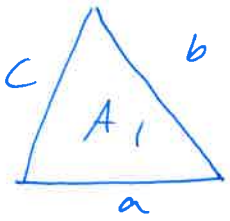
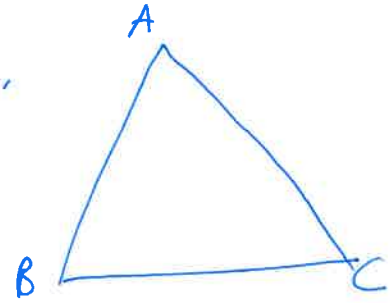
LAUSE (HERON, METRIKA 1.8)

$$p = \frac{a+b+c}{2}$$

$$A_1 = \sqrt{p(p-a)(p-b)(p-c)}$$

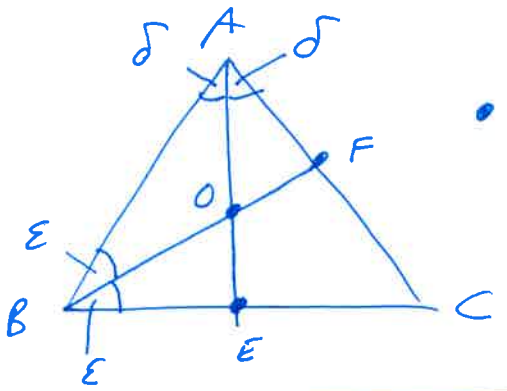


Tod.



PIIRRETÄÄN YMPYRÄ KOLMION $\triangle ABC$ SISÄLLE SEURAAVASTI:

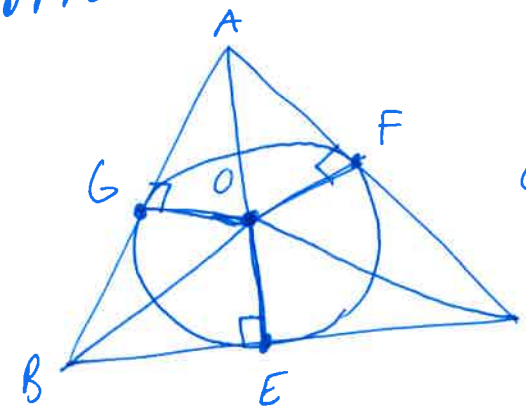
- PIIRRETÄÄN KULMIEN PUOLITTAJAT $\sphericalangle BAC$ JA $\sphericalangle CBA$
- MERKITÄÄN PISTEET O, E, F
- PIIRRETÄÄN YMPYRÄ $C(O, OE)$

