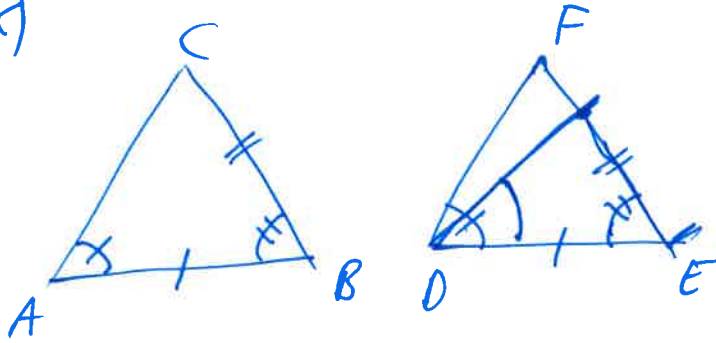


LAUSE 1.26 (KSK JA KKS)

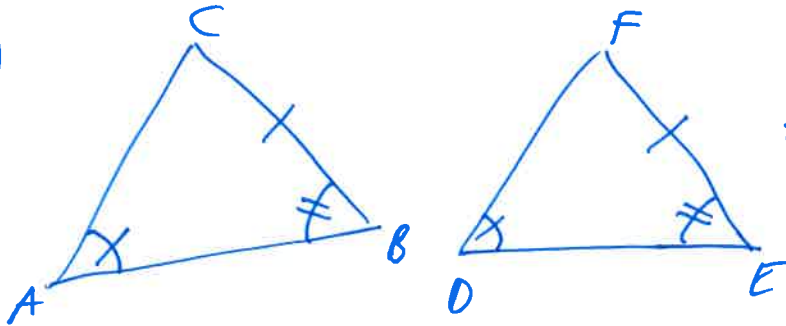
6

(KSK)



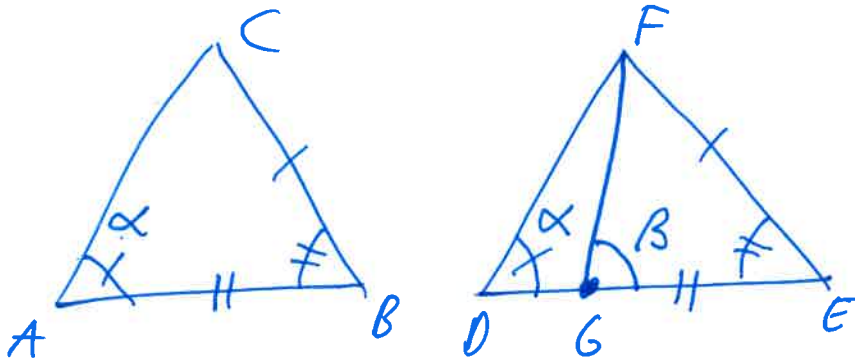
$\Rightarrow \triangle ABC \cong \triangle DEF$

(KKS)



$\Rightarrow \triangle ABC \cong \triangle DEF$

Tod (KKS) AT $AB < DE$.



OTA GE DE SITTEN, ETÄ $GE = AB$.
PIIRRI GF.

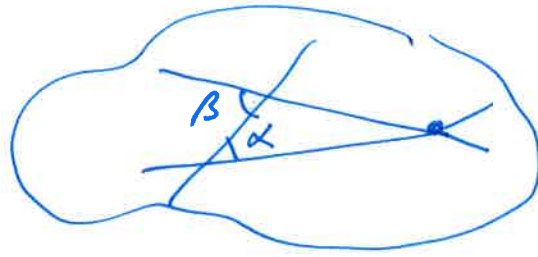
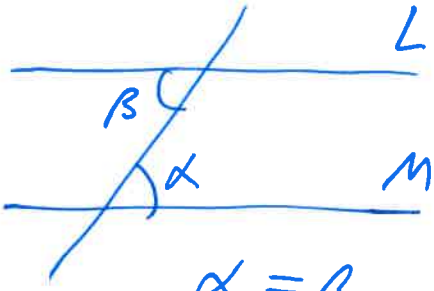
SKS $\Rightarrow \triangle ABC \cong \triangle GEF$

