SOME PHYSICS

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Abstract. Here are some physics lectures.

Contents

Most files are here:
http://integraali.com/stuff/

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1. Mechanics

1.1. **Newtonian mechanics.** Analyze the forces.

1.2. **Lagrangian.** Write

$$L = T - U,$$

where T is the kinetic energy and U is the potential energy. Then use the equation

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}}$$

for each variable q.

1.3. **Hamiltonian mechanics.** Write

$$H = \text{total energy}.$$

For each variable q_a, search the generalized momenta $\dot{p_a}$ by setting

$$p_a = \frac{\partial H}{\partial \dot{q_a}}$$

Then use the equations

$$\boxed{\begin{cases} \dot{q_a} = \dfrac{\partial H}{\partial p_a} \\ \dot{p_a} = -\dfrac{\partial H}{\partial q_a} \end{cases}}$$

1.4. **Examples of Hamiltonian mechanics.** The Hamiltonian of a mass m in a string of length l is

$$H = mg\frac{x^2}{2L} + \frac{1}{2}m\dot{x}^2.$$

Let

$$p = \frac{\partial}{\partial \dot{x}}H = m\dot{x}$$

and substitute it to the equation to obtain

$$H = mg\frac{x^2}{2L} + \frac{1}{2m}p^2.$$

We use the equations

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{1}{m}p = \dot{x}$$

and

$$m\ddot{x} = \dot{p} = -\frac{\partial H}{\partial x} = -m\frac{g}{l}x$$

to obtain the set of equations

$$\begin{cases} \dot{x} = \dfrac{1}{m}p \\ \dot{p} = -m\dfrac{g}{l}x. \end{cases}$$

We obtain

$$\ddot{x} + \frac{g}{l}x = 0.$$

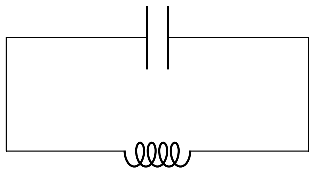
2. Electronics

2.1. **Coulomb's law.** The matter consists of atoms and atoms consist of a positively charged nucleus and negatively charged electrons. Electron has a charge

$$e = 1.6022 \times 10^{-19} \text{ C}.$$

All other charges in nature are multiples of e: a charge Q may be written $Q = ne$ for some integer n.

2.2. **LC-circuit.**



A Newtonian solution

The charge in the capacitor is

$$Q = CU_C.$$

The voltage in the capacitor is caused by the inductor

$$U_C = -L\dot{I}.$$

We obtain

$$\frac{Q}{C} + L\dot{I} = 0,$$

which yields

$$\dot{I} + \frac{1}{LC}Q = 0,$$

that is,

$$\ddot{Q} + \frac{1}{LC}Q = 0.$$

We write this as

$$\ddot{Q} + \omega_0^2 Q = 0, \quad \omega_0 = \frac{1}{\sqrt{LC}}.$$

A Lagrangian solution

The energies of the capacitor and inductor are

$$E_C = \frac{1}{2C}Q^2; \quad E_L = \frac{1}{2}LI^2.$$

The charge Q is a coordinate with derivative I. Thus E_C represents the potential energy and E_L the kinetic energy. Thus the Lagrangian

$$L = T - U = E_L - E_C = \frac{1}{2}L\dot{Q}^2 - \frac{1}{2C}Q^2.$$

We will apply

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}}\right) = \frac{\partial L}{\partial Q}$$

to obtain

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}}\right) = \frac{d}{dt}\left(L\dot{Q}\right) = L\ddot{Q} = \frac{\partial L}{\partial Q} = -\frac{1}{C}Q.$$

This yields

$$\ddot{Q} + \frac{1}{LC}Q = 0,$$

as before.

