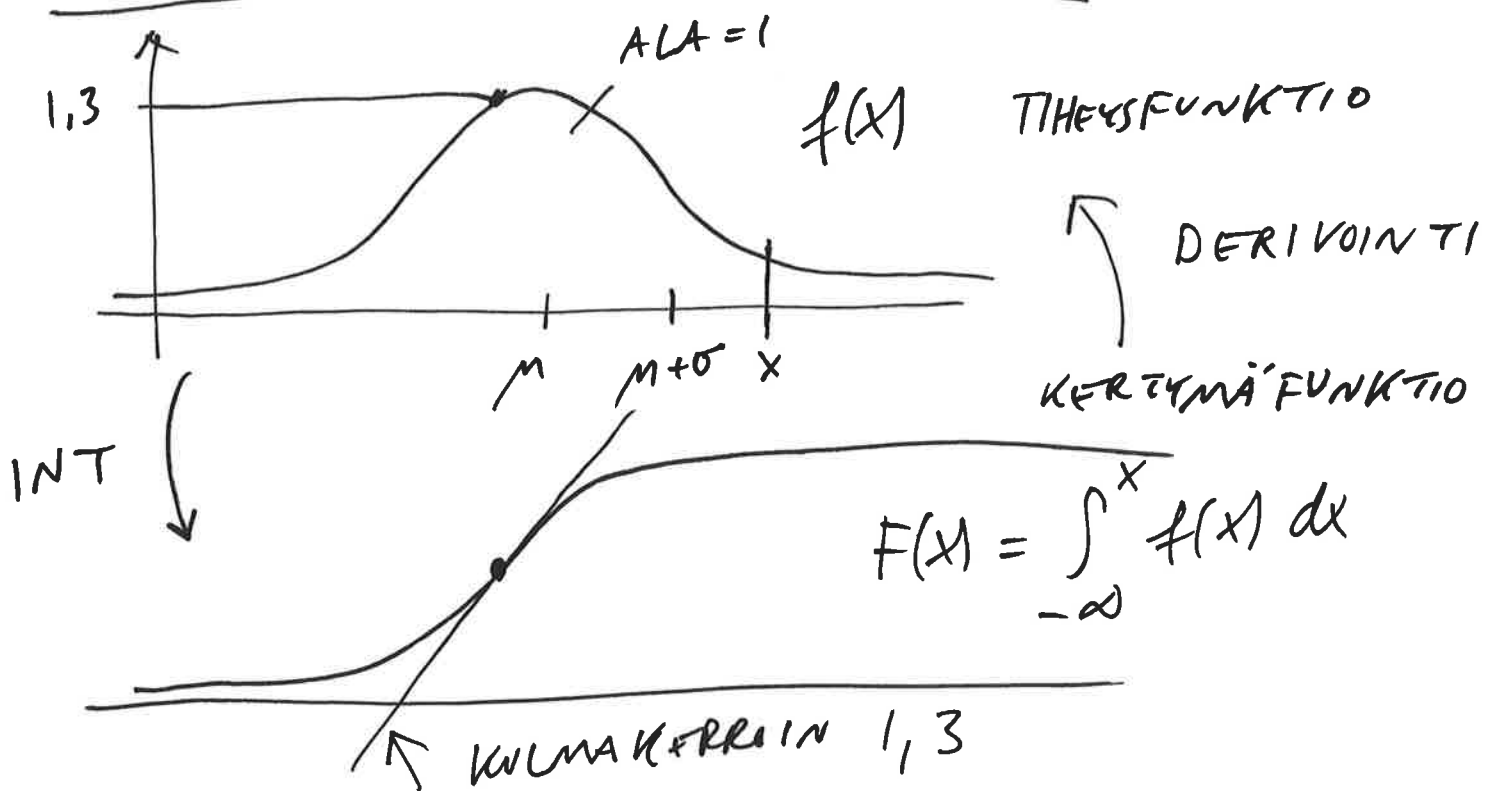
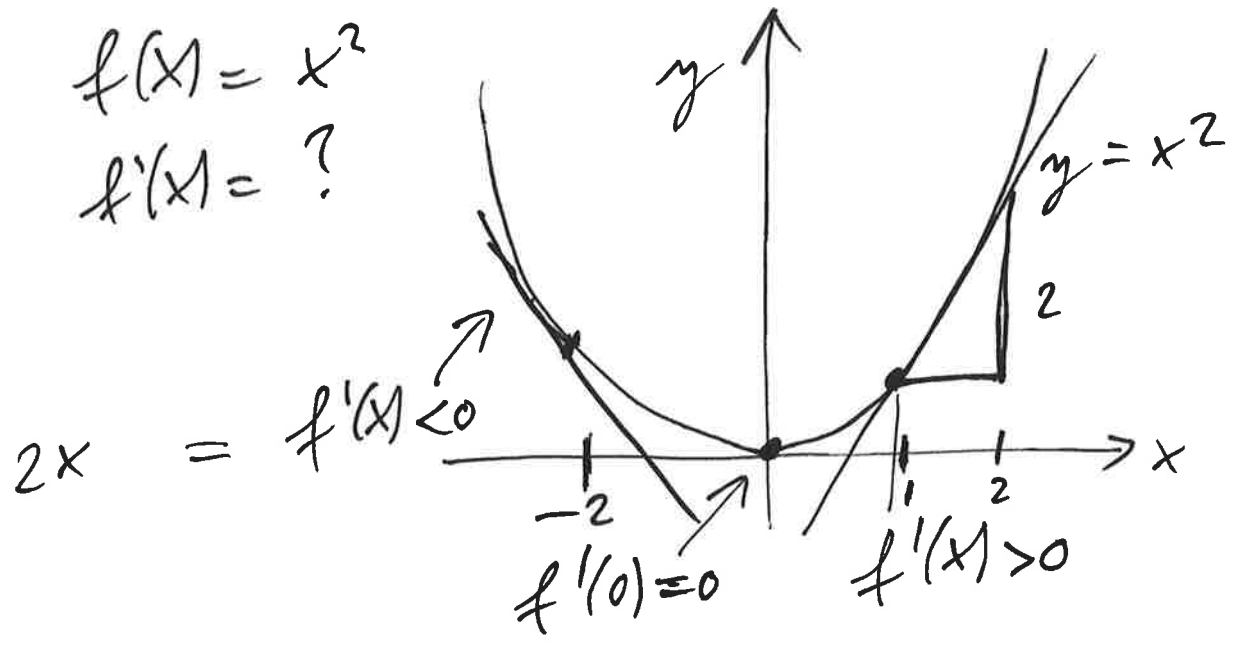