$\Rightarrow \alpha = \beta$ (VASTINKULMINA)

TOISAALTA ULKOKULMA EPÄYHTÄLÖN
NOJALLA $\alpha < \beta$. RR

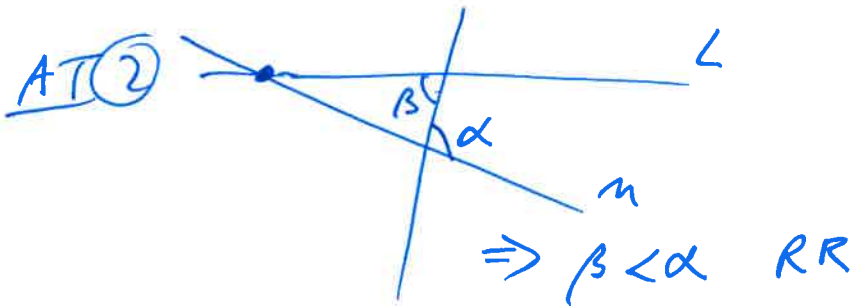
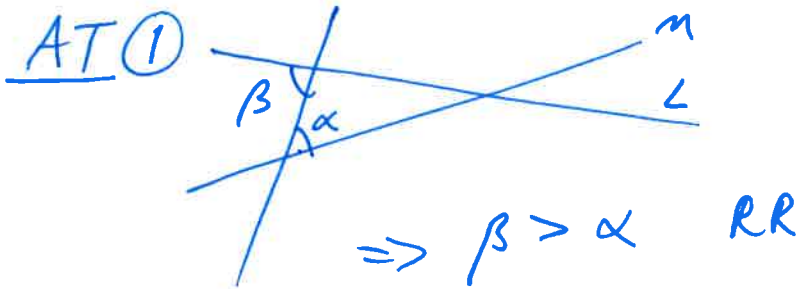


LAUSE 1.27 (VUOROKULMA LAUSE)

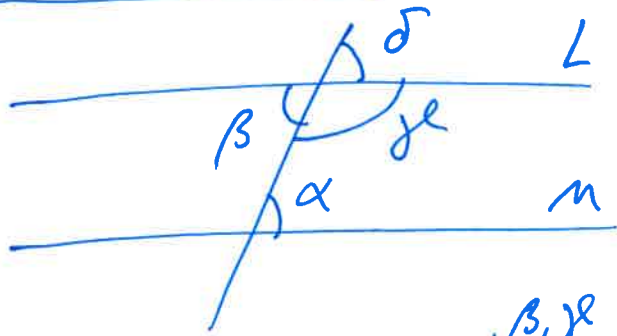


$\alpha = \beta \Rightarrow L \parallel M$ (eli $L \cap M = \emptyset$
tai $L = M$)

Tod. ULKO KULMA EPÄYHTÄISIN NOJALLA



LAUSE 1.28



(a) $\alpha + \gamma = 180^\circ \Rightarrow L \parallel M$

(b) $\alpha = \delta \Rightarrow L \parallel M$

(c) $\alpha = \beta \Rightarrow L \parallel M$ ok

Tood.

(a)

$\beta = 180^\circ - \gamma = \alpha \Rightarrow L \parallel M$

β, γ VIERUSKULMIA

L1.27

RISTIKULMIA

OLETUS

(b)

