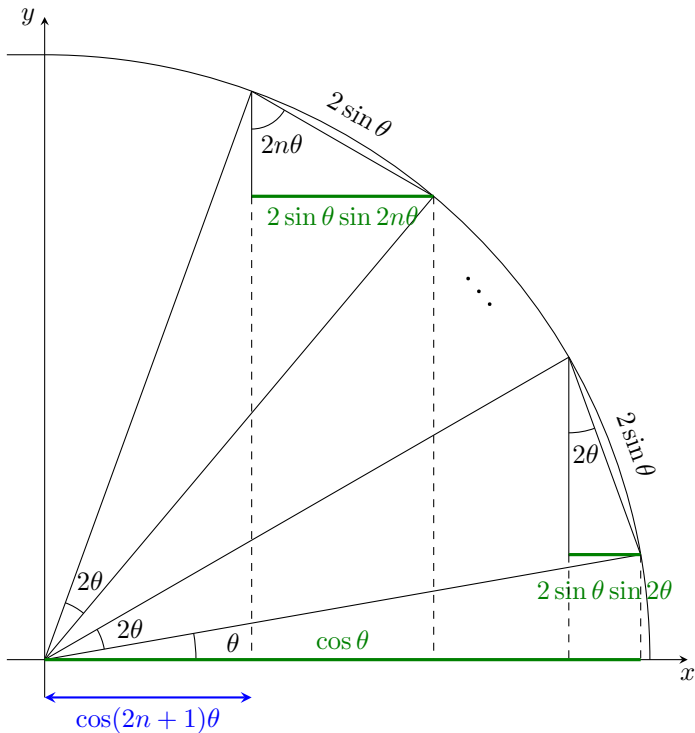


$$y = \sin(3x) = f(x)$$

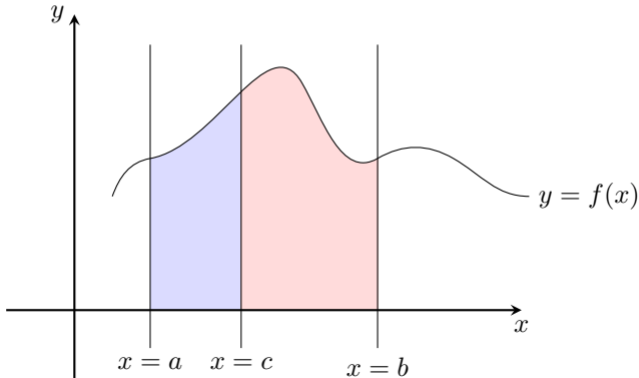
$$y = -\frac{1}{3} \cos(3x) = \int f(x) dx$$

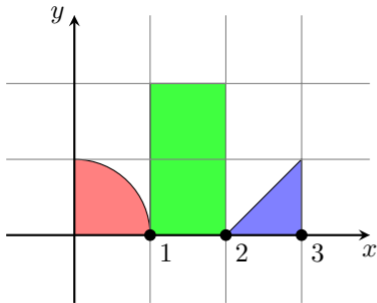


$$\cos(2n+1)\theta = \cos \theta - 2 \sin \theta \sum_{k=1}^n \sin 2k\theta$$

$$\sum_{k=1}^n \sin 2k\theta = \frac{\cos \theta}{2 \sin \theta} - \frac{\cos(2n+1)\theta}{2 \sin \theta} = \frac{1}{2} \cot \theta - \frac{\cos(2n+1)\theta}{2 \sin \theta}$$

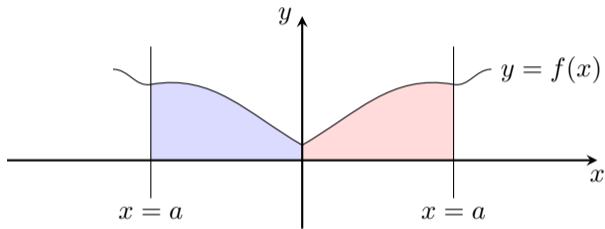
$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$





