Example exam solutions 2

Monday, October 7, 2024 11:15 AM

1. Find $f'(x)$, when

   (a) $f(x) = 4x^5 - \sqrt{2x} + \frac{1}{x^3}$

   (b) $f(x) = 2\sin(3x) - 7e^{x^4}$

   (c) $f(x) = e^x \ln(x)$

$\sqrt{x} = x^{\frac{1}{2}}$ $\quad \frac{1}{x^p} = x^{-p}$

$D\, x^p = p\, x^{p-1}$

Sol (a) $D\ 4x^5 - (2x)^{\frac{1}{2}} + x^{-3} = 20x^4 - (2x)^{-\frac{1}{2}} \cdot \frac{1}{2} \cdot 2 - 3x^{-4}$

(b) $D\ 2\sin(3x) - 7e^{x^4}$

$= 2\cos(3x)\, D3x - 7e^{x^4} \cdot Dx^4$

$= 6\cos(3x) - 28x^3 e^{x^4}$

$D\, e^x = e^x$

$D\sin(x) = \cos(x)$

$D\sin(ax) = a\cos(x)$

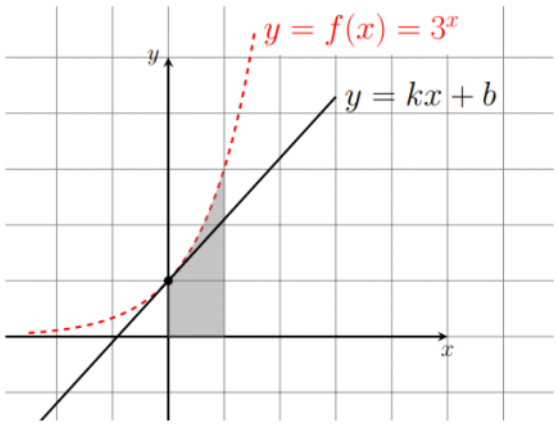
$D\, e^{f(x)} = f'(x) e^{f(x)}$

(c) $D\ e^x \cdot \ln(x)$

$= e^x \ln(x) + e^x \cdot \frac{1}{x}$

$D\, f \cdot g = f'g + fg'$

$D \ln(x) = \frac{1}{x}$



Kuva 1: A graph of a function, a tangent line and a shaded area. Gridlines are one unit apart.

2. Find the equation $y = kx + b$ of the line in Figure 1.

SOL. $f(x) = 3^x \longrightarrow f(0) = 3^0 = 1$

$f'(x) = 3^x \ln(3)$

$f'(0) = \ln(3)$

$a = 0$

$y = kx + b = f'(a)x + f(a)$,

$y = \ln(3)\,x + 1$

3. Calculate the shaded area in Figure 1.

SOL $\int_0^1 3^x \, dx = \left[\frac{3^x}{\ln(3)} \right]_{x=0}^{x=1} = \frac{3^1}{\ln(3)} - \frac{3^0}{\ln(3)} = \frac{2}{\ln(3)}$

4. Find x > 0 which is the maximum of $f(x) = x^8 e^{-3x}$.

SOL $f(x) = x^8 e^{-3x}$

$f'(x) = 8x^7 e^{-3x} + x^8 e^{-3x} \cdot (-3)$

$= x^7 e^{-3x}(8 - 3x) = 0$

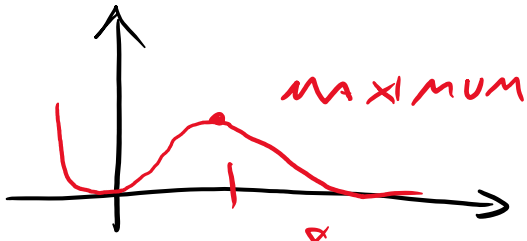
$\rightarrow x = 0$ OR $e^{-3x} = 0$ OR $8 - 3x = 0$

EXCLUDED — NOT POSSIBLE — $x = \frac{8}{3}$

EXCLUDED

NOT POSSIBLE

$x = \frac{8}{3}$



5. Calculate

(a) $\int 4x + \sqrt{x}dx$

(b) $\int \cos(2x)dx$

(c) $\int_2^3 e^{-x} + 1dx$

$$\int x^p\,dx = \frac{x^{p+1}}{p+1}, \text{ IF } p \neq -1$$

SOLS. (a) $\int 4x^1 + x^{\frac{1}{2}}\,dx = \frac{4x^2}{2} + \frac{2}{3}x^{\frac{3}{2}}$

(b) $\int \cos(2x)\,dx = \frac{1}{2}\sin(2x)$

$$\int \cos(ax) = \frac{\sin(ax)}{a}$$

$$\int g'(x) f'(g(x))\,dx = f(g(x))$$

(c) $\int_2^3 e^{-x} + 1\,dx$

$= \left[-e^{-x} + x \right]_{x=2}^{x=3}$

$= (-e^{-3} + 3) - (-e^{-2} + 2)$

$= e^{-2} - e^{-3} + 1$