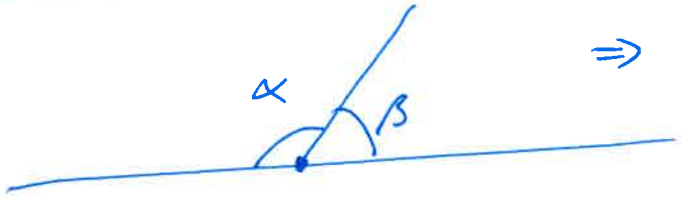
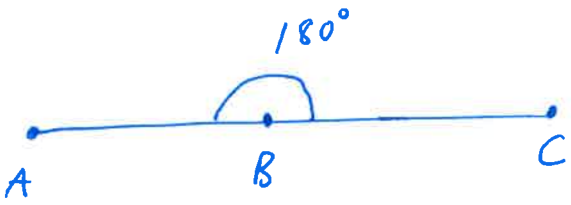
LAUSE 1.13 (VIERUSKULMALAUSE)

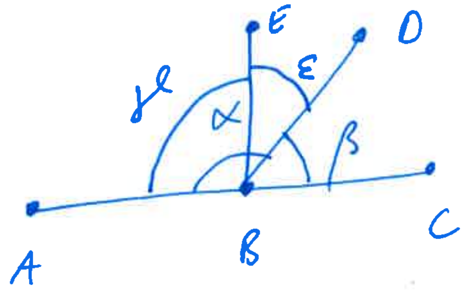


$\Rightarrow \alpha + \beta = 180°$

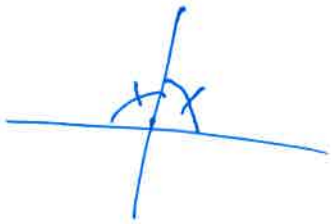
Tod. [ANTIIKIN KREIKKALAISIA MIETITYTTI, ETTÄ



ONKO TÄMÄ KUVIO "KULMA", SIINÄHÄN ON VAIN SUORA VIIVA.]



PIIRRÄ $EB \perp \overleftrightarrow{AC}$



$$EB \perp \overleftrightarrow{AC} \Leftrightarrow \gamma = (\varepsilon + \beta)$$

$$\Leftrightarrow \gamma \ \& \ (\varepsilon + \beta) \text{ SUORIA KULMIA}$$

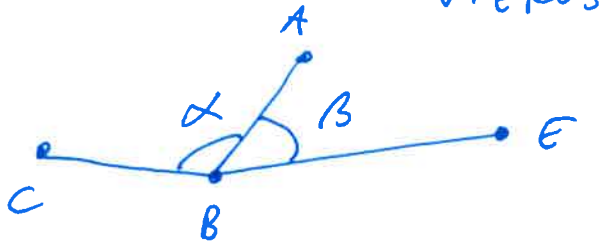
$$\Rightarrow \gamma = 90° \ \& \ (\varepsilon + \beta) = 90°$$

$$\Rightarrow 180° = \gamma + (\varepsilon + \beta) = (\gamma + \varepsilon) + \beta$$

$$= \alpha + \beta.$$

$\square$

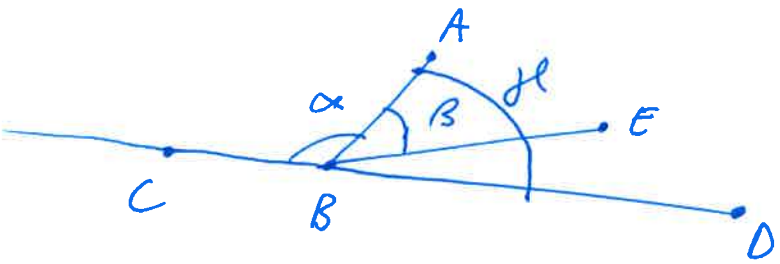
LAUSE 1.14 (KÄÄNTEINEN VIERUSKULMALAUSE)



$$\alpha + \beta = 180^\circ$$

$$\Rightarrow E \in \overleftrightarrow{CB}.$$

Tod. ANTITEESI: $E \notin \overleftrightarrow{CB}$.