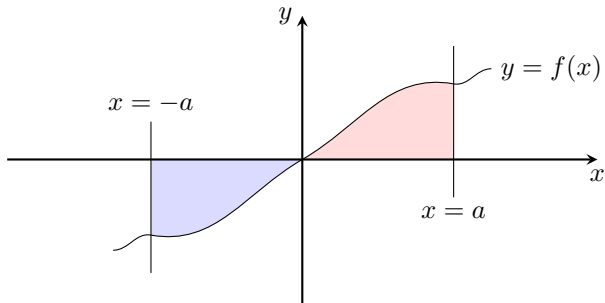
$$\begin{aligned} & \int_0^3 f(x) dx \\ &= \int_0^1 \sqrt{1-x^2} dx + \int_1^2 2 dx + \int_2^3 (x-2) dx \\ &= \frac{\pi}{4} + 2 + \frac{1}{2} \approx 3.035 \end{aligned}$$

Jos $f(-x) \equiv f(x)$, niin $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$.

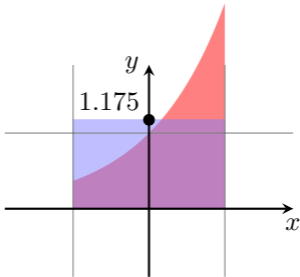


$$A = \text{[blue area]} + \text{[red area]} = 2 \times \text{[blue area]}$$

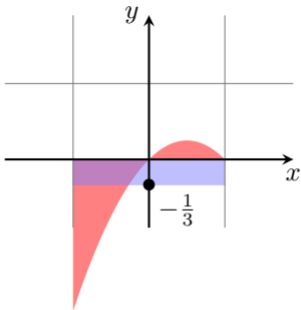
Jos $f(-x) \equiv -f(x)$, niin $\int_{-a}^a f(x) dx = 0$.



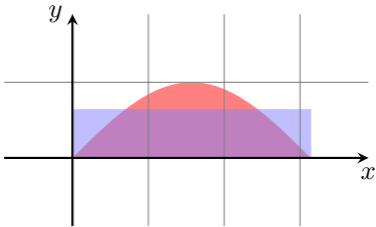
$$A = \text{red area} - \text{blue area} = 0$$



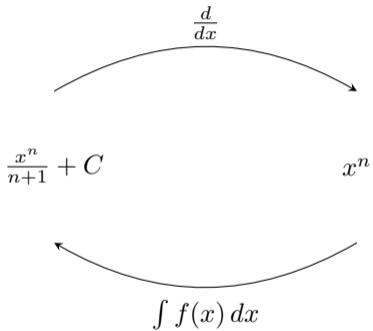
$$\int_{-1}^1 \exp(x) dx \approx 2.35 \approx 2 \cdot 1.175$$

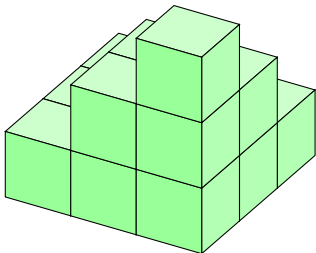
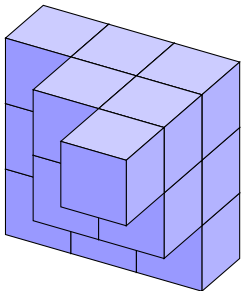
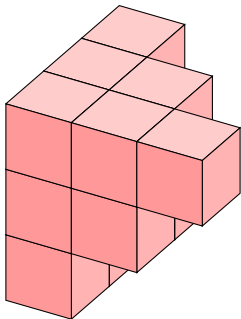


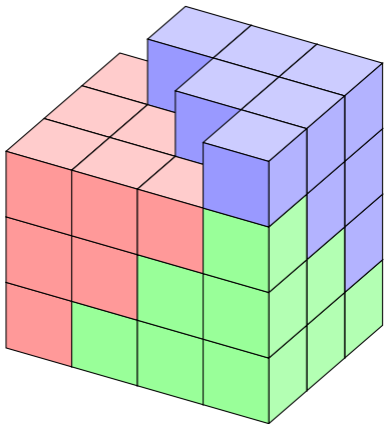
$$\int_{-1}^1 -x^2 + x = -\frac{2}{3} dx = -\frac{1}{3} \cdot 2$$