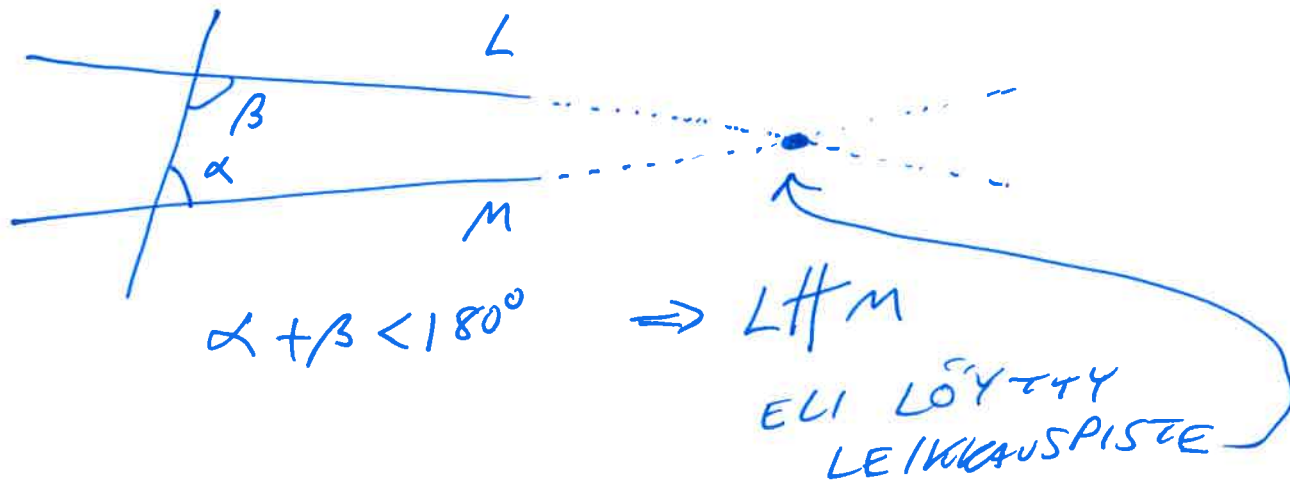
$\delta = \beta = \alpha \Rightarrow L \parallel M$

L1.27



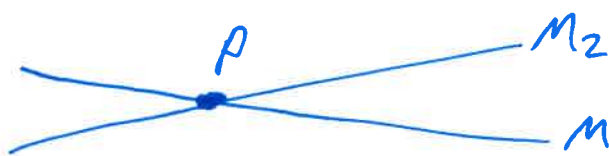
OTE TAN SEURAAVAKSI KÄYTÖÖN
 EUKLEIDEN PARALLELAKSIOMA

EUKLEIDES:



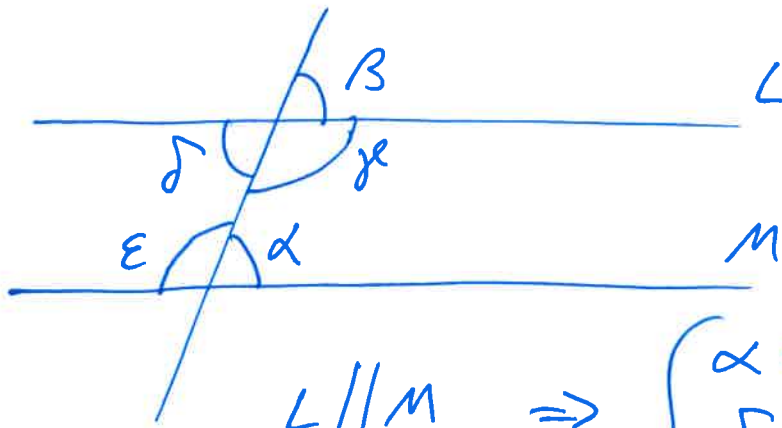
[\Leftrightarrow]

PLAY FAIR:



$$\left[\begin{array}{l} P \notin L \\ P \in M \parallel L \\ P \in M_2 \not\parallel L \end{array} \Rightarrow M = M_2 \right]$$