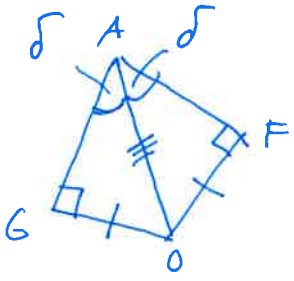


VÄITE: $F \in C(O, OE)$ JA $AB \cap C(O, OE)$ KOOSTUU YHDESTÄ PISTEESTÄ, OLKOON SE G

(TODISTUS HT) JA $\sphericalangle FCO = \sphericalangle OCE$

LOPPUTULOS





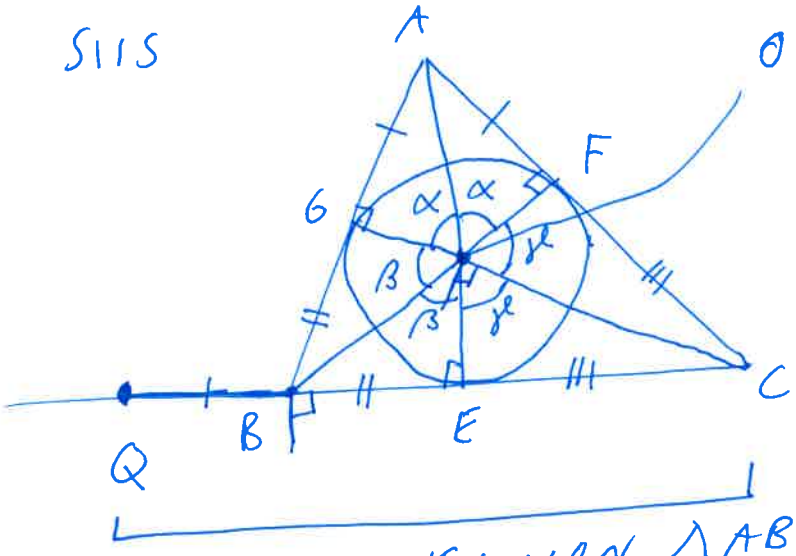
SSK $\Rightarrow$

$\triangle AOG \cong \triangle AOF$

VASTAAVASTI

$\begin{cases} \triangle BOE \cong \triangle BOG \\ \triangle COF \cong \triangle COE \end{cases}$

SIIS



$2(\alpha+\beta+\gamma) = 360°$

$\Rightarrow \alpha+\beta+\gamma = 180°$

PUOLET KOLMION $\triangle ABC$ PIIRISTÄ

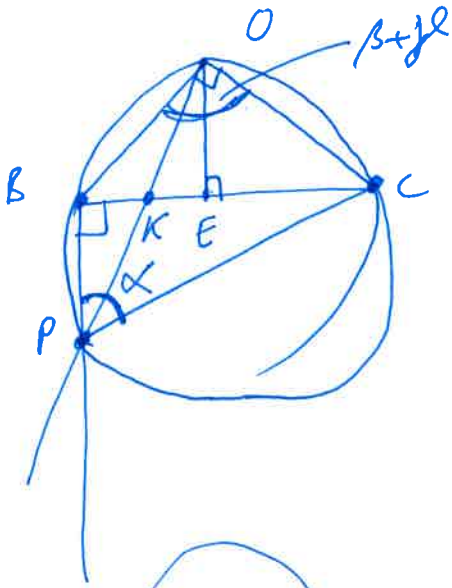
SELVÄSTI KOLMION ALA $= QC \cdot EO$

TULISI VIELÄ OSOITTAA

$$QC \cdot EO = \sqrt{QC \cdot BC \cdot QE \cdot CQ \cdot BE \cdot BQ \cdot EC}$$

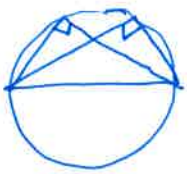
$\in$ \in
$\ni$ \ni

PIIRRETÄÄN SITTEN $\begin{cases} L \perp BC \\ L \ni B \end{cases}$ JA $\begin{cases} M \perp OC \\ M \ni O \end{cases}$



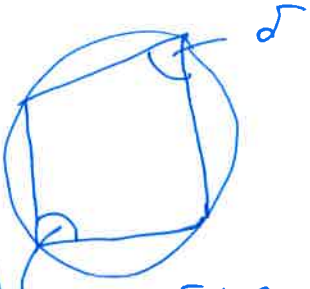
JA OLKOON $P = L \cap M$.

THALES:



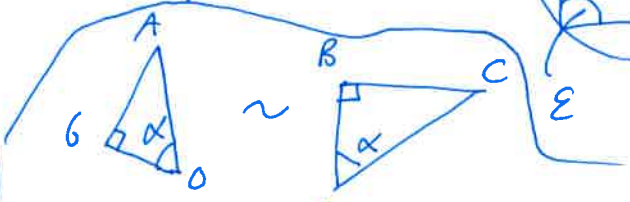
$\Rightarrow P, B, O, C$ SAMALLA YMPYRÄLLÄ!