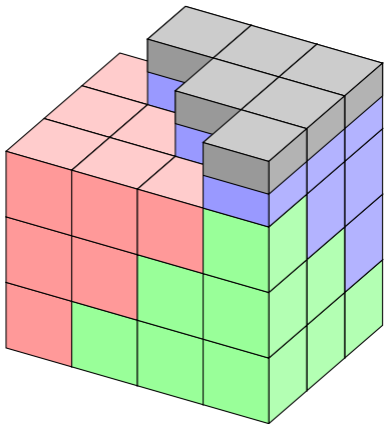


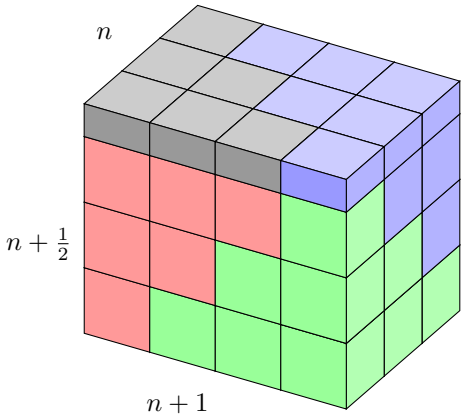
$$\int_0^{\pi} \sin(x) dx = 2 = \pi \cdot \frac{2}{\pi}$$



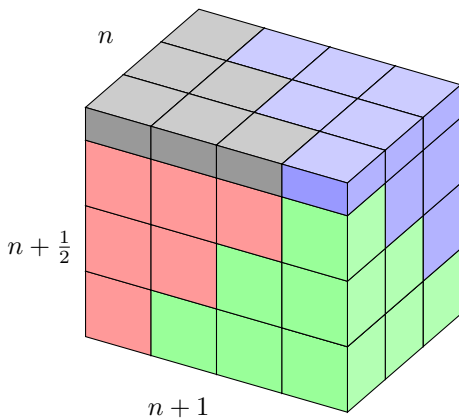
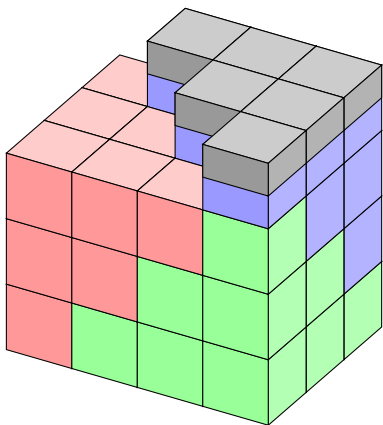
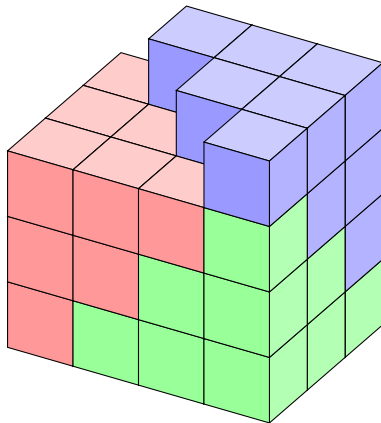
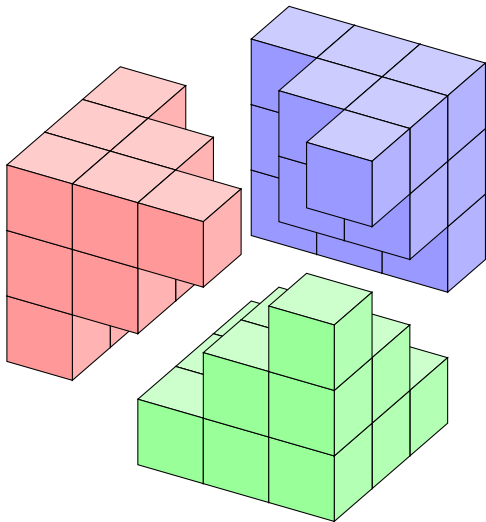