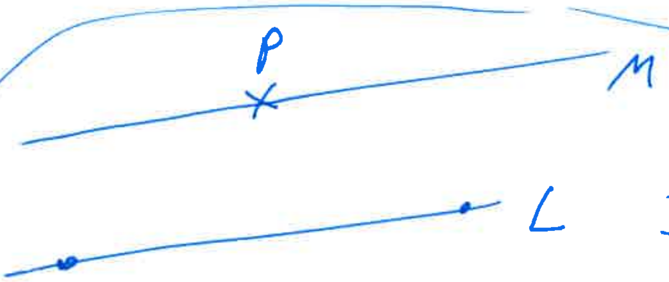
OLETETAAN ESIM. ETTÄ ON NÄIN, $D \in \overleftrightarrow{CB}$.

KOSKA $AB \perp \overleftrightarrow{CD}$, NIIN

$$\alpha + \gamma = 180^\circ = \alpha + \beta \quad \| -\alpha$$

$$\Rightarrow \quad \gamma = \beta$$

$\Rightarrow$ RISTIRIITA, SILLÄ KUVASSA $\gamma > \beta$. □



$\exists M \parallel L$

M YKSIKÄSITTEINEN

PARALLEELIAKSIOOMA

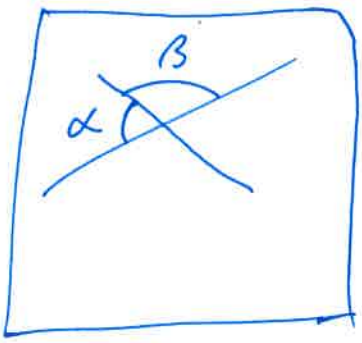
1 kpl

YHDENSUUNTAISIA

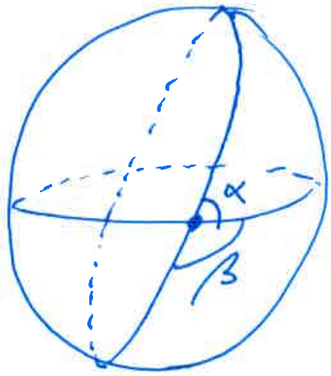
HUOM. TÄHÄNASTISISSA TULOKSISSA EI OLE KÄYTETTY PARALLEELIAKSIOOMAA, SIIS TULOKSET PÄTEVÄT:

- EUKLIDISESSA TASOSSA
- PALLOGEOMETRIASSA
- HYPERBOLISESSA GEOMETRIASSA

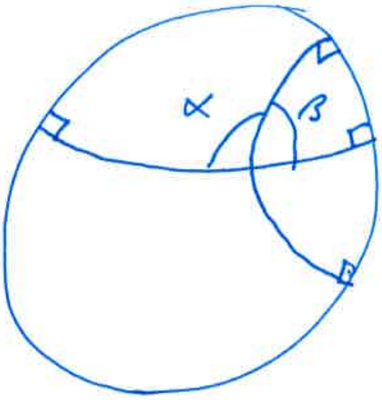
SIIS ESIM. LAUSE 1.13:



$\alpha+\beta=180^\circ$

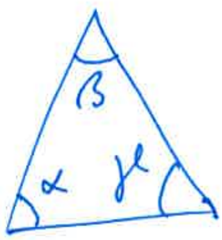


$\alpha+\beta=180^\circ$

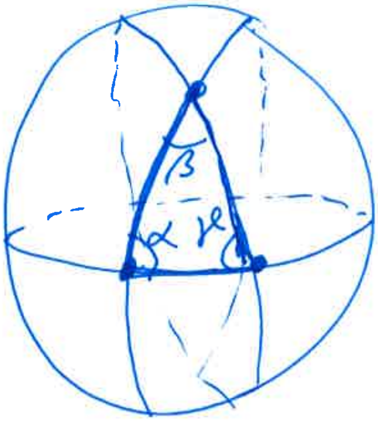


$\alpha+\beta=180^\circ$

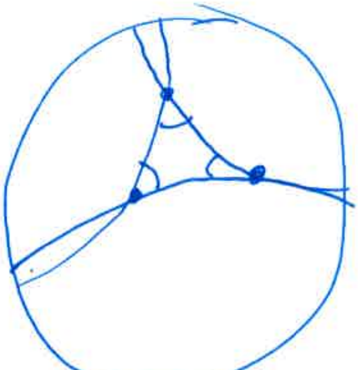
MUTTA ESIM.



