SOME MATLAB FILES

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Abstract. Here are some of my Matlab files, and some calculations.

Contents

Most files are here:
`http://integraali.com/stuff/`

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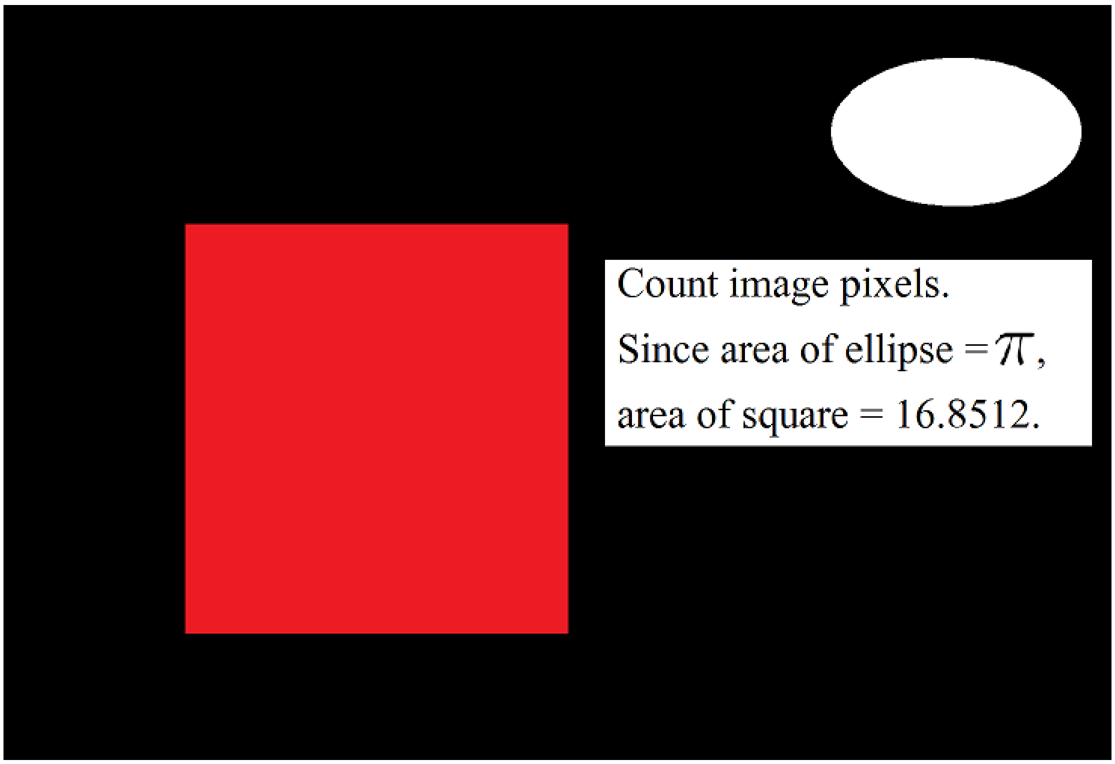
1. Image editing

1.1. **Area calculator.** Counts pixels in a picture. Given a picture with two domains E, D in different colors and a black background, the function counts the pixels in each domain and calculates the quotient $\operatorname{area}(E)/\operatorname{area}(D)$.

Second part of the function assumes that $\operatorname{area}(D) = \pi$ and calculates $\operatorname{area}(E)$ by the formula $\operatorname{area}(E) = \pi * \operatorname{area}(E)/\operatorname{area}(D)$.