A Hamiltonian solution

The total energy of the system is

$$H = \frac{1}{2}L\dot{Q}^2 + \frac{1}{2C}Q^2.$$

We have a coordinate Q. The generalized momentum is

$$P = \frac{\partial H}{\partial \dot{Q}} = L\dot{Q}.$$

We can write H as

$$H = \frac{1}{2L}P^2 + \frac{1}{2C}Q^2.$$

We obtain the first order equations

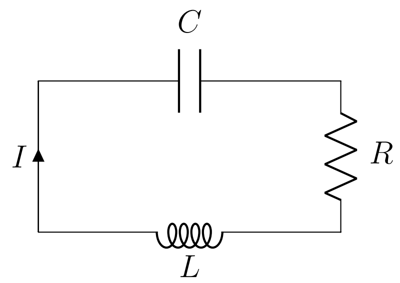
$$\begin{cases} \dot{Q} = \dfrac{\partial H}{\partial P} = \dfrac{1}{L}P \\ \dot{P} = -\dfrac{\partial H}{\partial Q} = -\dfrac{1}{C}Q. \end{cases}$$

These may be combined to a second order equation by differentiating the first equation to obtain

$$\ddot{Q} = \frac{1}{L}\dot{P} = \frac{1}{L}\left(-\frac{1}{C}Q\right) = -\frac{1}{LC}Q,$$

as before.

2.3. **RLC-circuit.**



A Newtonian solution

Similarly as before

$$U_C + U_R = -L\dot{I}.$$

We obtain

$$\frac{Q}{C} + RI = -L\dot{I},$$

which yields

$$\ddot{Q} + \frac{R}{L}\dot{Q} + \frac{1}{LC}Q = 0.$$

A Hamiltonian solution

The total energy of the system is

$$H = \frac{1}{2}L\dot{Q}^2 + \frac{1}{2C}Q^2.$$

We choose the variable Q and obtain the momentum

$$P = \frac{\partial H}{\partial \dot{Q}} = L\dot{Q}.$$

Hence

$$H = \frac{1}{2L}P^2 + +\frac{1}{2C}Q^2.$$

We obtain the first order equations

$$\begin{cases} \dot{Q} = \dfrac{\partial H}{\partial P} = \dfrac{1}{L}P \\ \dot{P} = -\dfrac{\partial H}{\partial Q} = -\dfrac{1}{C}Q. \end{cases}$$

The resistor will cause a loss

$$F_R = -R\dot{Q},$$

which we add to the derivative of the momentum

$$\dot{P} = -\frac{1}{C}Q - R\dot{Q}.$$

We obtain

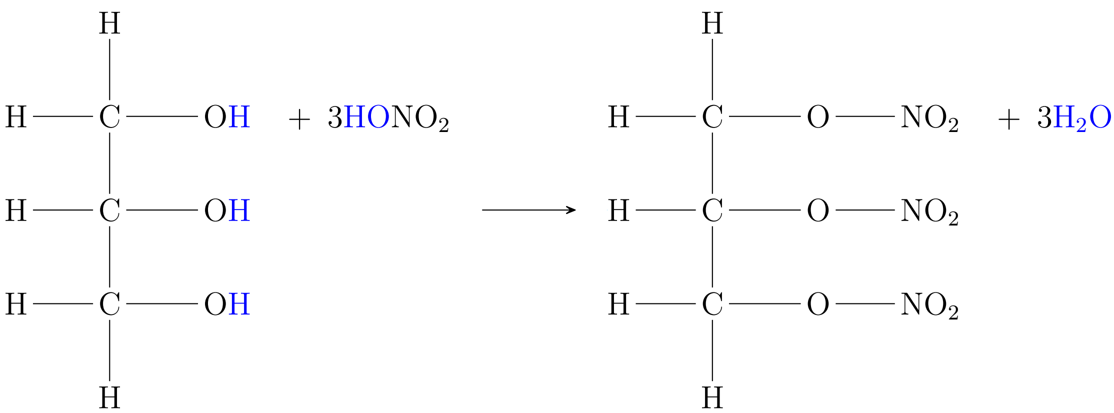
$$\ddot{Q} = \frac{1}{L}\dot{P} = -\frac{1}{LC}Q - \frac{R}{L}\dot{Q},$$

that is,

$$\ddot{Q} + \frac{R}{L}\dot{Q} + \frac{1}{LC}Q = 0,$$

as in the Newtonian solution.

2.4. **Chemistry.** You can make nitroglyserin or trinitroglyserin as follows:



The reaction for this is

$$C_3H_5[OH]_3 \;+\; 3HNO_3 \longrightarrow C_3H_5[ONO_2]_3 \;+\; 3H_2O$$

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