$\alpha+\beta+\gamma=180^\circ$

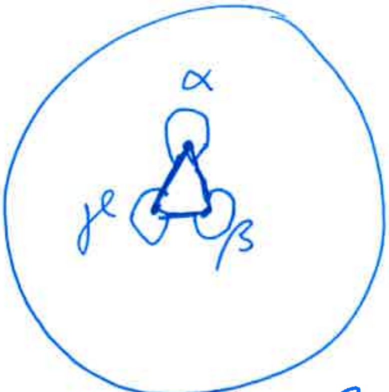


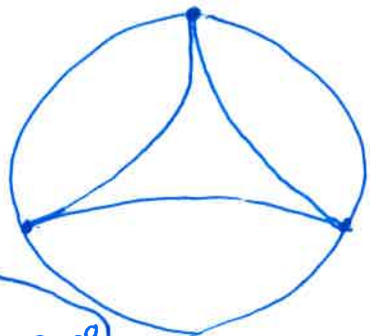
$\alpha+\beta+\gamma>180^\circ$



$\alpha+\beta+\gamma<180^\circ$

VOI OLLA JOPA

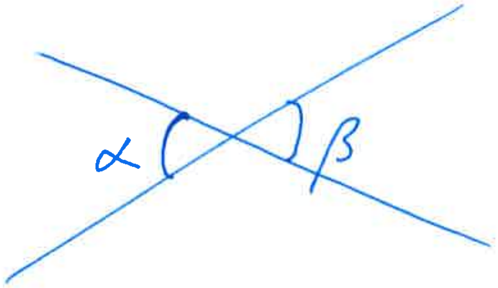




$\alpha+\beta+\gamma\approx 0$

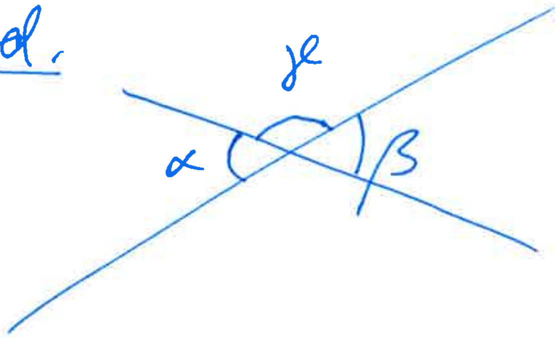
$\alpha+\beta+\gamma \approx 3\cdot 360^\circ - \overbrace{180^\circ}^{=\frac{1}{2}\cdot 360^\circ} = \frac{5}{2}\cdot 360^\circ$

LAUSE 1.15 (RISTIKULMALAUSE)



$\Rightarrow \quad \alpha = \beta$

TOD.

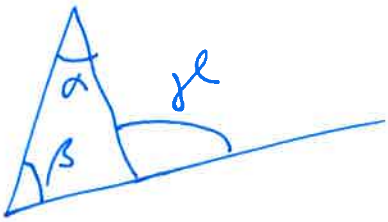


VIERUSKULMALAUSE $\Rightarrow$

$$\begin{array}{rl} & \alpha + \gamma = 180^\circ \\ - & \gamma + \beta = 180^\circ \\ \hline & \alpha - \beta = 0 \end{array}$$

$\Rightarrow \quad \alpha = \beta.$ $\square$

LAUSE 1.16 (ULKOKULMAEPÄYHTÄLÖ)



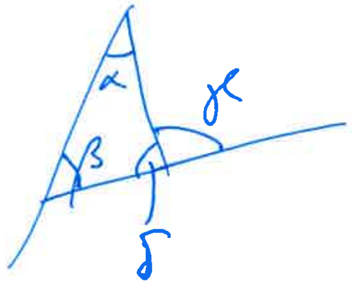
$\Rightarrow \quad \gamma > \alpha$

$\gamma > \beta.$

HUOM. TÄMÄ TULOS PÄTEE EUK., PALLO- JA HYP. GEOMETRIASSA.

