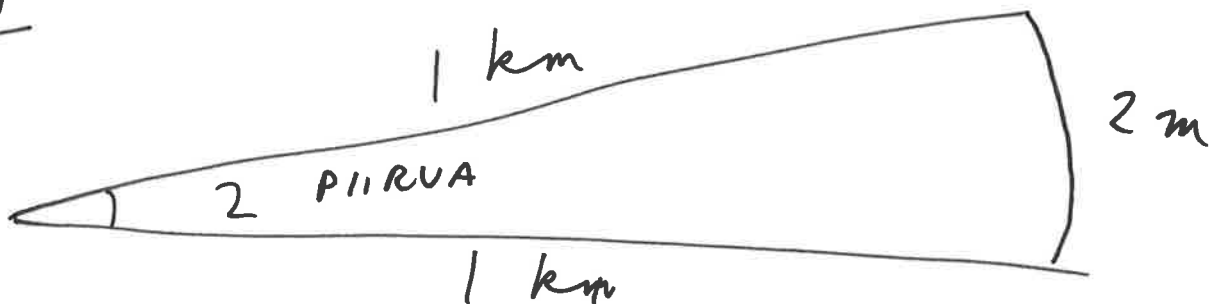


$$\text{Koko kehä} \quad 2\pi \cdot 1 = 2\pi$$

$$= 2 \cdot 3,14 = 6,28$$

$$\text{Pituus } x = \frac{\alpha}{360^\circ} 2\pi$$

## PIIRU



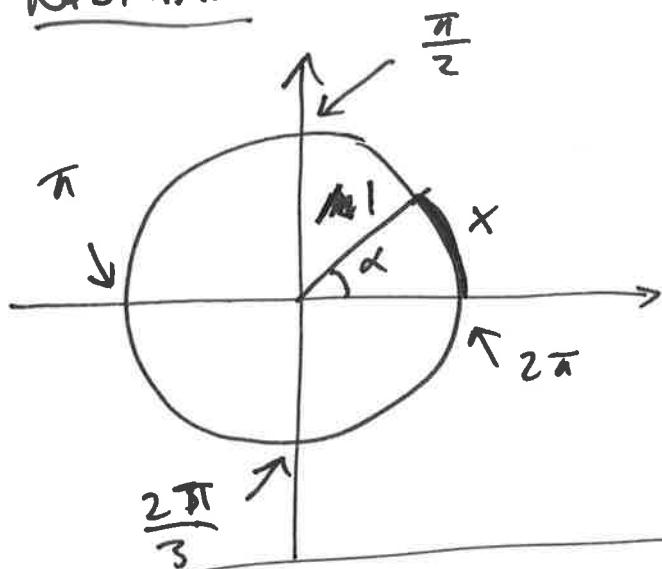
$$\text{Koko ympyrä:} \quad 6283 \text{ PIIIRUA}$$