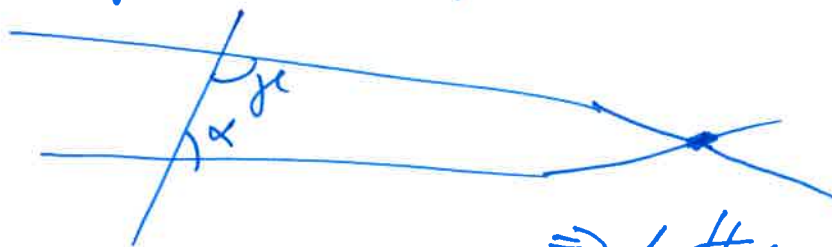
LAUSE 1.29 (KÄÄNTÄINEN VUOROKULMA LAUSE)



$$L \parallel M \Rightarrow \begin{cases} \alpha = \beta \\ \delta = \epsilon \\ \alpha + \beta = 180^\circ \end{cases}$$

Tod. Omitetaan $\alpha + \beta = 180^\circ$,

AT ①
PAR. AKS.
⇒ $\alpha + \beta < 180^\circ \Rightarrow$

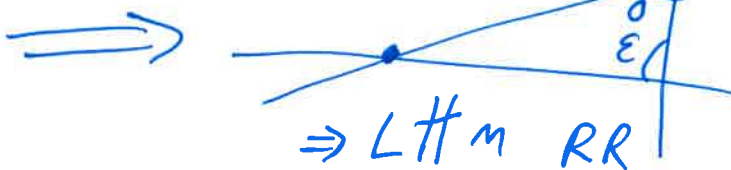


⇒ $L \nparallel M$ RR

AT ② $\alpha + \beta > 180^\circ$

$$\begin{aligned} \Rightarrow \epsilon + \delta &= (180^\circ - \alpha) + (180^\circ - \beta) \\ &= 360^\circ - \underbrace{(\alpha + \beta)}_{> 180^\circ} < 180^\circ \end{aligned}$$

PAR. AKS.



Siis $\alpha + \gamma = 180^\circ$, Osoiteaan $\begin{cases} \alpha = \beta \\ \alpha = \delta \end{cases}$.

$$\alpha + \gamma = 180^\circ$$

$$\beta + \gamma = 180^\circ \quad (\text{VIERUSKULMIA})$$

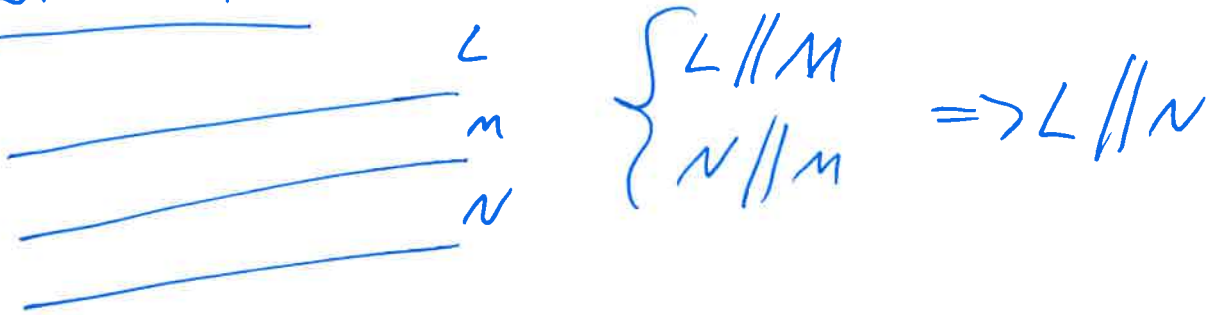
$$\alpha - \beta = 0$$

$$\Rightarrow \alpha = \beta = \delta$$

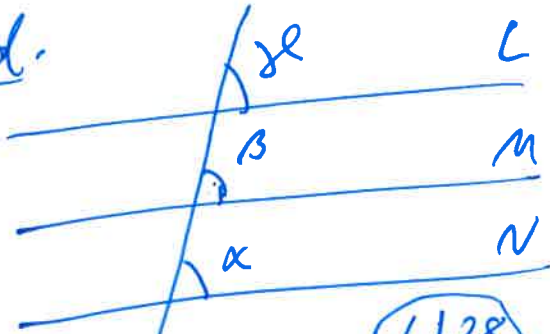
↑ ↑
RISTIKULMIA



LAUSE 1,30



Tod.