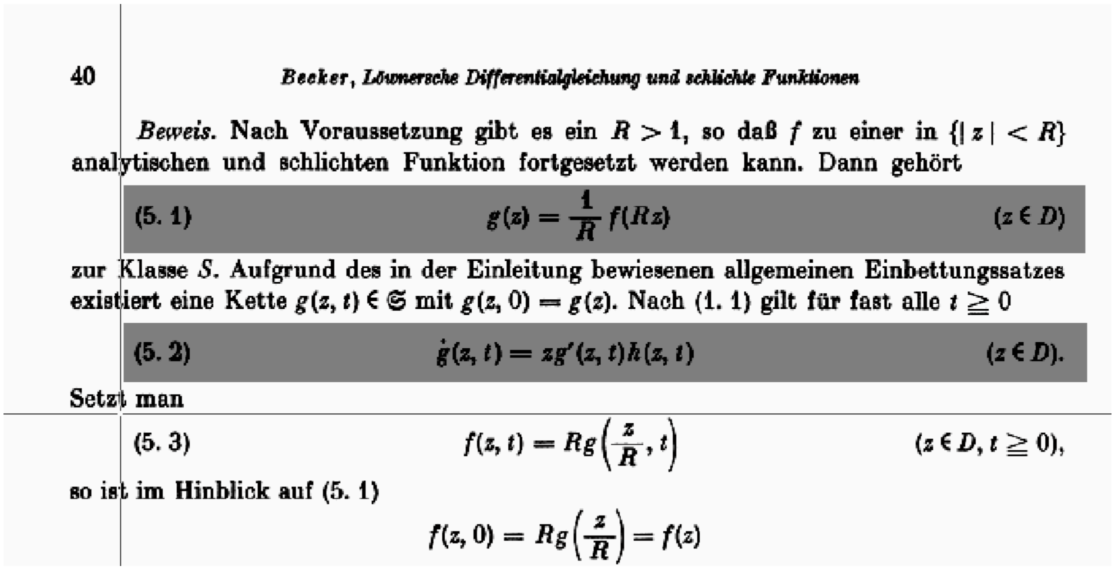


Download in:

`https://se.mathworks.com/matlabcentral/fileexchange/62210-area-pix-picture-filename-`

1.2. **Automatic picture snipping tool.** An automatic snipping tool for pictures and formulas. Saves the formula images automatically to .PNG and .EPS format.

If a pdf document is translated through OCR-software and Google Translate into a Word/LaTeXfile, the formulas can be added in a simple way.



40 *Becker, Löwnersche Differentialgleichung und schlichte Funktionen*

Beweis. Nach Voraussetzung gibt es ein $R > 1$, so daß f zu einer in $\{|z| < R\}$ analytischen und schlichten Funktion fortgesetzt werden kann. Dann gehört

$$g(z) = \frac{1}{R} f(Rz) \qquad (z \in D) \tag{5.1}$$

zur Klasse S. Aufgrund des in der Einleitung bewiesenen allgemeinen Einbettungssatzes existiert eine Kette $g(z,t) \in \mathfrak{S}$ mit $g(z,0) = g(z)$. Nach (1. 1) gilt für fast alle $t \geqq 0$

$$\dot{g}(z,t) = z g'(z,t) h(z,t) \qquad (z \in D). \tag{5.2}$$

Setzt man

$$f(z,t) = R g\left(\frac{z}{R}, t\right) \qquad (z \in D, t \geqq 0), \tag{5.3}$$

so ist im Hinblick auf (5. 1)