$$\text{TAI} \quad 6000 \text{ PIIIRUA}$$

$$\text{TYKISTÖSSÄ:} \quad \text{VÄTEVÄ}$$

RADIAANI

$$\alpha = \cancel{x}$$



$$\text{PITUUS } x = \alpha$$

YKSIKKÖYMPYRÄSSÄ:  $r=1$  JA  $\alpha = x$

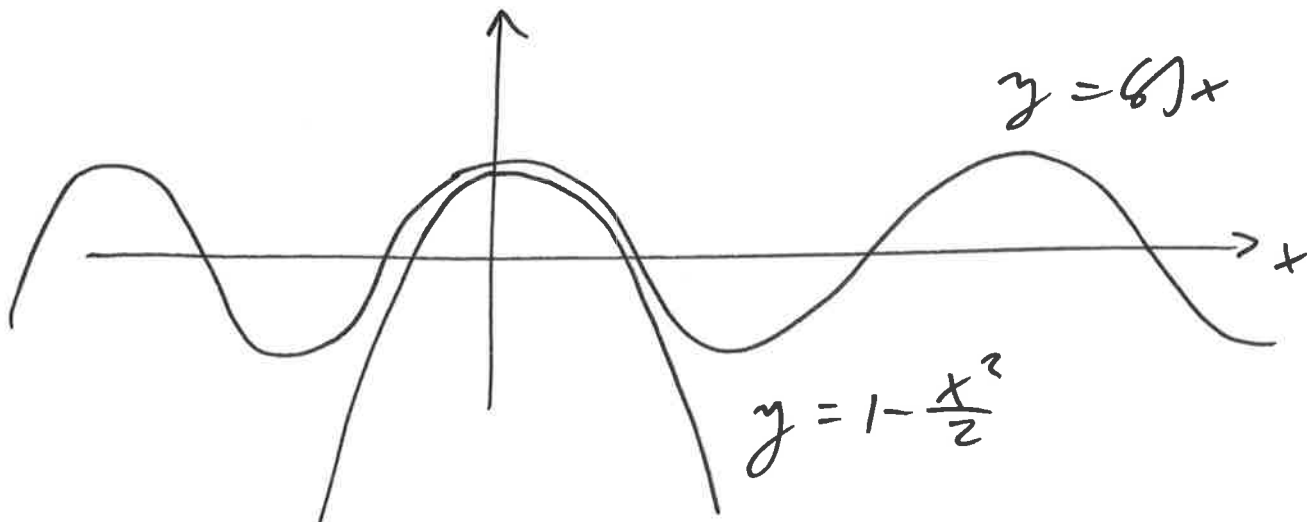
PARAS KOLMEYKSIKKÖ:

YKSIIN KERTAISIA KAIVOJA

$$\sin x \approx x - \frac{x^3}{3 \cdot 2 \cdot 1} + \frac{x^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} - \dots$$

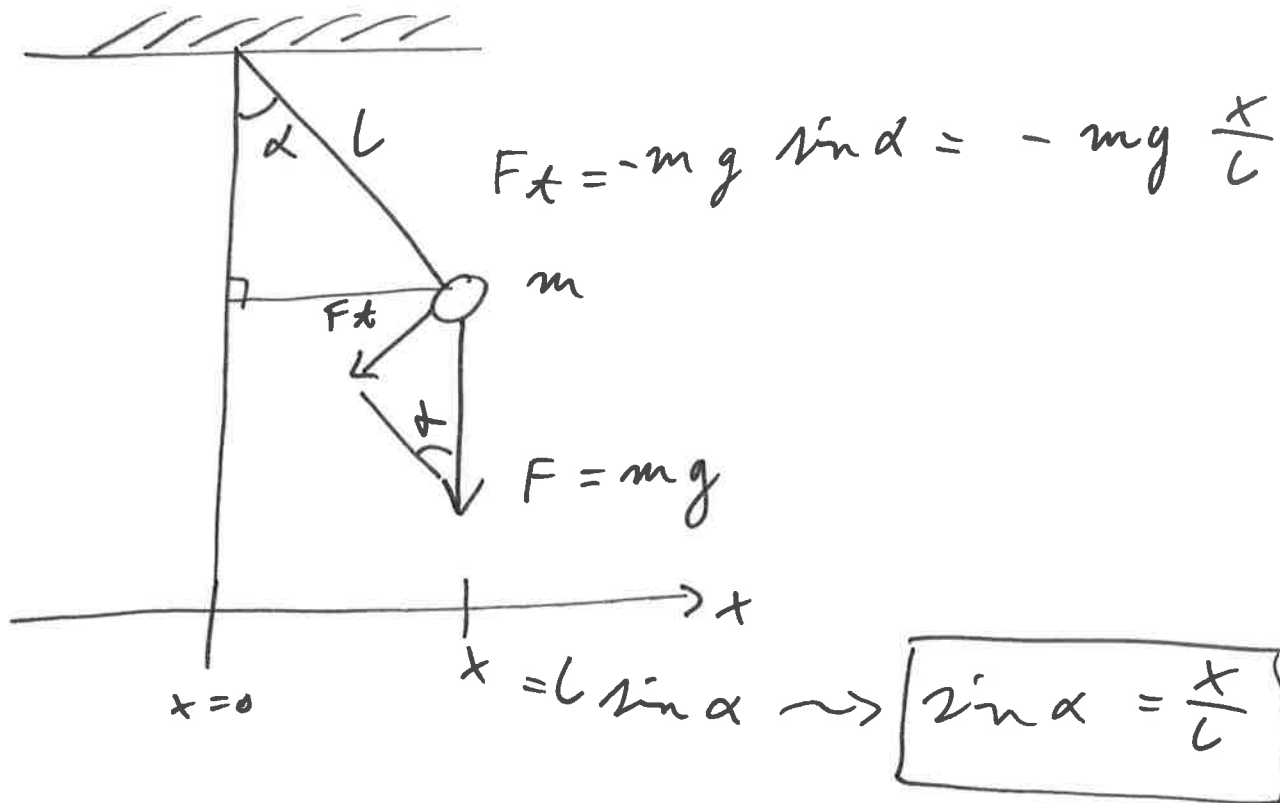
$$\cos x = 1 - \frac{x^2}{2 \cdot 1} + \frac{x^4}{4 \cdot 3 \cdot 2 \cdot 1} - \dots$$