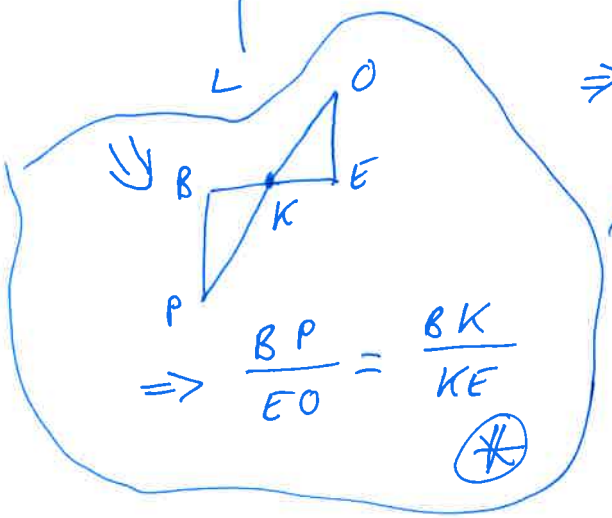
HARJOITUS 4:



$\delta + \varepsilon = 180°$

$\Rightarrow$ SIIS $\sphericalangle CPB = \alpha$





$\Rightarrow \frac{BP}{EO} = \frac{BK}{KE}$ (*)

$\Rightarrow \frac{BC}{BP} = \frac{AG}{GO} = \frac{BQ}{EO}$

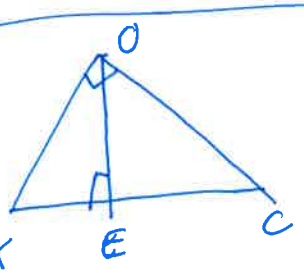
$\Rightarrow \frac{BC}{BQ} = \frac{BP}{EO} \overset{(*)}{=} \frac{BK}{KE}$

$$\Rightarrow \frac{CB}{BQ} = \frac{BK}{KE} \quad \| + 1$$

$$\Rightarrow \frac{CB}{BQ} + \frac{BQ}{BQ} = \frac{BK}{KE} + \frac{KE}{KE}$$

$$\Rightarrow \frac{CQ}{BQ} = \frac{BE}{KE}$$

$$\Rightarrow KE = BE \times \frac{BQ}{CQ}$$



HARJOITUKSET:

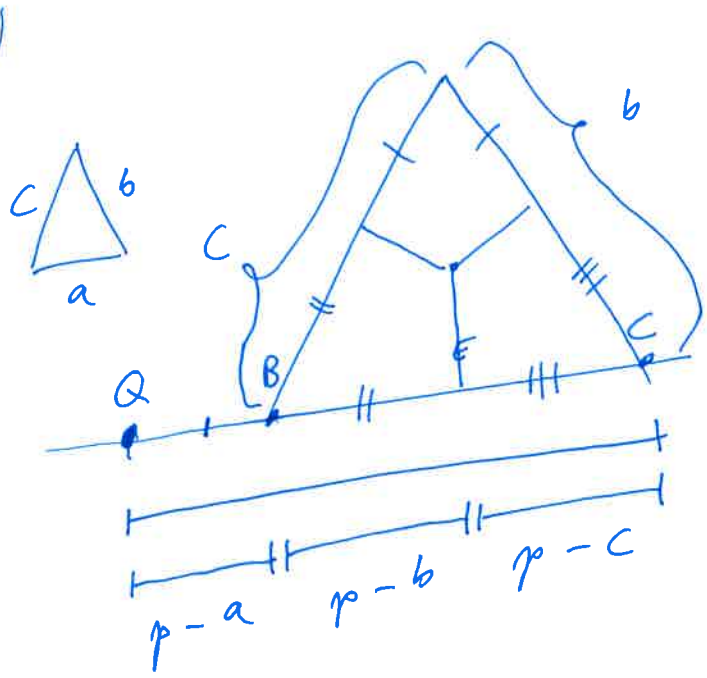
$$EO = \sqrt{KE \times EC}$$

$$\Rightarrow EO = \sqrt{BE \cdot \frac{BQ}{CQ} \cdot EC}$$

$$ALA = CQ \cdot EO$$

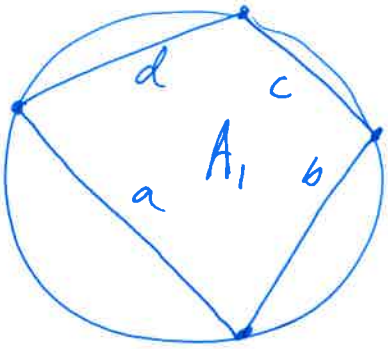
$$= \sqrt{CQ^2} \cdot \sqrt{BE \cdot \frac{BQ}{CQ} \cdot EC}$$

