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Plan to develope my research

I am interested to continue my work in all the topics mentioned below. E.g. a discussion with visiting researchers lead to [4]. Many papers similar to [9] can be simplified with methods simimlar to those in [1]. Studies on Yamanoi's results and 3D graphics have proven to be fruitful. **I have been employed in other peoples projects (for details, see my teaching portfolio) and haven't got personal outside funding.**

About my publications

In [1], I studied the equation

$$f'' + A_1(z)\exp\left(\frac{a_1}{(1-z)^{p_1}}\right)f' + A_0(z)\exp\left(\frac{a_0}{(1-z)^{p_0}}\right)f = 0, \qquad (*)$$

where A_j are analytic in $\mathbb{D}\cup\{1\}$, and $p_j \in (0,\infty)$, and $a_j \in \mathbb{C}$. Here $\mathbb{C}$ is the complex plane and $\mathbb{D} = \{z \in \mathbb{C}\,;\, |z| < 1\}$ is the unit disc. It is noteworthy, that behavior of A_j in other parts of $\partial\mathbb{D}$ is not known.

I found cases of suitable constants a_j, and p_j such that all non-trivial solutions of $(*)$ have order of growth

$$\sigma(f) = \lim_{r\to 1^-} \frac{\log\log M(r,f)}{-\log(1-r)} \geq p_0,$$

where $M(r,f) = \max_{|z|=r}|f(z)|$. The main tool was a localization mapping $T : \mathbb{D} \to \mathbb{D}$ such that $T(\mathbb{D})$ is a drop-shaped area touching the point $z = 1$. The motivation of was to show that results in [9] can be proved and improved by using some suitable localization mapping.

In [2], we studied the equation

$$f'' + A(z)f = 0, \qquad (1)$$

where A is analytic in $\mathbb{D}$. We found that

$$\sup_{z\in\mathbb{D}} |A(z)|(1-|z|^2)^2 \log\frac{e}{1-z} < 1 \quad \text{implies} \quad \sup_{z\in\mathbb{D}} |f'(z)|(1-|z|^2) < \infty,$$

that is, f belongs to the Bloch space. Similarly,

$$\sup_{z\in\mathbb{D}} |A(z)|(1-|z|^2)^2 < p(p+1) \quad \text{implies} \quad \sup_{z\in\mathbb{D}} |f(z)|(1-|z|^2)^p < \infty,$$

that is, f belongs to the H^∞_p space. Both results were shown to be sharp. The results were generalized for the nth order equation. The main method was integration, that is, expressing f in terms of $f^{(n)}$ by using the Fundamental Theorem of calculus. A suitable weight function was introduced and studied.

In [3] we studied the equation (1). We found sufficient conditions for A such that all solutions of (1) belong to certain function space such as H^∞, BMOA or the Bloch space. The main method was to use representation theorems for the function spaces. Also the distribution of zeros of solutions of (1) and a third order equation was studied.

In [4], we studied the equation

$$f'' + A(z)f = 0$$

placing the solutions in the Fock space. The methods were mostly same than in [3].

In [6], we studied locally univalent analytic functions f that satisfy

$$\left|\frac{zf''(z)}{f'(z)}\right|(1-|z|^2) < 1 + C(1-|z|), \quad z \in \mathbb{D},$$

for some $C \in (0,\infty)$. We found that such f is univalent in certain horodiscs. That is, f attains each complex value at most once in such horodiscs. Becker's univalence criterion states that if $(*)$ holds for $C = 0$, then f is univalent in $\mathbb{D}$.

In [5] we studied finite valence criteria for locally univalent functions $f : \mathbb{D} \to \mathbb{D}$. We studied the case where f is analytic in $\mathbb{D}$, and the case where f is harmonic and orientation preserving in $\mathbb{D}$. In the latter case, $f = g + \overline{h}$ for some analytic functions g and h and $|h(z)| < |g(z)|$ for $z \in \mathbb{D}$. The main tool was to use the localization idea from the recent paper [10].

In [7] and [8], I helped with the complex analytic tools. The topic of the papers is photonics.

About my postdoc period 12/2018-7/2019

We studied two papers [11] and [12] by K. Yamanoi. We held a seminar discussing the details of these papers; the materials can be found in [13]. Currently, one PhD student is studying the analogues of the results for the difference operator. I am his second supervisor.

Let me say briefly what are [11] and [12] are about. In [11, Theorem 1.2] where, essentially, the number of poles and a-points of a meromorphic function are bounded by the number of zeros of its k^{th} derivative. This allows Yamanoi to prove the Mues' and Gol'dberg conjectures. The proof is divided into two theorems: we discussed the entire proof of [11, Theorem 1.3] in our seminar; whereas the proof of [11, Theorem 1.4] requires advanced techniques.

To discuss the content of [12], recall that the Second main theorem of Nevanlinna theory reads

$$\sum_{j=1}^{k} \frac{1}{2\pi} \int_0^{2\pi} \log^+ \frac{1}{|f(re^{i\theta} - a_j|} \, d\theta \leq 2T(r,f) - N_1(r,f) + S(r,f).$$

In [12, Theorem 1], Yamanoi lets $k = k(r)$ with suitable growth rate, replaces the sum with a maximum over a_j, and places the maximum inside the integral. Consequently, instead of an inequality, an asymptotic equality is attained. For details, see [12].

About programming 3D visualizations

I gave a talk [15] on my approach to create 3D visualizations by using the JSXGraph JavaScript library. Inspired by this, the developers are implementing the 3D features in the library. For an announcement, see [14].

References

My publications

[1] Huusko, J.-M., *Localisation of Linear Differential Equations in the Unit Disc by a Conformal Map*, Bull. Aust. Math. Soc. 93 (2016), 260–271.

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[7] Pesonen, H., A. Halder, J.-M. Huusko, A.T. Friberg, T. Setälä and J. Turunen, *Spatial coherence effects in second-harmonic generation of scalar light fields*, J. Opt., Volume 23, Number 3.

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Other

[9] Hamouda S., *Properties of solutions to linear differential equations with analytic coefficients in the unit disc*, Electron. J. Differential Equations 177 (2012), 1-8.

[10] Becker J. and Ch. Pommerenke, *Locally univalent functions and the Bloch and Dirichlet norm*, Comput. Methods Funct. Theory 16, 43–52 (2016)

[11] Yamanoi, K., *Zeros of higher derivatives of meromorphic functions in the complex plane*, Proc. London Math. Soc. (3) 106 (2013) 703-780.

[12] Yamanoi, K., *On a reversal of the second main theorem for meromorphic functions of finite order*, The proceedings of the 19th ICFIDCAA "Topics in Finite or Infinite Dimensional Complex Analysis", (2013), 75–83.

[13] Notes and exercises of the Yamanoi seminar `http://integraali.com/yamanoi/`

[14] Release of JSXGraph 1.4.3 `https://jsxgraph.org/wp/2022-04-14-release-of-version-1.4.3/`

Talks

[15] Huusko, J.-M., *JSXGraph and 3D graphics*, JSXGraph conference, 6.10.2021 (abstract).