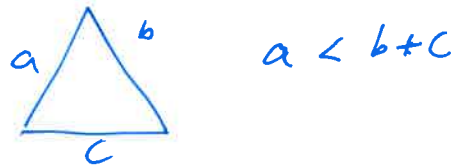


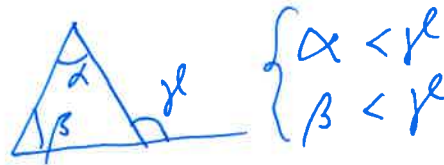
EUKLIIDINEN GEOMETRIA
 KERTAUS TEHTÄVIEN MALLIRATKAISUJA

K1. (a) KOLMIOEPÄYHTÄLÖ:



$$a < b + c$$

(b) ULKO KULMA EPÄYHTÄLÖ:



$$\begin{cases} \alpha < \gamma \\ \beta < \gamma \end{cases}$$

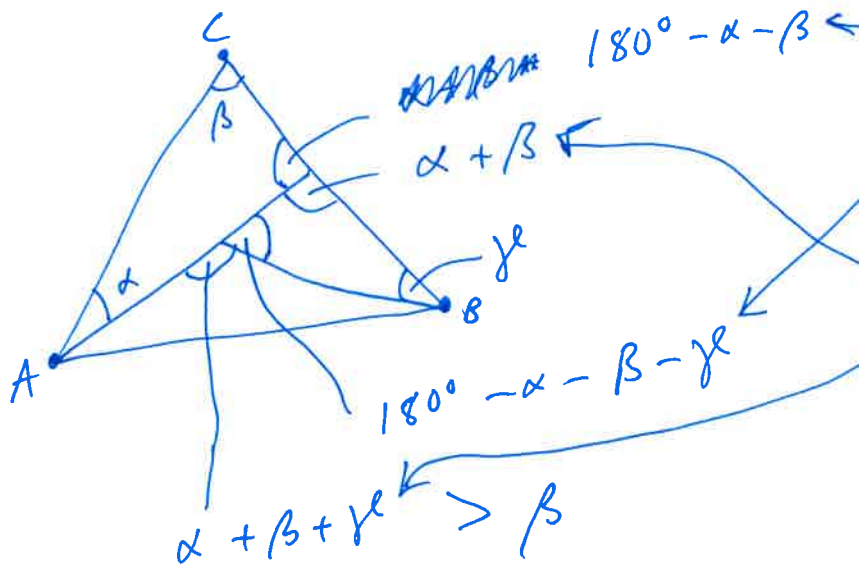
(c) KOLMIOEPÄYHTÄLÖ

$$\Rightarrow \begin{cases} AD + DE < AC + CE \\ DB < DE + EB \end{cases}$$

$$+ \hline AD + \cancel{DE} + DB < AC + CE + \cancel{DE} + EB$$

$$\Rightarrow AD + DB < AC + CB$$

(d)