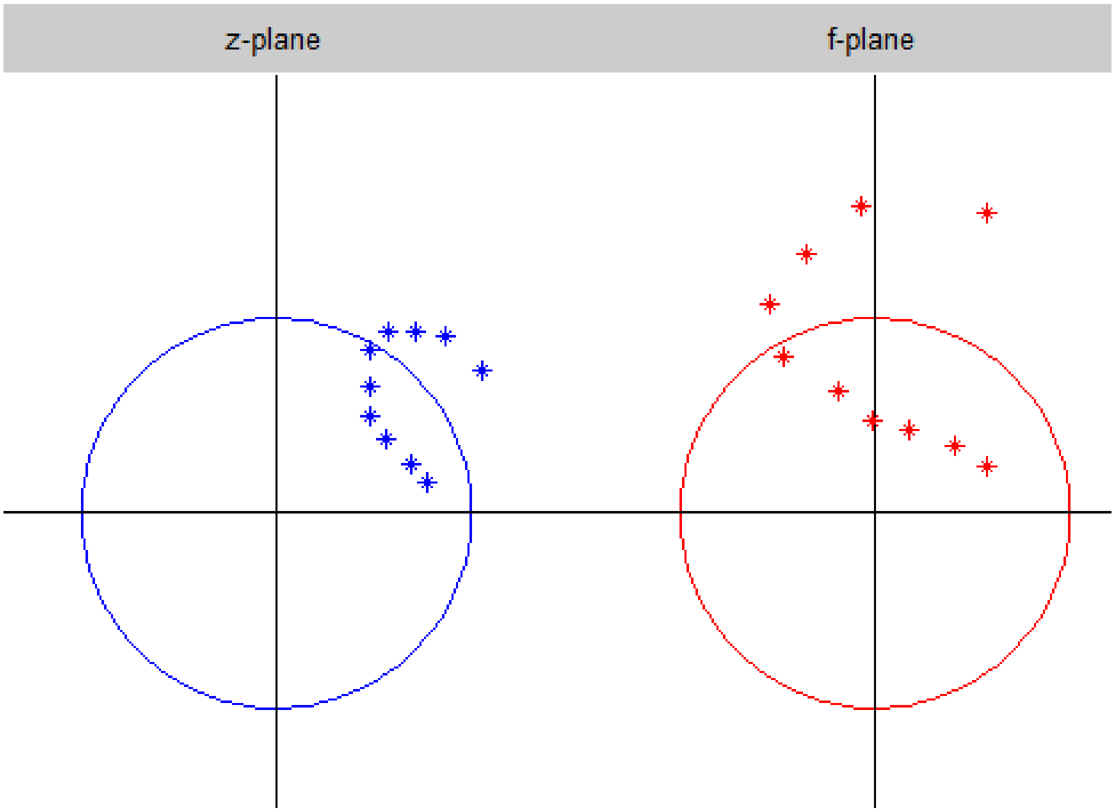
$$f(z,0) = R g\left(\frac{z}{R}\right) = f(z)$$

Download in:

https://se.mathworks.com/matlabcentral/fileexchange/62211-formula-snip-filename-filetype-opt-