$$\left. \begin{array}{l} L // M \Rightarrow \gamma = \beta \\ N // M \Rightarrow \alpha = \beta \end{array} \right\} \Rightarrow \gamma = \alpha \Rightarrow L // N. \quad \square$$

VÄLIKUUVENNYS

$\alpha, \beta, \gamma, \delta, \dots$ = "ALPHA, BETA, GAMMA"
→ ALPHABET

H, I, J, K, L, M, N = "EL, EM, EN"
→ "ELEMENTUM"

= AAKKOSET LATINAANSI

→ EUKLEIDEEEN ALKEET LATINAANSI

= "EUCLID ELEMENTA"

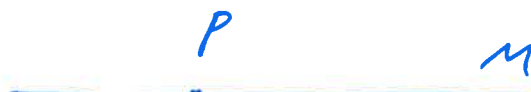
TIMO ERKAMA

$$\int_{-\pi}^{\pi} \cos(\underbrace{mx}_{\text{"EN"}}) \sin(\underbrace{mx}_{\text{"ÄM"}}) dx$$

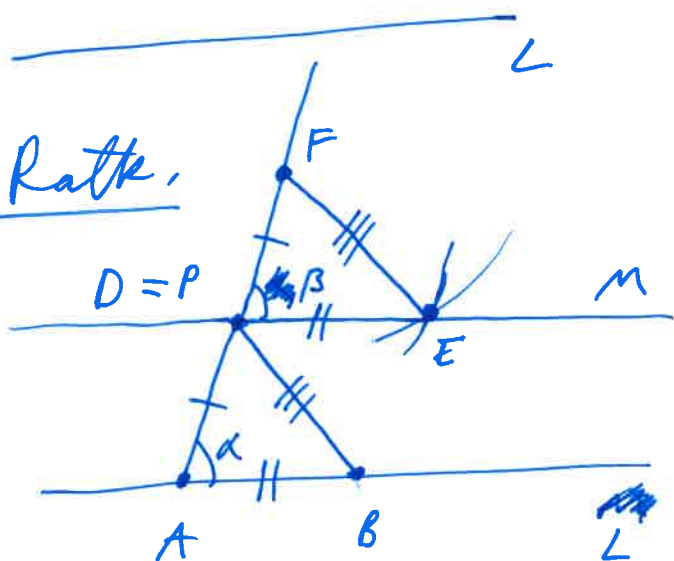
TEHTÄVÄ 1.31

P

?
=>



$L \parallel M$

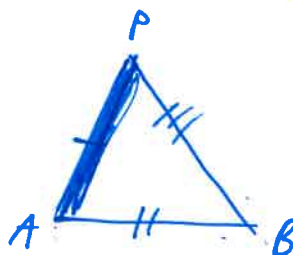


- OTA PISTEET $A, B \in L$.
- PIIRRI \vec{AP} ,
- LAI TA KOLMION $\triangle ABP$ KOPIO $\triangle DEF$ NIIN ETÄ