DERIVOINTI JA INTEGROINTI



- SIIS
- INTEGRAALIFUNKTIO KUVAA KERTYMÄÄ
 - DERIVAATTA-FUNKTIO KUVAA MUUTOSNOPEUTTA

ESIM, $f(x) = x^2$
 $f'(x) = ?$



$2x = f'(x)$

LASKETAAN

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} (2x + h) = 2x$$

x KASVAA

$f'(x)$ KASVAA

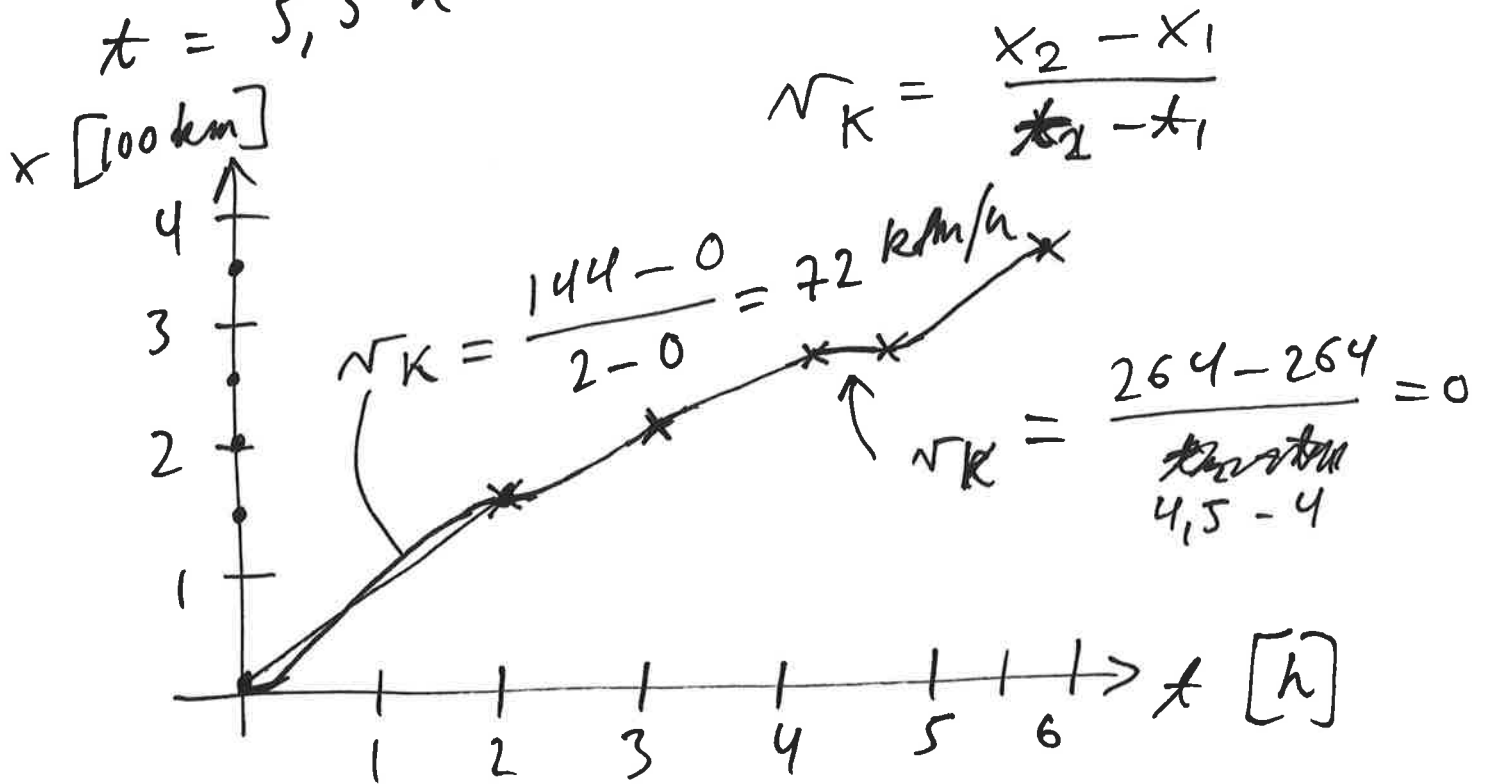