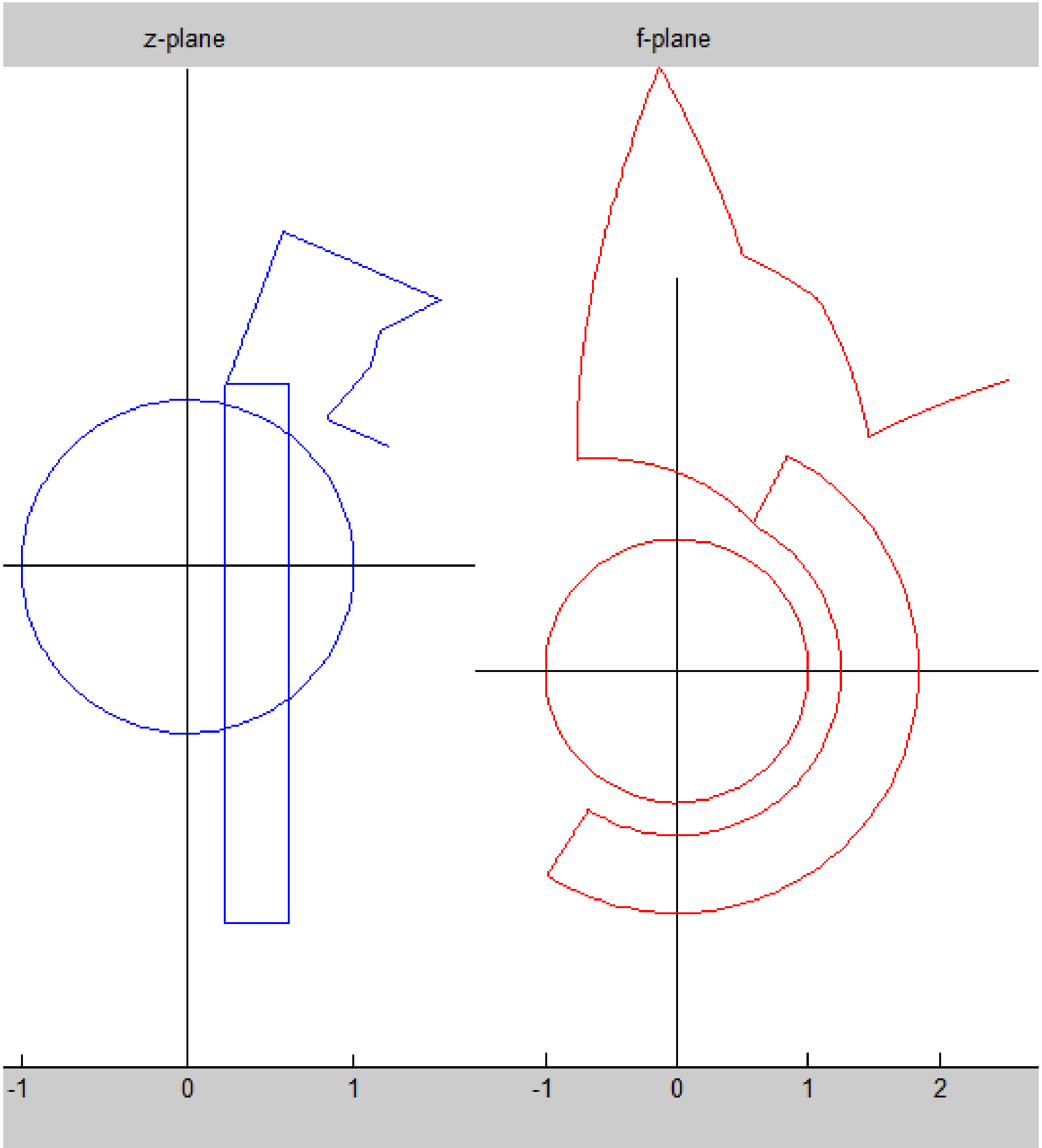
2. Complex analysis

2.1. **Complex map tool 1.** Maps dots by a complex map $z \mapsto f(z)$. Dots are given by graphical input. In the example picture, the map $z \mapsto z^2$ is applied.



Download in:
`https://se.mathworks.com/matlabcentral/fileexchange/62212-complex1-f-`

2.2. **Complex map tool 2.** Maps an polygonal chain by a complex map $z \mapsto f(z)$. Vertices are given by graphical input. In the example picture, the map $z \mapsto e^z$ is applied.



Download in:
`https://se.mathworks.com/matlabcentral/fileexchange/62213-complex2-f-`