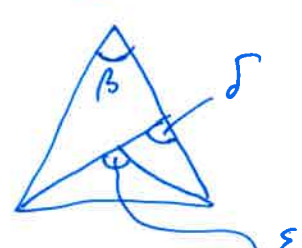
KOLMION
 KULMIEN
 SUMMA 180°

VIEROSKULMAT
 YHTEENSÄ 180°

$$\alpha + \beta + \gamma > \beta$$

$$\Rightarrow \underline{\angle ADB > \angle ACB}$$

TAPA 2



$$\beta < \delta < \epsilon \Rightarrow \angle ADB < \angle ACB$$

↑ ↑
 ULKO KULMA EPÄYHTÄLÖ

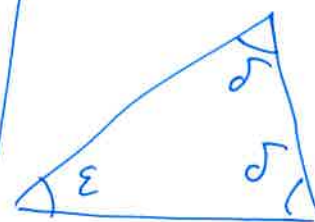
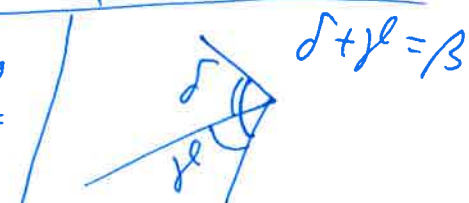
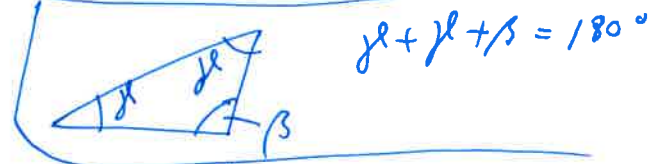
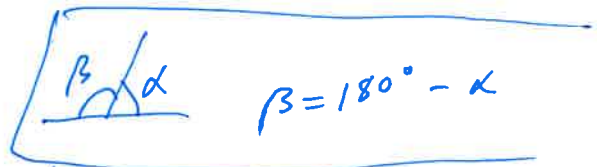
$$K2, \quad 5\alpha = 360^\circ \Rightarrow \alpha = \underline{\underline{72^\circ}}$$

$$\Rightarrow \beta = 180^\circ - \alpha = \underline{\underline{108^\circ}}$$

$$\Rightarrow \gamma = \frac{180^\circ - \beta}{2} = \underline{\underline{36^\circ}}$$

$$\Rightarrow \delta = \beta - \gamma = 108^\circ - 36^\circ = \underline{\underline{72^\circ}}$$

$$\Rightarrow \varepsilon = 180^\circ - 2\delta = \underline{\underline{36^\circ}}$$



$$\varepsilon + \delta + \delta = 180^\circ$$

K3. (a) SSS - YHTENEVYYS LAUSE TARKOITTA, ETTE JOS KOLMIOLLA

$\triangle ABC$ JA $\triangle DEF$ ON KOLME SIVUA PARITAIN YHTÄ PITKÄÄ

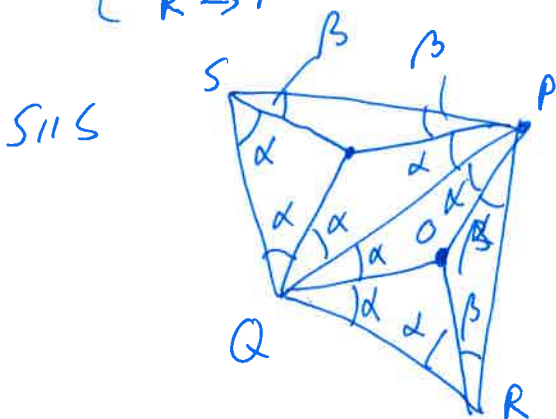
[ESIM. $\left. \begin{matrix} AB = DE \\ BC = EF \\ CA = FD \end{matrix} \right] \text{ NIIN KOLMIOT OVAT YHTENEVIÄ.}$

(b) $S, P, R \in C(Q, QP) \Rightarrow QS = QP = QR$ (1)
 $S, R \in C(P, PS) \Rightarrow SP = RP$ (2)

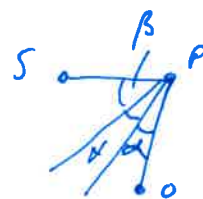
TÄRKEÄ SIIS

$\triangle SQP$ $\triangle PQR$
 $\left\{ \begin{matrix} SP = RP & (2) \\ QP = QR & (1) \\ SQ = PQ & (1) \end{matrix} \right. \xrightarrow{SSS} \underline{\underline{\triangle SQP \cong \triangle PQR}}$

(c) KOPPIOIDAN KOLMIO $\triangle OPR$ NIIN, ETTE OLKON O:N KOPIO O'.
 SIIS $Q, P, R \in C(O, OP) \Rightarrow OP = OR = OQ$
 ~~$OP = OQ = OR = OQ$~~