$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\frac{d}{dx} x^2 = 2x$$

ESIM, KESKINNOPEUS

AUTON LÄHTEET PÖHJOISEEN

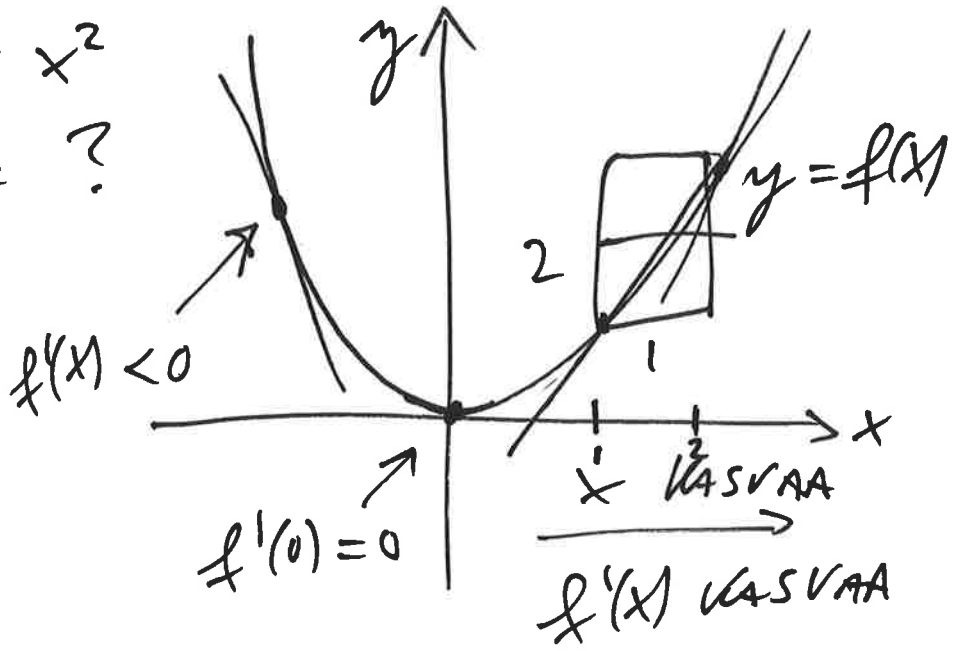
$t = 0$ h	KUOPIO	} 144	144
$t = 2$ h	JOENSUU		
$t = 3$ h	KOLI	} 60	204
$t = 4$ h	NURMES		
$t = 4,5$ h	NURMES	} 60	264
$t = 5,5$ h	KUHMO		