3. Text editing

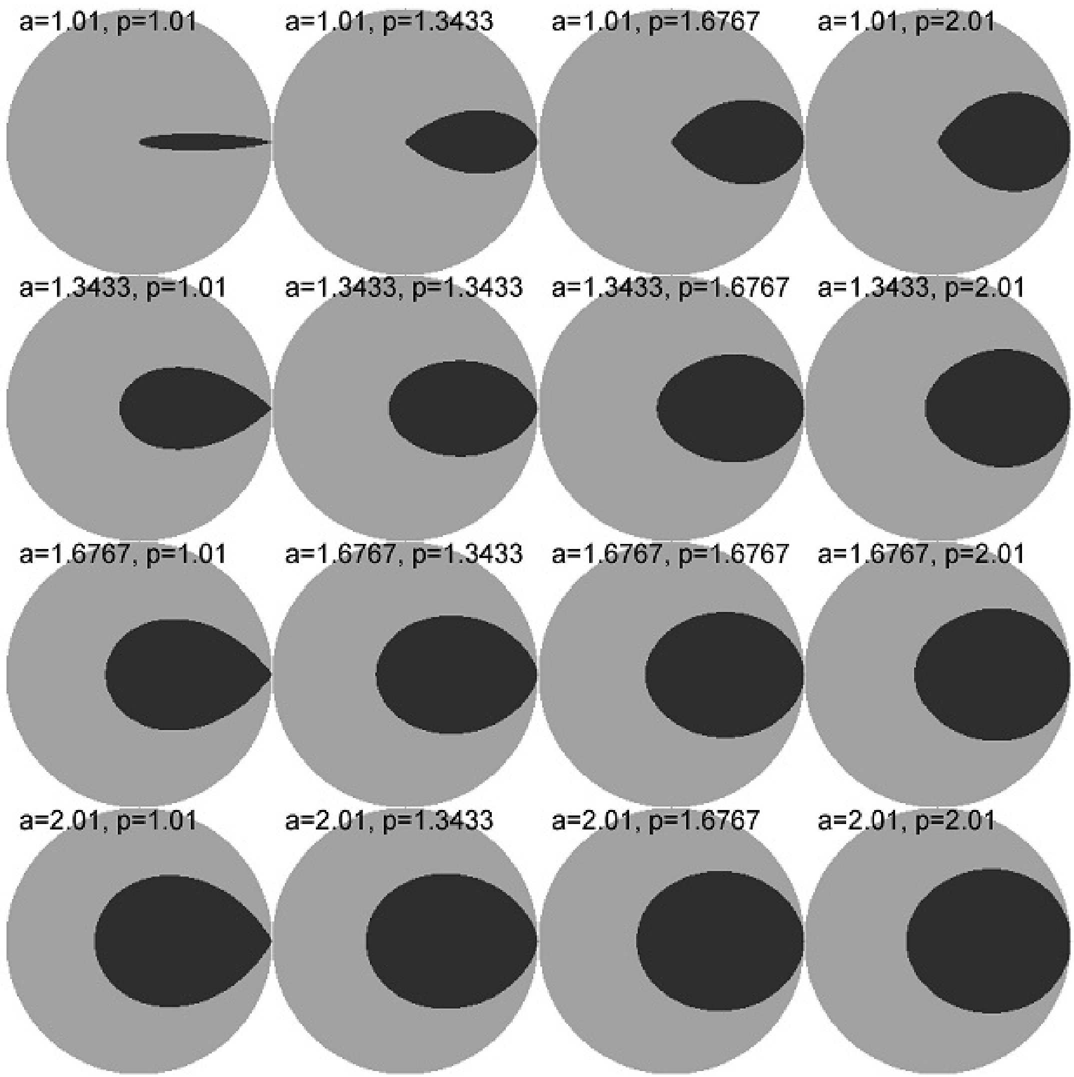
3.1. **Genealogy.** I have written some genealogy details to a file. The function showgenealogy.mat shows the information nicely. The graphics need to be improved.

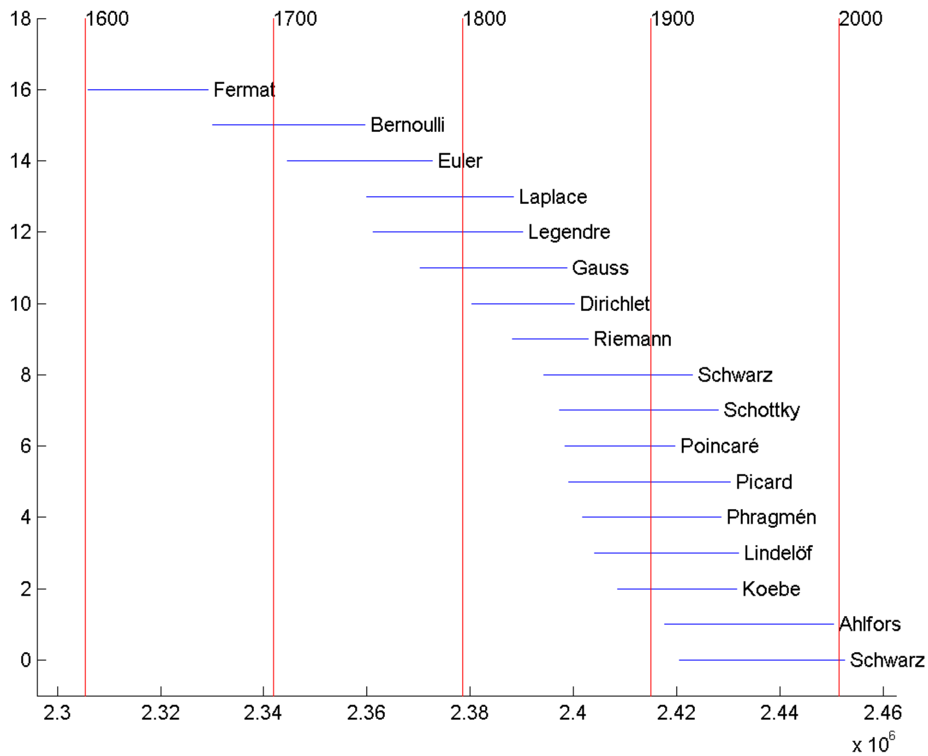
```
>>  load huusko_genealogy.mat
 [D] =showgenealogy(H,3)

D =

    'Iisak Huusko'             'Greta Sofia Boström'
    'Aleksanteri Huusko'       'Hanna Huusko os. Meriläinen'
    '04.05.1895'                                          []
    '08.08.1948'                                          []
    'Sanny Karoliina'          'Rauha Heikkinen os. Huusko'
    'Oskar Aukusti'            'Rauni Piirainen os. Huusko'
    'Arwid'                    'Helmi Huusko'
    'Alma'                     'Helvi Huusko'
    'Nanni Sofia'              'Sirkka Niskanen os. Huusko'
    'Olga'                     'Kauko Huusko'
    'Konsta'                   'Erkki Huusko'
    'Kalle'                    'Eeva Mertanen os. Huusko'
    'Aleksanteri'                                         []
    'Iines Maria'                                         []
    'Akseli'                                              []
    'Frans Eerik'                                         []
```

