Exam in Exam room, created 18.10.2024

Mathematics for Programmers, ID00EK08-3001

Basic math

1. (a) Simplify the expression

$$\left(\frac{3}{2} + \frac{2}{5}\right) \Big/ \left(3 - \frac{1}{6}\right)$$

to the form $\frac{p}{q}$, where p and q are integers.

(b) Simplify by opening the parentheses

$$(4x - 3)^2 - 3(5 - 2x).$$

2. (a) Solve the quadratic equation
$$x^2 - 7x + 10 = 0.$$

(b) Solve the unknowns x and y from the pair of equations

$$\begin{cases} 5x + 2y & = 6 \\ 2x - y & = 3. \end{cases}$$

3. (a) Solve x when $2^{7x} = 3^{x+2}$.

(b) Solve x when $2\ln(5x^3) - 5\ln(x) = \ln(7x + 3)$.

Calculus

4. Find $f'(x)$, when

(a) $f(x) = 3x^4 - \sqrt{5x} + \frac{1}{x^7}$

(b) $f(x) = 3\sin(2x) - 3e^{x^3}$

(c) $f(x) = \cos(x)\ln(x)$

5. Find $x > 0$ which is the maximum of $f(x) = x^5 e^{-7x}$.

6. Calculate

(a) $\int 3x + \sqrt{x}dx$

(b) $\int \cos(3x)dx$

(c) $\int_0^1 e^{-x} + 1dx$

Calculus formulas

$$y - y_1 = k(x - x_1), \quad y = kx + b, \quad k = \frac{y_2 - y_1}{x_2 - x_1} = f'(x_1)$$

$$\int_a^b f(x)dx = \Big|_a^b F(x) = F(b) - F(a), \quad F'(x) = f(x)$$

Differentiation	Integration

$$
\begin{aligned}
Dx^n &= nx^{n-1} \\
De^x &= e^x \\
Db^x &= b^x \ln(b) \\
D\ln(x) &= \tfrac{1}{x} \\
D\ln|x| &= \tfrac{1}{x} \\
D\log_a(x) &= \tfrac{1}{x\ln(a)} \\
D\log_a|x| &= \tfrac{1}{x\ln(a)} \\
D\sin(x) &= \cos(x) \\
D\cos(x) &= -\sin(x) \\
D\tan(x) &= 1 + \tan^2(x) \\
Dx\ln(x) - x &= \ln(x)
\end{aligned}
$$

$$
\begin{aligned}
\int x^n dx &= \tfrac{x^{n+1}}{n+1} + C \\
\int e^x dx &= e^x + C \\
\int b^x dx &= \tfrac{b^x}{\ln(b)} \\[1em]
\int \tfrac{1}{x} dx &= \ln|x| + C \\[1em]
\int \cos(x)dx &= \sin(x) + C \\
\int \sin(x)dx &= -\cos(x) + C \\
\int 1 + \tan^2(x)dx &= \tan(x) + C \\
\int \ln(x)dx &= x\ln(x) - x + C
\end{aligned}
$$

$$
\begin{aligned}
D\arcsin(x) &= \tfrac{1}{\sqrt{1-x^2}} \\
D\arccos(x) &= \tfrac{1}{-\sqrt{1-x^2}} \\
D\arctan(x) &= \tfrac{1}{1+x^2} \\
D\sinh(x) &= \cosh(x) \\
D\cosh(x) &= \sinh(x) \\
D\tanh(x) &= \tfrac{1}{\cosh^2(x)}
\end{aligned}
$$

$$
\begin{aligned}
\int \tfrac{1}{\sqrt{1-x^2}} &= \arcsin(x) + C \\
\int \tfrac{1}{-\sqrt{1-x^2}} &= \arccos(x) + C \\
\int \tfrac{1}{1+x^2} &= \arctan(x) + C
\end{aligned}
$$

Differentiation	Integration

$$Df(g(x)) = f'(g(x))g'(x) \qquad \int f'(g(x))g'(x)dx = f(g(x)) + C$$

Special cases

$$
\begin{aligned}
D\ln(f(x)) &= \tfrac{f'(x)}{f(x)} \\
De^{f(x)} &= e^{f(x)}f'(x)
\end{aligned}
\qquad
\begin{aligned}
\int \tfrac{f'(x)}{f(x)}dx &= \ln(f(x)) + C \\
\int f'(x)e^{f(x)}dx &= e^{f(x)} + C
\end{aligned}
$$

$$
\begin{aligned}
Dfg &= f'g + fg' \\
D(f/g) &= (gf' - fg')/g^2
\end{aligned}
\qquad
\int f'g\,dx = fg - \int fg'dx
$$

