

# ALGEBRA A

8 VIIKKOA

TENTTI 29.-31.10. ?

MAX 30 p

ARVOSANA

50% = 15 p → 1

60% → 2

70% → 3

80% → 4

90% = 27 p → 5

DE MOT 10% = 3 p

## Lemma 1.2 tod

Nyt  $a \mid b$ . Siis  $b = a \cdot k$ ,  $k \in \mathbb{Z}$ .

Koska  $b \neq 0$ , niin  $k \neq 0$  eli  $|k| \geq 1$ .

Siis  $|b| = |a k| = |a| |k| \geq |a| \cdot 1 = |a|$

eli  $|b| \geq |a|$ .

□

## T1 ESIJÄ' JAKOYHTÄLÖ

(a) 59 JAETAAN 16:LLÄ

(b)  $n+1$  JAETAAN  $n$ :LLÄ ( $n > 1$ )

$$59 = 16q + r \quad 0 \leq r < 16$$

Ratk. (a)  $59 = \underbrace{16 \cdot 3}_{=48} + 11 = \cancel{16 \cdot 4 + (-5)}$

(b)  $n+1 = n \cdot 1 + 1$

Satz 1.3. (yuklitolite-nyy)

$$a, b \in \mathbb{Z}, \quad b > 0 \quad \text{jö}$$

$$a = b q_1 + r_1, \quad 0 \leq r_1 < b$$

$$a = b q_2 + r_2, \quad 0 \leq r_2 < b,$$

0 biteton  $q_1 = q_2$  jö  $r_1 = r_2$ .

Rek.  $b q_1 + r_1 = a = b q_2 + r_2$

$$\Rightarrow b(q_1 - q_2) = r_2 - r_1,$$

Kittes abottas  $r_1 = r_2$ .

AI.  $r_1 \neq r_2, \Rightarrow q_1 \neq q_2$  jö

$$b \leq b |q_1 - q_2| = |r_2 - r_1| \textcircled{x} < b$$

Jö  $r_2 > r_1$ , min

$$0 \leq |r_2 - r_1| = r_2 - r_1 < b$$

RR  $\textcircled{x}$  in konth.

NYÖ'S  $r_1 < r_2 \Rightarrow RR$



T2 MIKSI EI VOI OLLA

$$z^2 = 4k + 2, \quad z \in \mathbb{Z}, k \in \mathbb{Z} ?$$

Ratk. 1<sup>o</sup> Jos  $z = 2k + 1$ , niin

$$\begin{aligned} z^2 &= (2k+1)^2 = 4k^2 + 2 \cdot 2k \cdot 1 + 1^2 \\ &= 4k^2 + 4k + 1 \\ &= 4(k^2 + k) + \underline{1} \end{aligned}$$

2<sup>o</sup> Jos  $z = 2k$ , niin  $\neq 2$

$$z^2 = (2k)^2 = 4k^2 = 4k^2 + \underline{0}$$

T3 MÄÄRÄÄ EUKLEIDEEN ALGORITMILU

$$\text{Nyt } (1414, 666)$$

$$1414 = 2 \cdot 666 + 82$$

$$666 = \underbrace{8 \cdot 82}_{656} + 10$$

$$82 = 8 \cdot 10 + \underline{2}$$

$$10 = \underline{5} \cdot \underline{2} + 0$$

Sis  $\text{Nyt } (1414, 666) = 2,$