Download in:
http://integraali.com/stuff/showgenealogy.m

4. Pictures

4.1. **Various pictures.** During my academics, I have made the following pictures:


a=1.01, p=1.01
a=1.01, p=1.3433
a=1.01, p=1.6767
a=1.01, p=2.01
a=1.3433, p=1.01
a=1.3433, p=1.3433
a=1.3433, p=1.6767
a=1.3433, p=2.01
a=1.6767, p=1.01
a=1.6767, p=1.3433
a=1.6767, p=1.6767
a=1.6767, p=2.01
a=2.01, p=1.01
a=2.01, p=1.3433
a=2.01, p=1.6767
a=2.01, p=2.01



Download in:
`http://integraali.com/stuff/mathematicians/`

5. Some calculations

5.1. Hyperbolic geometry. Let

$$\varphi_a(z) = \frac{a-z}{1-\overline{a}z}$$

for $a, z \in \mathbb{D}$, be the disc automorphism. Then $\varphi_a^{-1} = \varphi_a$. A hyperbolic segment between two points $a, b \in \mathbb{D}$, can be parametrized by

$$\langle a, b\rangle = \{\varphi_a\left(\varphi_a(b)t\right) \,:\, 0 \le t \le 1\}\,.$$

The hyperbolic midpoint of $\langle a, b\rangle$, denoted by $\zeta = \varphi_a(\varphi_a(b)t)$, satisfies

$$|\varphi_a(\zeta)| = |\varphi_b(\zeta)|\,.$$

We wish to calculate a formula for ζ. By choosing $a = 0$, we obtain $\varphi_a(z) = -z$, $\zeta = bt$ and

$$|b|t = \left|\frac{b-bt}{1-|b|^2t}\right| = |b|\frac{1-t}{1-|b|^2t}.$$

This implies that

$$t = \frac{1-\sqrt{1-|b|^2}}{|b|^2} = \frac{1}{1+\sqrt{1-|b|^2}} \tag{5.1}$$

In the general case, map the segment $\langle a, b\rangle$ by φ_a, so that points a, ζ, b map to points $0, \varphi_a(\zeta), \varphi_a(b)$. Since the automorphism preserves hyperbolic distances, $\varphi_a(\zeta)$ is the midpoint of $[0, \varphi_a(b)]$ and we obtain by (5.1) that

$$\varphi_a(\zeta) = \varphi_a(b)t,$$

that is,

$$\zeta = \varphi_a\left(\varphi_a(b)t\right), \quad t = \frac{1}{1+\sqrt{1-|\varphi_a(b)|^2}},$$

that is,

$$\zeta = \varphi_a\left(\frac{\varphi_a(b)}{1+\sqrt{1-|\varphi_a(b)|^2}}\right). \tag{5.2}$$

