Example exam solutions

Monday, October 7, 2024 11:03 AM

1. (a) Simplify the expression

$$A = \left(\frac{1}{7} - \frac{1}{10}\right) \Big/ \left(\frac{1}{7} + \frac{1}{2}\right)$$

to the form $\frac{p}{q}$, where p and q are integers.

Sol. $A = \left(\frac{10}{70} - \frac{7}{70}\right) \Big/ \left(\frac{2}{14} + \frac{7}{14}\right)$

$= \frac{3}{70} \Big/ \frac{9}{14} = \frac{3}{70} \cdot \frac{14}{9}$

$= \frac{3 \cdot 14}{70 \cdot 9}$ $\left[= \frac{3 \cdot 2 \cdot 7}{2 \cdot 5 \cdot 7 \cdot 3 \cdot 3} = \frac{1}{15}\right]$

NOT REQUIRED

(b) Simplify by opening the parentheses

$$(3x+2)^2 - 2(6x-2) + 5$$

Sol. $= (3x+2)(3x+2) - 2(6x-2) + 5$

$= 9x^2 + 6x + 6x + 4 - 12x + 4 + 5$

$= 9x^2 + 13$

2. (a) Solve the quadratic equation $2x^2 - 8x + 6 = 0$.

Sol.

$2x^2 - 8x + 6 = 0 \quad \| :2$

$x^2 - 4x + 3 = 0$

$\begin{cases} a = 1 \\ b = -4 \\ c = 3 \end{cases}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \dfrac{4 \pm \sqrt{4}}{2} = \dfrac{2(2 \pm 1)}{2}$

$= 2 \pm 1 \rightarrow \begin{cases} x_1 = 3 \\ x_2 = 1 \end{cases}$

(b) Solve the unknowns x and y from the pair of equations

$$\begin{cases} 6x + 7y = 4 \\ x + 2y = 2. \end{cases} \rightarrow x = 2 - 2y$$

$6x + 7y = 4$

$6(2 - 2y) + 7y = 4$

$12 - 12y + 7y = 4 \quad \| -12$

$-5y = -8$

$-5y = -8$

$y = \frac{8}{5}$

THEN $x = 2 - 2y = 2 - 2 \cdot \frac{8}{5} = \frac{10}{5} - \frac{16}{5} = -\frac{6}{5}$

ANSWER $\begin{cases} x = -\frac{6}{5} \\ y = \frac{8}{5} \end{cases}$

3. (a) Solve x when $5^{4x} = 3^{x+2}$.

SOL. $5^{4x} = 3^{x+2}$ $\quad \| \ln(\ )$

$\ln(5^{4x}) = \ln(3^{x+2})$

$4x \ln(5) = (x+2)\ln(3)$

$x(4\ln(5) - \ln 3) = 2\ln(3)$

$x = \frac{2\ln(3)}{4\ln(5) - \ln(3)}$