$D = P, DE \subset \vec{AP}$

PIIRRI! $\vec{DE} = M = \vec{PE}$

$\alpha = \beta \xrightarrow{L. 1.29} L \parallel M$

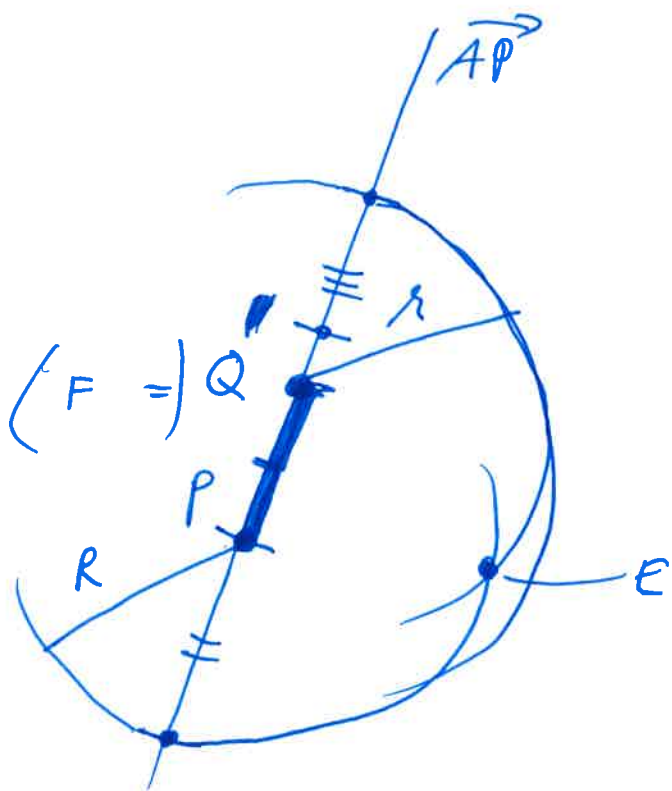


ALUKSI $PQ = AP$

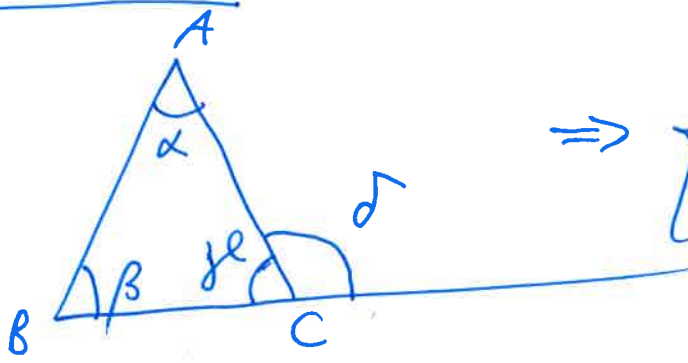
SITTEEN $C(P, AB)$

$C(Q, PB)$

SITTEEN $E = C(P, AB) \cap C(Q, PB)$



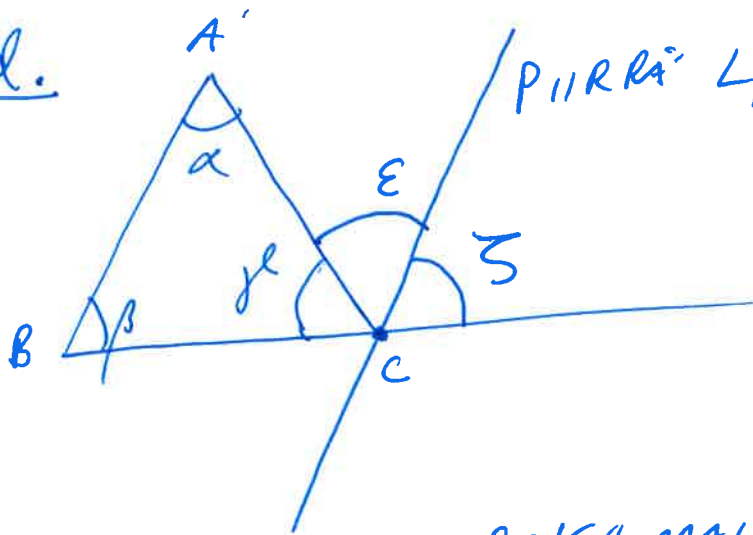
LAUSE 1.32 (KULMASUMMALAUSE)



$$\Rightarrow \begin{cases} \alpha + \beta + \gamma = 180^\circ \\ \alpha + \beta = \delta \end{cases}$$

$$\Rightarrow \begin{cases} \delta > \alpha \\ \delta > \beta \end{cases}$$

Tod.