ESIM,

$$f(x) = x^2$$

$$f'(x) = ?$$



LASUKETUAN

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h = 2x$$

$$\Rightarrow \frac{d}{dx} x^2 = 2x$$

ESIM,

$$f(x) = ax, \text{ a vakio}$$

$$f'(x) = ?$$

$$\frac{d}{dx} ax = a$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{a(x+h) - ax}{h}$$

$$= a \lim_{h \rightarrow 0} \frac{x+h-x}{h} = a \lim_{h \rightarrow 0} 1 = a$$

$$\frac{d}{dx} x^2 = 2x^{2-1}$$

$$\frac{d}{dx} ax = a$$

$$\frac{d}{dx} a x^1 = a \frac{d}{dx} x^1$$

$$= a \cdot 1 \cdot x^{1-1}$$

$$= a \cdot 1 \cdot x^0$$

$$= a \cdot 1 \cdot 1$$

$$= a$$

$$\boxed{\frac{d}{dx} x^1 = 1}$$

$$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$$

$$\begin{aligned} 2^3 &= 8 \\ 2^2 &= 4 \\ 2^1 &= 2 \\ 2^{\frac{1}{2}} &= \sqrt{2} = 1,41 \\ 2^0 &= 1 \end{aligned}$$

$$\frac{d}{dx} \frac{1}{x} = \frac{d}{dx} x^{-1}$$

$$= -1 \cdot x^{-1-1} = -x^{-2} = -\frac{1}{x^2}$$

$$\rightarrow \boxed{\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}}$$

ESIM. PU TOAMINEN

$x(t)$ PAIKKA [m] HETKELLÄ t

$v(t) = \dot{x}(t) = \frac{d}{dt} x(t)$ NOPEUS [m/s]

$a(t) = v'(t) = \frac{d}{dt} v(t) = x''$ KIIHTYVYYS [m/s²]

LÄHELLÄ MAAN PINTAA KIIHTYVYYS ON
VAKIO $= g = 9,81 \text{ m/s}^2$.

\Rightarrow $a(t) = g$
INT \downarrow $v(t) = gt$
INT \downarrow $x(t) = \frac{1}{2}gt^2$

$\frac{d}{dt} 9,81 t^1 = 9,81 \cdot 1 \cdot t^0 = 9,81 \text{ ak}$

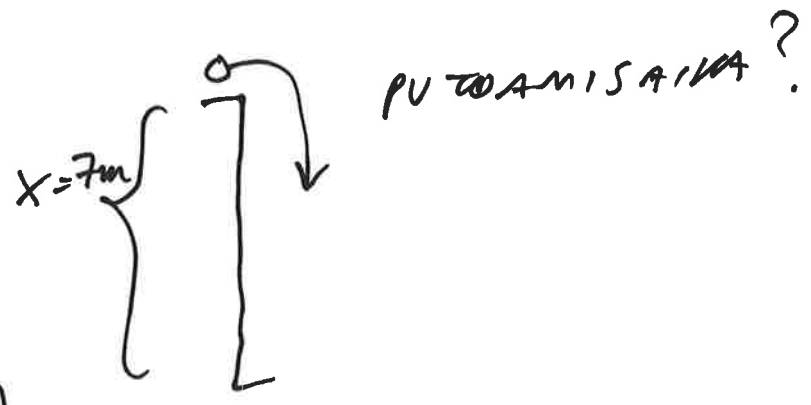
$\frac{d}{dt} \frac{1}{2} g t^2 = \frac{1}{2} g \cdot 2 t^{2-1} = g t \text{ ak}$