$$x \approx 0 \Rightarrow \cos x \approx 1 - \frac{x^2}{2}$$



# HEILURI

## VOIMA KUVIO



|   |                        |                 |                |
|---|------------------------|-----------------|----------------|
| $\left\{ \begin{array}{l} x = x \\ v = x' \\ a = x'' \end{array} \right.$ | $= \frac{d}{dt} x$     | $[x] = m$       | PAIKKA         |
|   | $= \frac{d^2}{dt^2} x$ | $[x'] = m/s$    | NOPEUS         |
|   |                        | $[x''] = m/s^2$ | VEIKHTY VAKUUS |

Newton:  $F = ma = mx''$

$\rightarrow mx'' = Ft = -mg \frac{x}{L} \rightarrow$

MASSA  
ET  
VAIKUTTA

$\Rightarrow x'' + \frac{g}{L} x = 0$

$\underbrace{\frac{g}{L}}_{=\omega^2}$

$\Rightarrow x'' + \omega^2 x = 0, \quad \omega = \sqrt{\frac{g}{L}}$

ENERGIA UCVIO

$$\alpha = \frac{x}{L}$$

$$= \frac{1}{2} \frac{x^2}{L} mg$$

$$U_{POT} = mgh = mgL \frac{\alpha^2}{2}$$

$$h = L - L \cos \alpha \approx L - L \left(1 - \frac{\alpha^2}{2}\right)$$

$$= L \frac{\alpha^2}{2} = \frac{1}{2} \frac{x^2}{L}$$

$$U_{POT} = 0$$

$$E_{LIKE} = \frac{1}{2} m v^2 = \frac{1}{2} m (x')^2$$

$$\alpha = \frac{x}{L}$$

$$E_{TOT} = mgh + \frac{1}{2} m v^2$$

$$= mg \frac{x^2}{2L} + \frac{1}{2} m (x')^2$$

$$= mgL \frac{\alpha^2}{2} + \frac{1}{2} m L^2 (\alpha')^2$$

$$L = mg \frac{x^2}{2L} - \frac{1}{2} m (x')^2$$

$$\text{LAGRANGE: } \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) = \frac{\partial L}{\partial q}$$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial x'} \right) = \frac{d}{dt} (-m x') = -m x''$$

$$= \frac{\partial L}{\partial x} = mg \frac{x}{L}$$

$$\Rightarrow x'' + \frac{g}{L} x = 0$$

$$x'' + \omega^2 x = 0, \quad \omega = \sqrt{\frac{g}{L}}$$

$$YH \ddot{A} \ddot{C} \ddot{O} \ddot{N} \quad x'' + \omega^2 x = 0$$

RATKAISU

$$x(t) = A \sin(\omega t) + B \cos(\omega t),$$

$$x(t) = C \sin(\omega t + \varphi_0)$$

[A, B, C, ~~x~~  $\varphi_0$  VAKIOITTA]

$$\sin(t) \quad \text{JAKSO} \quad 2\pi$$

$$\sin(2t) \quad \text{JAKSO} \quad \frac{2\pi}{2} = \pi$$

$$\sin(100t) \quad \text{JAKSO} \quad \frac{2\pi}{100} = \frac{\pi}{50}$$

$$\sin(\omega t) \quad \text{JAKSO} \quad \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L}}} = 2\pi \sqrt{\frac{L}{g}}$$