PIIRRA: $l \parallel AB$ JOSLE $C \in l$
(TEHT. 1.31)

KÄÄNTEINEN VUOROKULMALAUSE

$$\Rightarrow \begin{cases} \alpha = \epsilon & (\text{VUOROKULMIA}) \\ \beta = \zeta & (\text{SAMAN KOHTAISIA KULMIA}) \end{cases}$$

$$\Rightarrow \delta = \epsilon + \zeta = \alpha + \beta$$

VIERUSKULMILLE γ JA δ PIITEE

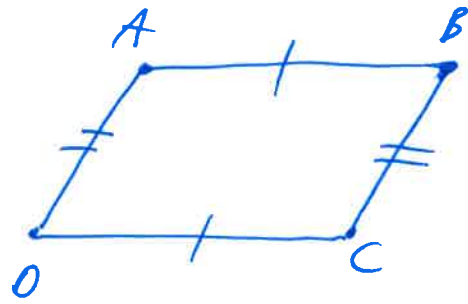
$$180^\circ = \gamma + \delta = \gamma + \alpha + \beta.$$



LAUSE 1.33

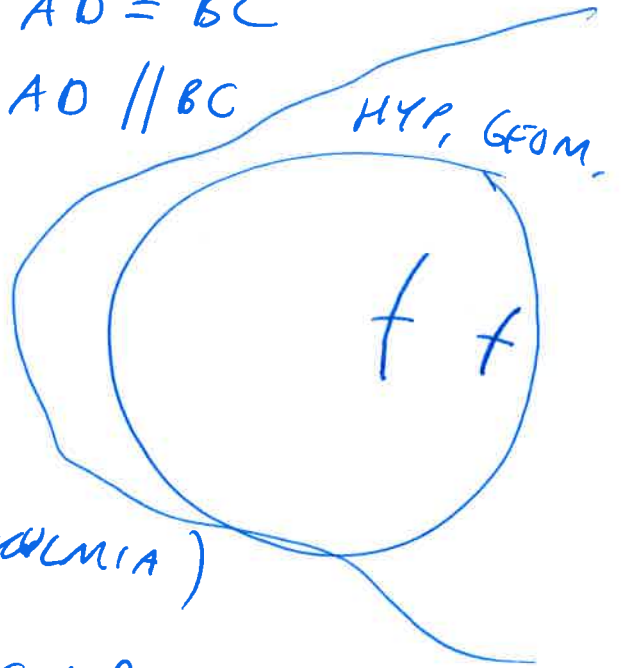
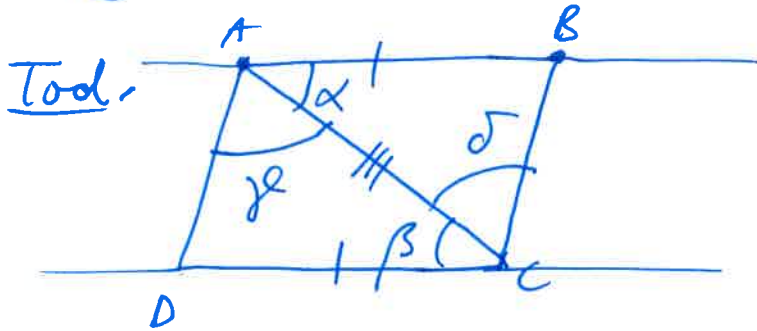


\Rightarrow



$$\begin{cases} AB = DC \\ AB \parallel DC \end{cases}$$

$$\begin{cases} AD = BC \\ AD \parallel BC \end{cases}$$



$$L. 1.29 \Rightarrow \alpha = \beta \text{ (VUOROKULMIA)}$$

$$SKS \Rightarrow \triangle ACB \cong \triangle CAD$$

$$\Rightarrow BC = AD \text{ (VASTIN SIVUINA)}$$

$$\Rightarrow \gamma = \delta \text{ (VASTIN KULMINA)}$$

$$\Rightarrow AD \parallel BC.$$

