Exam in Exam room, created 17.1.2025

Mathematics for Programmers, ID00EK08-3001

Basic math

1. (a) Simplify the expression
$$\left(\frac{2}{3}+\frac{3}{4}\right)\Big/\left(6-\frac{1}{3}\right)$$
to the form $\frac{p}{q}$, where p and q are integers.

   (b) Simplify by opening the parentheses
$$(2x-3)^2-2(1-6x).$$

2. (a) Solve the quadratic equation
$$x^2-5x+4=0.$$

   (b) Solve the unknowns x and y from the pair of equations
$$\begin{cases} 5x+y &= 6 \\ -4x+y &= -6 \end{cases}$$

3. (a) Solve x when $2^{7x}=5^{x+2}$.

   (b) Solve x when $2\ln(5x)-\ln(x)=\ln(4x+7)$.

Calculus

4. Find $f'(x)$, when
   (a) $f(x)=3x^4-\sqrt{5x}+\frac{1}{x^7}$
   (b) $f(x)=2\sin(3x)-7e^{x^4}$
   (c) $f(x)=\cos(x)\ln(x)$

5. Find $x>0$ which is the maximum of $f(x)=x^5e^{-7x}$.

6. Calculate
   (a) $\int 3x+\sqrt{2x}dx$
   (b) $\int \cos(3x)dx$
   (c) $\int_0^1 e^{-x}+1dx$

Calculus formulas

$$y - y_1 = k(x - x_1), \quad y = kx + b, \quad k = \frac{y_2 - y_1}{x_2 - x_1} = f'(x_1)$$

$$\int_a^b f(x)dx = \Big|_a^b F(x) = F(b) - F(a), \quad F'(x) = f(x)$$

Differentiation		**Integration**	
Dx^n	$= nx^{n-1}$	$\int x^n dx$	$= \frac{x^{n+1}}{n+1} + C$
De^x	$= e^x$	$\int e^x dx$	$= e^x + C$
Db^x	$= b^x \ln(b)$	$\int b^x dx$	$= \frac{b^x}{\ln(b)}$
$D\ln(x)$	$= \frac{1}{x}$		
$D\ln\lvert x\rvert$	$= \frac{1}{x}$	$\int \frac{1}{x} dx$	$= \ln\lvert x\rvert + C$
$D\log_a(x)$	$= \frac{1}{x\ln(a)}$		
$D\log_a\lvert x\rvert$	$= \frac{1}{x\ln(a)}$		
$D\sin(x)$	$= \cos(x)$	$\int \cos(x)dx$	$= \sin(x) + C$
$D\cos(x)$	$= -\sin(x)$	$\int \sin(x)dx$	$= -\cos(x) + C$
$D\tan(x)$	$= 1 + \tan^2(x)$	$\int 1 + \tan^2(x)dx$	$= \tan(x) + C$
$Dx\ln(x) - x$	$= \ln(x)$	$\int \ln(x)dx$	$= x\ln(x) - x + C$
$D\arcsin(x)$	$= \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}$	$= \arcsin(x) + C$
$D\arccos(x)$	$= \frac{1}{-\sqrt{1-x^2}}$	$\int \frac{1}{-\sqrt{1-x^2}}$	$= \arccos(x) + C$
$D\arctan(x)$	$= \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}$	$= \arctan(x) + C$
$D\sinh(x)$	$= \cosh(x)$		
$D\cosh(x)$	$= \sinh(x)$		
$D\tanh(x)$	$= \frac{1}{\cosh^2(x)}$		

Differentiation		**Integration**	
$Df(g(x))$	$= f'(g(x))g'(x)$	$\int f'(g(x))g'(x)dx$	$= f(g(x)) + C$
Special cases			
$D\ln(f(x))$	$= \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)}dx$	$= \ln(f(x)) + C$
$De^{f(x)}$	$= e^{f(x)}f'(x)$	$\int f'(x)e^{f(x)}dx$	$= e^{f(x)} + C$
Dfg	$= f'g + fg'$	$\int f'g dx$	$= fg - \int fg' dx$
$D(f/g)$	$= (gf' - fg')/g^2$		

Basic formulas

Fractions

$$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd},\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd},\quad \frac{a}{b}\Big/\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$$

Powers

$$a^b a^c = a^{b+c},\quad \frac{a^b}{a^c}=a^{b-c},\quad (a^b)^c=a^{bc},\quad (ab)^c=a^b a^c,\quad \left(\frac{a}{b}\right)^c=\frac{a^b}{b^c}$$

Roots

$$(a^b)^{\frac{1}{b}}=a^{b\cdot\frac{1}{b}}=a^1=a,\quad \text{if}\quad a>0,\quad \sqrt{a}=a^{\frac{1}{2}},\quad \sqrt[3]{a}=a^{\frac{1}{3}}$$