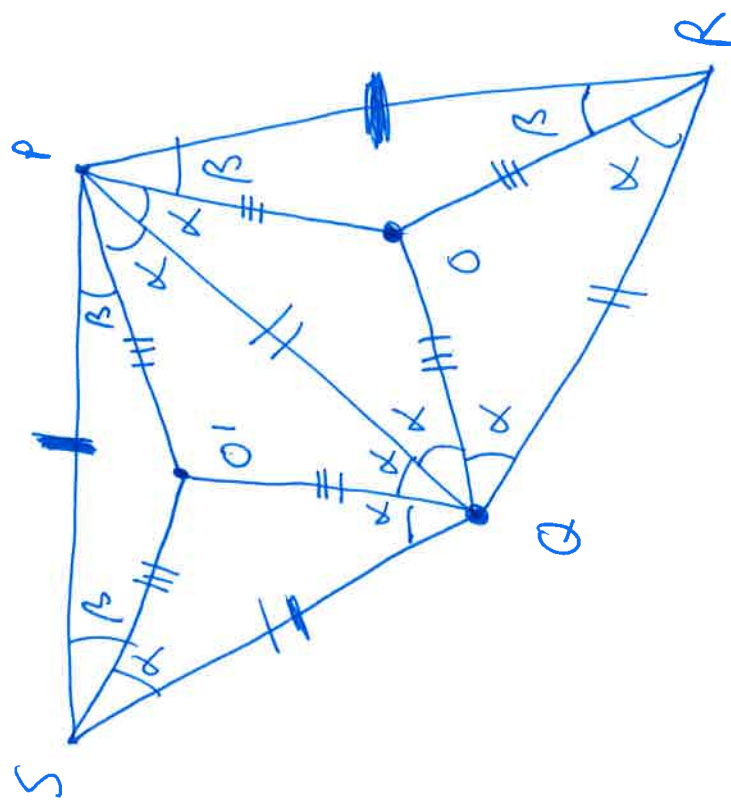
$\Rightarrow O'S = O'R = O'Q$



$\triangle QPR \Rightarrow 4\alpha + 2\beta = 180^\circ$
 $\Rightarrow 2\alpha + \beta = 90^\circ \Rightarrow \underline{\underline{SP \perp OP}}$

K3. ©

ISOMPANA



$$4\alpha + 2\beta = 180^\circ \quad || \cdot 2$$

$$2\alpha + \beta = 90^\circ$$

$$\Rightarrow SP \perp OP$$

$$S, P, R \in C(Q, QP)$$

$$\Rightarrow \cancel{SP} \quad QS = QP = QR$$

$$S, R \in C(P, PS)$$

$$\Rightarrow SP = PR$$

$$P, R \in C(O, OP)$$

$$\Rightarrow PO = OR = OQ$$

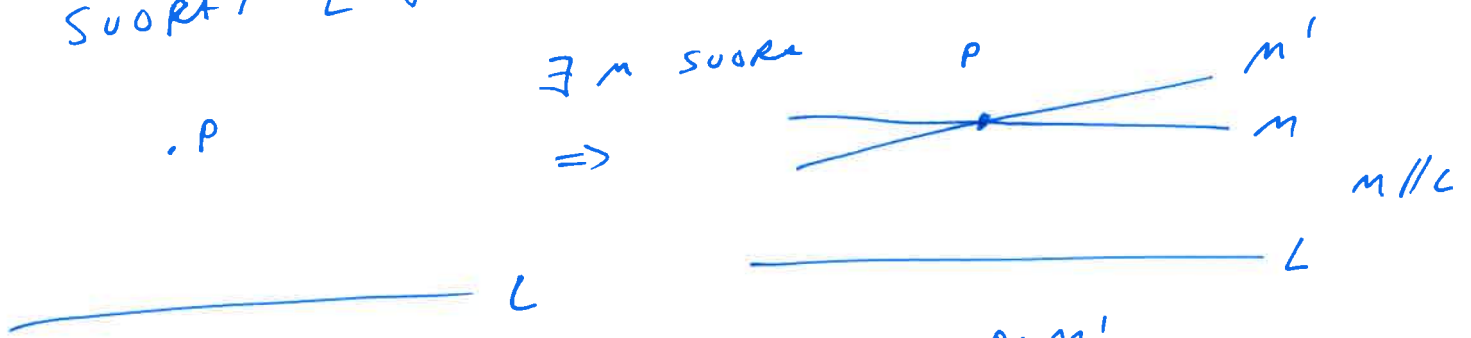
KOPIOI $\triangle POQ \rightarrow \triangle SO'P$,