$\Rightarrow t^2 = \frac{2x}{g}$
 $\Rightarrow t = \sqrt{\frac{2x}{g}}$

ESIM. SAVONIACLA

$t = \sqrt{\frac{2x}{g}}$ (EI RIIPU MASSASTA)

$= \sqrt{\frac{2 \cdot 7 \text{ m}}{9,81}} = \underline{\underline{1,19 \text{ s}}}$



ESIM . PUI JON TORNI $x = 75m$

PV TAMISAUKA

$$t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2 \cdot 75}{9,81}} = \underline{\underline{3,91 \text{ s}}}$$

VIRHE KAAVOJEN JOHTAMINEN DERIVOIMALLA

HALUTTAAN LASKEA SUURE

$$f = f(a, b, c)$$

JOS a, b & c MUUTTUVAAT VÄHÄN,
MITEN PALJON f MUUTTUU?

MERKITÄÄN PIENIÄ MUUTOKSIA

$$da, db, dc, df$$

$$[\Delta a, \Delta b, \Delta c, \Delta f]$$

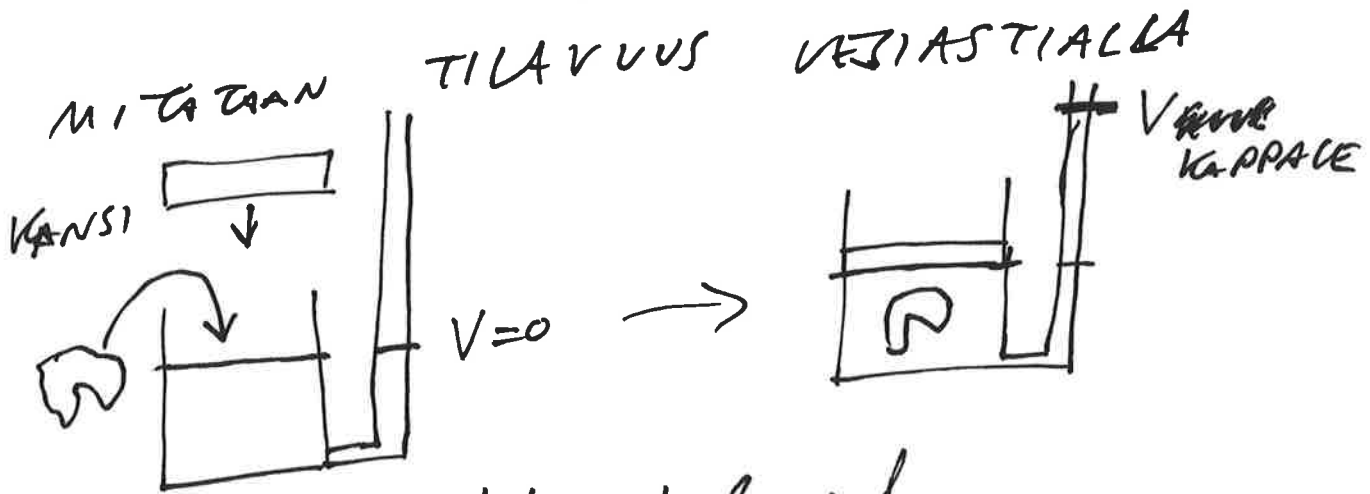
$$\rightarrow df = \frac{df}{da} da + \frac{df}{db} db + \frac{df}{dc} dc$$