SANOTAAN, ETTÄ TULOS KUULUU NEUTRAALIIN GEOMETRIAAN, JOSSA EI OTETA KANTAA SIIHEN MIKÄ VERSIO PARALLEELIAKSIOOMASTA ($P \notin \overleftrightarrow{AB} \Rightarrow$ YHDENSUUNTAISIA $L \parallel \overleftrightarrow{AB}$ JOLLE $P \in L$ JA $L \cap \overleftrightarrow{AB} = \emptyset$ ON 1 kpl / 0 kpl / ∞ kpl)

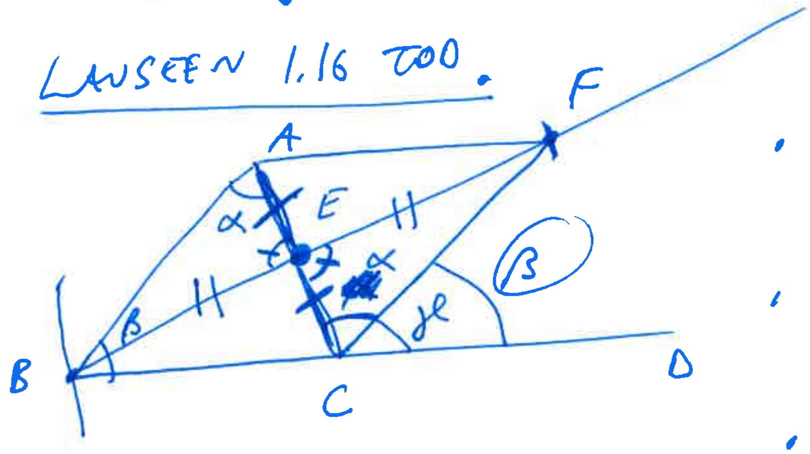
MYÖHEMMIN OSOITETAAN, ETTÄ EUKLIDISESSA GEOMETRIASSA



① $\alpha + \beta + \delta = 180°$

② $\gamma = \alpha + \beta \Rightarrow \begin{cases} \gamma > \alpha \\ \gamma > \beta \end{cases}$.

LAUSEEN 1.16 TOD.



$\alpha\ \cancel{\beta} < \gamma$

- PIIRRÄ $E \in \overleftrightarrow{AC}$, JOLLE $AE = EC$,
- PIIRRÄ $\overrightarrow{BE}$
- PIIRRÄ $F \in \overrightarrow{BE}$, JOLLE $BE = EF$
- PIIRRÄ CF

~~SKS~~ VIERUSKULMINA $\measuredangle AEB = \measuredangle CEF$

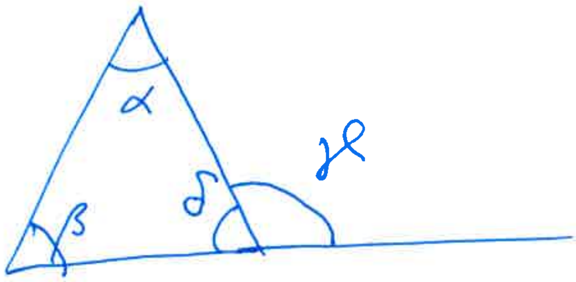
SKS $\Rightarrow \triangle EAB \cong ECF$

$\Rightarrow \alpha = \measuredangle FCE < \gamma$.

VASTAAVASTI TODISTETAAN, ETTÄ $\beta < \gamma$. $\square$

LAUSE 1.17 KOLMIOSSA KAHDEN KULMAN SUMMA ON $< 180°$.

Tod.