PIIRRE QO JA QO' ,

\rightarrow KULMIA α JA β KUTEN KUVASSA

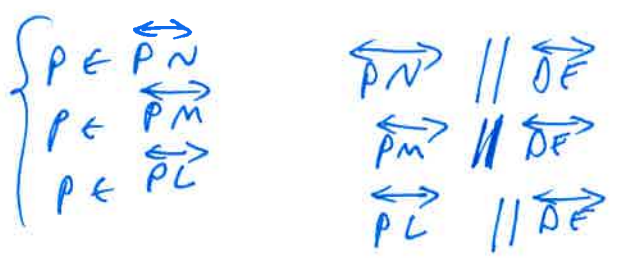
K4, KATSO HARJOITUS 4 TEHTÄVÄ 1
 K5, KATSO HARJOITUS 4 TEHTÄVÄ 2

K6. (a) VÄITETTÄ, ETTÄ
 JOS ON ANNETTU SUORA L JA PISTE P
 SUORAN L ULKOPUOLELTA, NIIN LÖYTYY
 TASAN YKSI SUORA M SITEN, ETTÄ PEM JA
 SUORAT L JA M OVA T YHDENSUUNTAISET.



LISÄKSI JOS PEM'
 M' // L
 NIIN M' = M.

(b) ERO 1 : SUORAN \vec{DE} ULKOPUOLISEN PISTEEN P
 KAUTTA LÖYDY Y USEITA
 YHDENSUUNTAISIA $\vec{DE} = \vec{LLP}$:

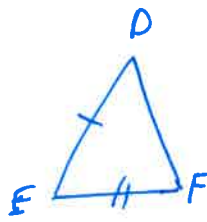
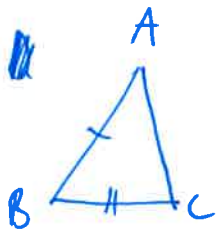


ERO 2 : KOLMION $\triangle ABC$ KULMIEN
 SUMMA ON $< 180^\circ$

(c) ERO 1 : KAIKKI "SUORAT" (= ISOYMPYRIIT)
 LEIKKAAVAT

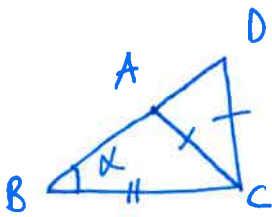
ERO 2 : KOLMION KULMIEN SUMMA
 ON AINA $> 180^\circ$

K7, (a)



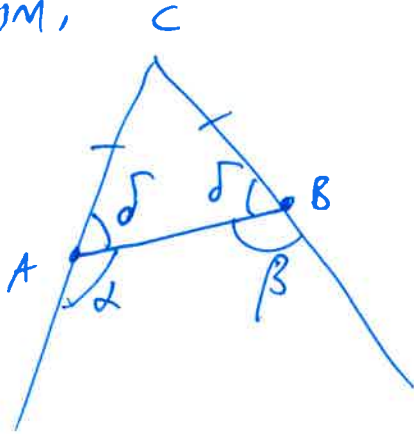
$$\left\{ \begin{array}{l} AB = DE \\ \sphericalangle CBA = \sphericalangle FED \\ BC = EF \end{array} \right. \quad \text{SKS} \quad \Rightarrow \quad \triangle ABC \cong \triangle DEF$$

(b) VÄITTE EI OLF TO TTA:



$$\left\{ \begin{array}{l} \sphericalangle CBA = \sphericalangle CBD \\ BC = BC \\ CA = CD \end{array} \right. \quad \Rightarrow \quad \triangle ABC \cong \triangle OBC$$

(c) ESIM,



KOSKA $CA = CB$, NIIN
 $\sphericalangle BAC = \sphericalangle CBA = \alpha$

VIERISKULMINA

$$\alpha + \gamma = 180^\circ$$

$$- \beta + \delta = 180^\circ$$

$$\alpha - \beta = 0 \quad \Rightarrow \quad \underline{\underline{\alpha = \beta}}$$