↑
MUUTOS-
NOPEUS

↑
MUUTOS

ESIM. MÄÄRITETTÄÄN METALLIN TIHEYS
PUNNITTAAN METALLI KAPPALF VAAKALLA

$$\rightarrow m = 7 \text{ kg} \pm 1 \text{ g}$$



$$\rightarrow V = 1 \text{ L} \pm 1 \text{ ml}$$

$$\rho = \frac{m}{V} = \frac{7 \text{ kg}}{1 \text{ L}} = 7 \text{ kg/L} \pm \boxed{\text{TÄRÄKÄÄVÄS?}}$$

$$\begin{aligned} \rightarrow d\rho &= \frac{d\rho}{dm} dm + \frac{d\rho}{dV} dV \\ &= \frac{1}{V} dm + -\frac{m}{V^2} dV \end{aligned}$$

$$\frac{d}{dm} \frac{m}{V} = \frac{1}{V}$$

$$\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}$$

$$|d\rho| \leq \left| \frac{1}{V} dm \right| + \left| -\frac{m}{V^2} dV \right|$$

$$= \frac{1}{V} dm + \frac{m}{V^2} dV$$

$$= \frac{1}{1 \text{ L}} \cdot 1 \text{ g} + \frac{7 \text{ kg}}{(1 \text{ L})^2} \cdot 1 \text{ mL}$$

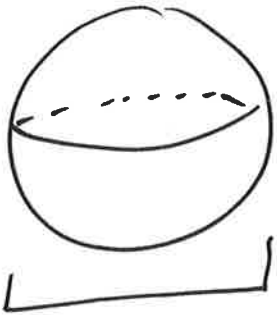
$$= 1 + 7 \frac{\text{g}}{\text{L}} = 8 \frac{\text{g}}{\text{L}}$$

$$\rightarrow \rho = 7 \text{ kg/L} \pm 8 \frac{\text{g}}{\text{L}} = 7000 \pm 8 \frac{\text{g}}{\text{L}}$$

$$\text{SIIS } \rho = 7 \frac{\text{kg}}{\text{L}} \pm 8 \frac{\text{g}}{\text{L}}$$

$$= (7000 \pm 8) \frac{\text{g}}{\text{L}}$$

TAPA II LASKE TÄN KUULAN TIIVYYS



$$d = \dots$$

$$r = \frac{d}{2}$$

$$r = 100 \text{ mm}$$

$$r = 1 \text{ dm}$$

$$dr = 0,1 \text{ mm}$$

$$\frac{dr}{r} = \frac{0,1 \text{ mm}}{100 \text{ mm}} = 0,001$$

$$V = \frac{4\pi r^3}{3}$$

$$\frac{dV}{dr} = \frac{3 \cdot 4\pi r^2}{3} = 4\pi r^2$$

$$dV = \frac{dV}{dr} dr$$

$$= 4\pi r^2 dr$$

$$= \frac{4\pi r^3}{3} \cdot 3 dr$$

$$= \frac{V}{r} \cdot 3 dr$$

$$dV = V \cdot 3 \frac{dr}{r}$$

SUHT. VIRH.

~~1007~~
TAPAK II MITA TAN LUBUKAN WA CLK415/14



$$d = 12,4 \text{ cm} \quad \Delta x = 1 \text{ mm}$$

$$r = \frac{d}{2} = 6,2 \text{ cm}$$

$$V = \frac{4\pi r^3}{3} = \frac{4 \cdot \pi \cdot 0,062^3}{3} = 0,998 \text{ L}$$

$$\frac{dV}{dr} = \frac{d}{dr} \left(\frac{4\pi r^3}{3} \right) = 4\pi r^2$$

$$dV = \frac{dV}{dr} dr = 4\pi r^2 dr$$

$$dV = V \cdot 3 \frac{dr}{r} = \frac{4\pi r^3}{3} \cdot 3 dr = \frac{V}{r} \cdot 3 \frac{dr}{r}$$

SUMT VI RHE

$$\begin{aligned} \Delta g &= 1 \frac{g}{L} + \left| \frac{dg}{dV} dV \right| = \text{frisan} \\ &= 1 \frac{g}{L} + \frac{m}{V^2} \cdot 4 \cdot 3 \frac{dr}{r} \\ &= 1 \frac{g}{L} + \frac{7 \text{ kg}}{0,998 \text{ L}} \cdot 3 \cdot \frac{1 \text{ mm}}{62 \text{ mm}} \\ &= 1 + 7,48 \frac{g}{L} \qquad = 48 \cdot 10^{-3} \\ &= \underline{\underline{337 \frac{g}{L}}} \qquad = 0,048 \end{aligned}$$

$$g = (7000 \pm 337) \frac{g}{L}$$