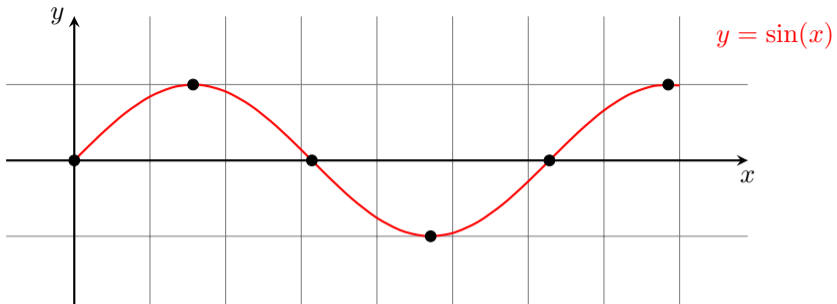


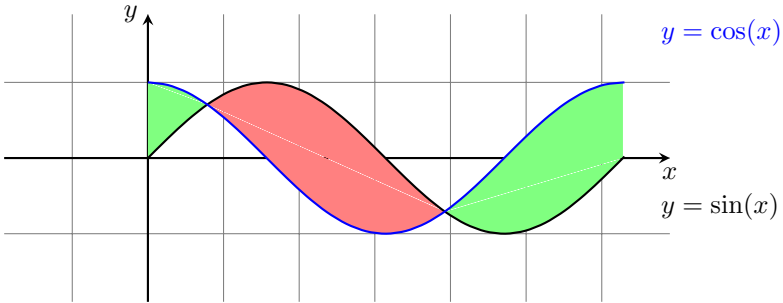


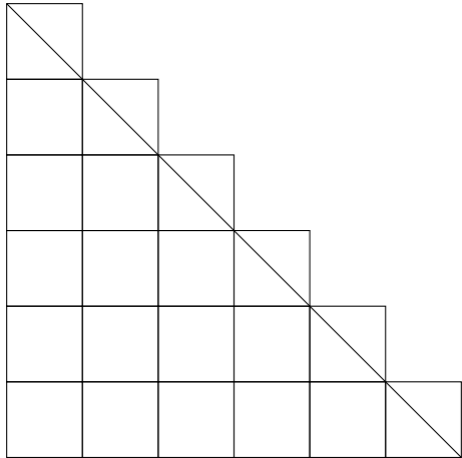
$$1^2 + 2^2 + \dots + n^2 = \frac{1}{3}n(n+1)(n + \frac{1}{2})$$



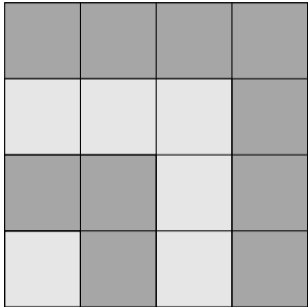
$$1^2 + 2^2 + \dots + n^2 = \frac{1}{3}n(n+1)(n+1/2)$$