$$= \sqrt{CQ \cdot BE \cdot BQ \cdot EC}$$



$$= \sqrt{p(p-a)(p-b)(p-c)}$$

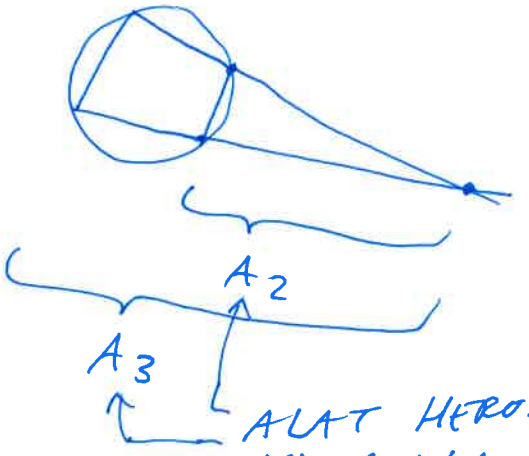
[BRAHMAGUPTA:



$$S = \frac{a+b+c+d}{2}$$

$$\sqrt{(s-a)(s-b)(s-c)(s-d)}$$

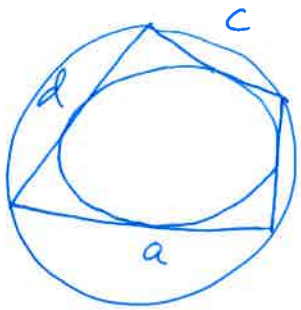
[TODISTUS NETISSÄ



ALAT HERONIN KAAVALLA

& PITKÄ PYÖRITTELY]

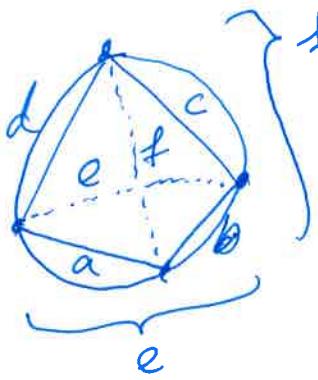
NETISSÄ



← YMPYRÄ SISÄLLÄ JA ULKOPUOLELLA

$$A_1 = \sqrt{a \cdot b \cdot c \cdot d}$$

VIIMEKSI

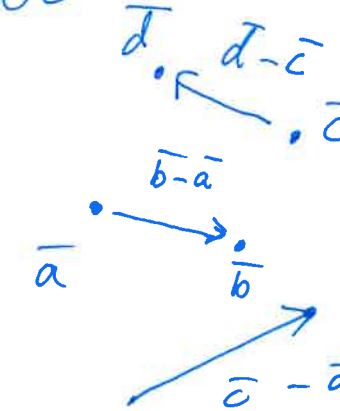


PTOLEMAIOS:

$$ac + bd = ef$$

* PTOLEMAIOKSEN EPÄYHTÄLÖ $\mathbb{R}^n$:SSÄ

OLKOOT $a, b, c, d \in \mathbb{R}^n$. [$a = \bar{a}$ ON VEKTORI $a = (a_1, \dots, a_n)$]



TÄLLÖIN

$$|b-a||c-d| + |c-b||a-d| \geq |c-a||d-b|.$$

Tod.

$$\begin{aligned}(b-a)(c-d) + (c-b)(a-d) &= bc - bd - ac + ad + ca - cd - ba + bd \\ &= c(b-d) + a(d-b) \\ &= (c-a)(b-d).\end{aligned}$$

SIIS $|(c-a)(b-d)| = |(b-a)(c-d) + (c-b)(a-d)|$

Δ-eg $\leq |(b-a)(c-d)| + |(c-b)(a-d)|$

$|(c-a)(d-b)| =$

?

$\Rightarrow$ VÄITE

$$\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cos\alpha$$

$$\Rightarrow |\bar{a} \cdot \bar{b}| = |\bar{a}||\bar{b}||\cos\alpha| \leq |\bar{a}||\bar{b}|$$

↑ VOI OLLA