Since ζ is the midpoint of a and b, formula (5.3) should remain the same, when points a and b are exchanged. By considering the mapping of $\langle a, b\rangle$ to both $[0, \varphi_a(b)]$ and $[0, \varphi_a(b)]$ and noting that $|\varphi_a(b)| = |\varphi_b(a)|$, this is really the case. Therefore

$$\zeta = \varphi_a\left(\frac{\varphi_a(b)}{1+\sqrt{1-|\varphi_a(b)|^2}}\right) = \varphi_b\left(\frac{\varphi_b(a)}{1+\sqrt{1-|\varphi_b(a)|^2}}\right). \tag{5.3}$$

We have

$$\zeta = \frac{a(1-\overline{a}b)-t(a-b)}{1-\overline{a}b-\overline{a}t(a-b)}, \quad t = \frac{1}{1+\sqrt{1-|\varphi_a(b)|^2}}.$$

6. Math ideas

Let A be analytic in $\mathbb{D}$ and let $\{f, g\}$ be a solution base for the equation

$$f'' + Af = 0$$

such that $W(f,g) = fg' - f'g \equiv 1$. Let

$$A \in H^\infty_{2+2\varepsilon} \setminus \bigcup_{0<p<2+2\varepsilon} H^\infty_p,$$

where $h \in H^\infty_p$ if and only if

$$\sup_{z\in\mathbb{D}} |h(z)|(1-|z|^2)^p < \infty.$$

Assume that there exists $\{z_n\} \subset \mathbb{D}$ such that $|z_n| \to 1^{-1}$ as $n \to \infty$ and

$$|A(z_n)| \geq \frac{1}{(1-|z_n|)^{2+2\varepsilon}}, \quad n \in \mathbb{N}.$$

Since

$$A = -\frac{f''}{f} = -\frac{f''}{f'}\frac{f'}{f},$$

we have

$$\max\left\{\left|\frac{f''(z_n)}{f'(z_n)}\right|, \left|\frac{f'(z_n)}{f(z_n)}\right|\right\} \geq \frac{1}{(1-|z_n|)^{1+\varepsilon}}.$$

The condition

$$\frac{f''(x)}{f'(x)} = \frac{1}{(1-x)^{1+\varepsilon}}$$

heuristically implies

$$(\log f(x))' = \frac{1}{(1-x)^\varepsilon} + C,$$

that is,

$$f(x) = e^{\frac{1}{(1-x)^\varepsilon}}.$$

7. Math sketches

Define the spherical distance

$$\sigma(z,w) = \frac{2}{\pi} \inf_{\gamma} \int_{\gamma} \frac{|d\zeta|}{1+|\zeta|^2}.$$

where γ is a curve joining z and w. Now $\sigma(0,1) = 1/2$ and $\sigma(0,\infty) = 1$. Moreover,

$$\sigma(0,z) = \frac{2}{\pi} \tan^{-1}(|z|).$$

Let $D \subset \widehat{\mathbb{C}}$ be a domain and $f : D \to \widehat{\mathbb{C}}$. Let $A \subset \widehat{\mathbb{C}}$ be a non-empty set. Define

$$M(r,f,A) = \max \{|f(z)| \,:\, \sigma(z,A) = r\}.$$

Hence, in case of an entire function

$$M_\infty(r,f) = \max_{|z|=r} |f(z)| = M(2(1-\tan^{-1}(r))/\pi, f, A)$$

and

$$M_\infty(r,f) = M(2(1/2 - tan^{-1}(r))/\pi, f, \partial\mathbb{D})$$

Since for $z \approx w$, we have $\sigma(z,w) \approx 2|z-w|/\pi$, we have

$$M(r,f,z_0) \sim \max \{|f(z)| \,:\, |z-z_0| = \pi r/2\}.$$

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