First degree equation

$$ax=b\quad\Leftrightarrow\quad x=\frac{b}{a}$$

Quadratic equation

$$ax^2+bx+c=0\quad\Leftrightarrow\quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

System of linear equations

$$\begin{cases} ax+by &= U \\ cx+dy &= V \end{cases}\quad\Rightarrow\quad \begin{array}{rl} adx+bdy &= Ud \\ -bcx-bdy &= -bV \\ \hline (ad-bc)x &= Ud-bV \end{array}\quad\Rightarrow\quad x=\frac{Ud-bV}{ad-bc}$$

and

$$\begin{cases} ax+by &= U \\ cx+dy &= V \end{cases}\quad\Rightarrow\quad \begin{array}{rl} -acx-bcy &= -Uc \\ acx+ady &= aV \\ \hline (ad-bc)y &= aV-Uc \end{array}\quad\Rightarrow\quad y=\frac{aV-Uc}{ad-bc}$$

Function $f(x)$ and inverse function $g(x)=f^{-1}(x)$

$$f(g(x))=x,\quad g(f(x))=x$$

Powers	**Logarithms**
$a^1=a$	$\log_a(a)=1$
$a^0=1$	$\log_a(1)=0$
$a^b=c \quad\Leftrightarrow\quad a=c^{\frac{1}{b}}$	$\log_a(c)=\frac{1}{\log_c(a)}$
$a^b a^c=a^{b+c}$	$\log_a(bc)=\log_a(b)+\log_a(c)$
$\frac{a^b}{a^c}=a^{b-c}$	$\log_a\frac{b}{c}=\log_a(b)-\log_a(c)$
$(a^b)^c=a^{bc}$	$\log_a(b)=\frac{\log_c(b)}{\log_c(a)}$
	$\log_a(b^c)=c\log_a(b)$

$$\log_{10}(x)\approx\frac{x-1}{x+1},\quad \frac{1}{5}\le x\le 5,$$

$$\log_e(x)\approx 2\frac{x-1}{x+1},\quad \frac{1}{3}\le x\le 3,$$

$$\log_2(x)\approx 3\frac{x-1}{x+1},\quad \frac{1}{2}\le x\le 2,$$

$$
\begin{array}{rlrlrl}
\log_2(2) & = 1 & \log_2(e) & \approx 1.4427 & \log_2(10) & \approx 3.3219 \\
\log_e(2) & \approx 0.6931 & \log_e(e) & = 1 & \log_e(10) & \approx 2.3026 \\
\log_{10}(2) & \approx 0.30103 & \log_{10}(e) & \approx 0.4343 & \log_{10}(10) & = 1
\end{array}
$$

Powers

$$a^b a^c = a^{b+c}, \quad \frac{a^b}{a^c} = a^{b-c}, \quad (a^b)^c = a^{bc}, \quad (ab)^c = a^b a^c, \quad \left(\frac{a}{b}\right)^c = \frac{a^b}{b^c}$$

Logarithms

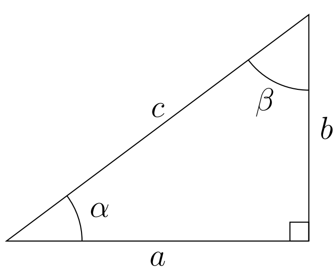
$$\ln(ab) = \ln(a) + \ln(b), \quad \ln(\frac{a}{b}) = \ln(a) - \ln(b), \quad \ln(a^b) = b\ln(a)$$

$$\log_a(x) = y \quad \Leftrightarrow \quad a^y = x$$

$$\log_a(1) = 0, \quad \log_a(a) = 1, \quad \log_a(a^x) = x, \quad a^{\log_a(x)} = x$$

$$
\begin{aligned}
\log_a(b^c) &= c\log_a(b) \\
\log_a(xy) &= \log_a(x) + \log_a(y) \\
\log_a\left(\frac{x}{y}\right) &= \log_a(x) - \log_a(y) \\
\log_a(x) &= \frac{\log_b(x)}{\log_b(a)}
\end{aligned}
$$

$$\mathrm{lb}(x) = \log_2(x), \quad \lg(x) = \log_{10}(x), \quad \ln(x) = \log_e(x), \quad e \approx 2,72$$



Trigonometry

$$c^2 = a^2 + b^2$$

$$\sin(\alpha) = \frac{b}{c}, \quad \cos(\alpha) = \frac{a}{c}, \quad \tan(\alpha) = \frac{b}{a},$$

$$\alpha = \arcsin\frac{b}{c}, \quad = \arccos\frac{b}{c}, \quad = \arctan\frac{b}{c},$$

$$\alpha = \mathrm{imag}(\ln(a + bi)) \cdot \frac{180^\circ}{\pi}$$