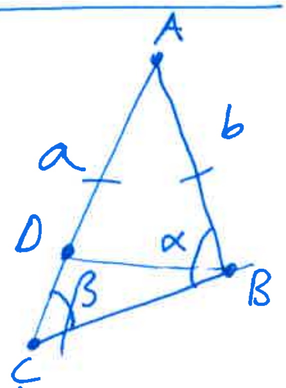


ULKOKULMAEPÄYHTÄLÖN NOJALLA

$$\alpha + \delta < \gamma + \delta = 180°.$$

$\square$

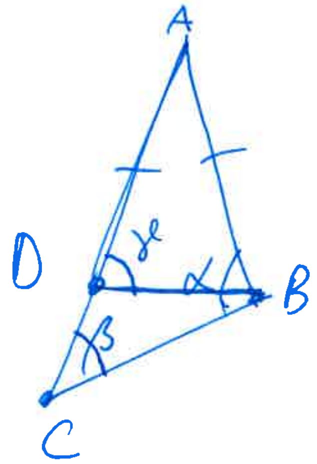
LAUSE 1.18



$a > b \Rightarrow \alpha > \beta.$

Tod. OLKOON ESIM. $AC > AB$.

OTETAAN $D \in AC$ JOLLE $AD = AB$.



KOLMIOSSA $\triangle BCD$

SISÄKULMA $\beta < \gamma$.

EDELLEEN $\gamma = \measuredangle ABD$.

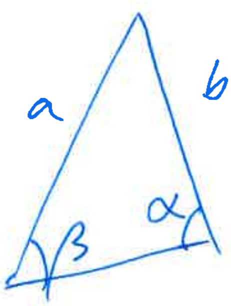
LISÄKSI KUVASTA $\measuredangle ABD < \alpha$.

SIIS $\beta < \gamma = \measuredangle ABD < \alpha$.

$\square$

LAUSE 1.19

$\alpha > \beta \Rightarrow a > b.$



Tod. AT. 1° $a = b \Rightarrow \alpha = \beta$ ↯

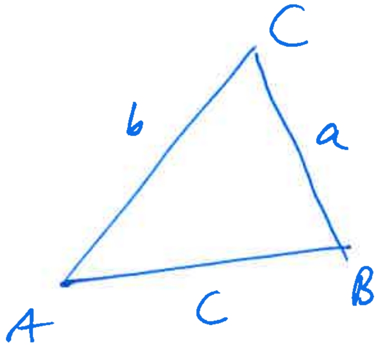
2° $a < b \overset{L1.18.}{\Rightarrow} \beta > \alpha$ ↯

1° & 2° $\Rightarrow$ ANTITEESI ON VÄÄRÄ

$\Rightarrow$ VÄITE ON TOTTA.

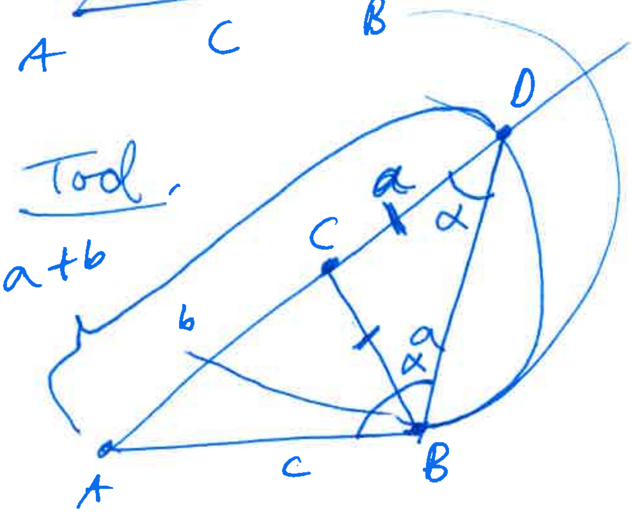
$\square$

LAUSE 1.20 (KOLMIOEPÄYHTÄLÖ)



PÄTEE
$$\begin{cases} a < b + c \\ b < a + c \\ c < b + a \end{cases}$$

Tod.



PIIRRÄ OHEINEN KUVIO.

$$CD = CB$$
$$\Rightarrow \measuredangle CDB = \measuredangle DBC$$
$$< \measuredangle DBA$$

SIIS KOLMION ΔDBA SIVUILLE PÄTEE (L 1.19 NOJALLA) $c < a + b$.

$\square$