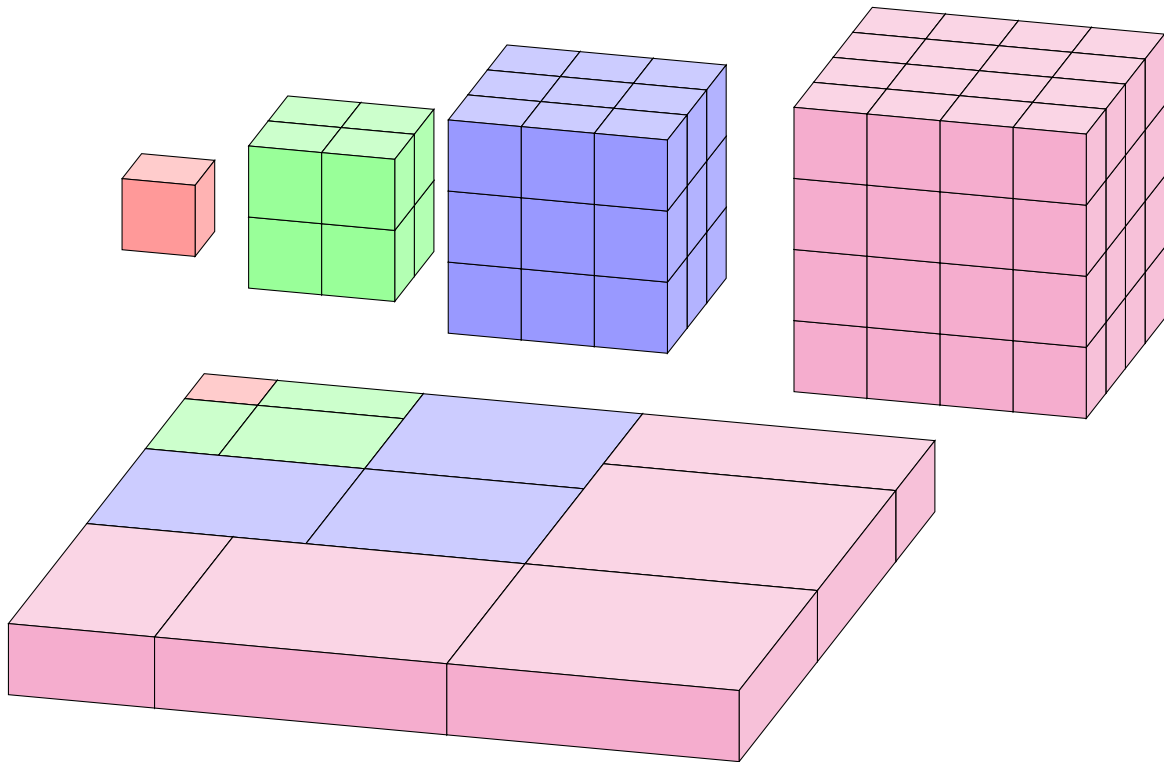




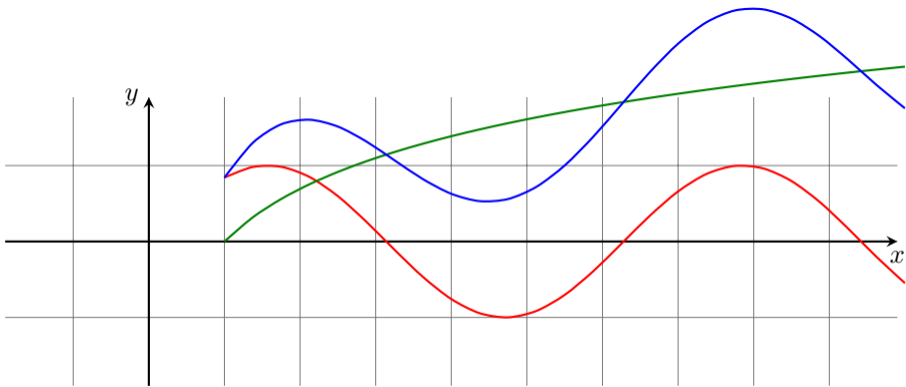
$$1 + 2 + \dots + n = \frac{n^2}{2} + \frac{n}{2} = \frac{n(n+1)}{2}$$



$$1 + 3 + 5 \dots + (2n + 1) = (n + 1)^2$$



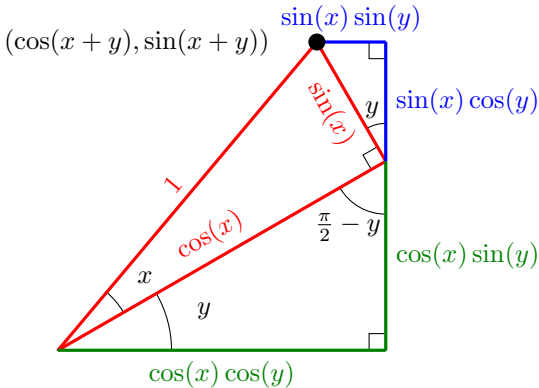
$$1^3 + 2^3 + \dots + n^3 = (1 + 2 + \dots + n)^2$$



$$y = \ln(x)$$

$$y = \ln(x) + \sin(x)$$

$$y = \sin(x)$$



$$\sin(x + y) = \cos(x) \sin(y) + \sin(x) \cos(y)$$

$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$