Basic formulas

Fractions

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}, \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \quad \frac{a}{b} \bigg/ \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Powers

$$a^b a^c = a^{b+c}, \quad \frac{a^b}{a^c} = a^{b-c}, \quad (a^b)^c = a^{bc}, \quad (ab)^c = a^b a^c, \quad \left(\frac{a}{b}\right)^c = \frac{a^b}{b^c}$$

Roots

$$(a^b)^{\frac{1}{b}} = a^{b \cdot \frac{1}{b}} = a^1 = a, \quad \text{if} \quad a > 0, \quad \sqrt{a} = a^{\frac{1}{2}}, \quad \sqrt[3]{a} = a^{\frac{1}{3}}$$

First degree equation

$$ax = b \quad \Leftrightarrow \quad x = \frac{b}{a}$$

Quadratic equation

$$ax^2 + bx + c = 0 \quad \Leftrightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

System of linear equations

$$\begin{cases} ax + by &= U \\ cx + dy &= V \end{cases} \Rightarrow \begin{array}{r} adx + bdy = Ud \\ -bcx - bdy = -bV \\ \hline (ad - bc)x = Ud - bV \end{array} \Rightarrow x = \frac{Ud - bV}{ad - bc}$$

and

$$\begin{cases} ax + by &= U \\ cx + dy &= V \end{cases} \Rightarrow \begin{array}{r} -acx - bcy = -Uc \\ acx + ady = aV \\ \hline (ad - bc)y = aV - Uc \end{array} \Rightarrow y = \frac{aV - Uc}{ad - bc}$$

Function $f(x)$ and inverse function $g(x) = f^{-1}(x)$

$$f(g(x)) = x, \quad g(f(x)) = x$$

Powers	**Logarithms**
$a^1 = a$	$\log_a(a) = 1$
$a^0 = 1$	$\log_a(1) = 0$
$a^b = c \Leftrightarrow a = c^{\frac{1}{b}}$	$\log_a(c) = \frac{1}{\log_c(a)}$
$a^b a^c = a^{b+c}$	$\log_a(bc) = \log_a(b) + \log_a(c)$
$\frac{a^b}{a^c} = a^{b-c}$	$\log_a \frac{b}{c} = \log_a(b) - \log_a(c)$
$(a^b)^c = a^{bc}$	$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$
	$\log_a(b^c) = c \log_a(b)$

$$\log_{10}(x) \approx \frac{x-1}{x+1}, \quad \frac{1}{5} \leq x \leq 5,$$

$$\log_e(x) \approx 2\frac{x-1}{x+1}, \quad \frac{1}{3} \leq x \leq 3,$$

$$\log_2(x) \approx 3\frac{x-1}{x+1}, \quad \frac{1}{2} \leq x \leq 2,$$

$$\begin{array}{lll}
\log_2(2) & = 1 & \log_2(e) & \approx 1.4427 & \log_2(10) & \approx 3.3219 \\
\log_e(2) & \approx 0.6931 & \log_e(e) & = 1 & \log_e(10) & \approx 2.3026 \\
\log_{10}(2) & \approx 0.30103 & \log_{10}(e) & \approx 0.4343 & \log_{10}(10) & = 1
\end{array}$$

Powers

$$a^b a^c = a^{b+c}, \quad \frac{a^b}{a^c} = a^{b-c}, \quad (a^b)^c = a^{bc}, \quad (ab)^c = a^b a^c, \quad \left(\frac{a}{b}\right)^c = \frac{a^b}{b^c}$$

Logarithms

$$\ln(ab) = \ln(a) + \ln(b), \quad \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b), \quad \ln(a^b) = b \ln(a)$$

$$\log_a(x) = y \quad \Leftrightarrow \quad a^y = x$$

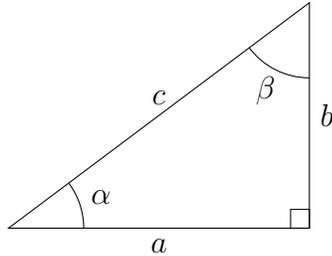$$\log_a(1) = 0, \quad \log_a(a) = 1, \quad \log_a(a^x) = x, \quad a^{\log_a(x)} = x$$

$$\log_a(b^c) = c \log_a(b)$$
$$\log_a(xy) = \log_a(x) + \log_a(y)$$
$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$
$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

$$\mathrm{lb}(x) = \log_2(x), \quad \lg(x) = \log_{10}(x), \quad \ln(x) = \log_e(x), \quad e \approx 2,72$$

Trigonometry

$$c^2 = a^2 + b^2$$

$$\sin(\alpha) = \frac{b}{c}, \quad \cos(\alpha) = \frac{a}{c}, \quad \tan(\alpha) = \frac{b}{a},$$

$$\alpha = \arcsin\frac{b}{c}, \quad = \arccos\frac{b}{c}, \quad = \arctan\frac{b}{c},$$

$$\alpha = \mathrm{imag}(\ln(a + bi)) \cdot \frac{180°}{\pi}$$