Formulas

Differentiation and integration

Differentiation		Integration	
Dx^n	$= nx^{n-1}$	$\int x^n dx$	$= \frac{x^{n+1}}{n+1} + C$
De^x	$= e^x$	$\int e^x dx$	$= e^x + C$
Db^x	$= b^x \ln(b)$	$\int b^x dx$	$= \frac{b^x}{\ln(b)}$
$D\ln(x)$	$= \frac{1}{x}$		
$D\ln\lvert x\rvert$	$= \frac{1}{x}$	$\int \frac{1}{x} dx$	$= \ln\lvert x\rvert + C$
$D\log_a(x)$	$= \frac{1}{x\ln(a)}$		
$D\log_a\lvert x\rvert$	$= \frac{1}{x\ln(a)}$		
$D\sin(x)$	$= \cos(x)$	$\int \cos(x) dx$	$= \sin(x) + C$
$D\cos(x)$	$= -\sin(x)$	$\int \sin(x) dx$	$= -\cos(x) + C$
$D\tan(x)$	$= 1 + \tan^2(x)$	$\int 1 + \tan^2(x) dx$	$= \tan(x) + C$
$Dx\ln(x) - x$	$= \ln(x)$	$\int \ln(x) dx$	$= x\ln(x) - x + C$
$D\arcsin(x)$	$= \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}$	$= \arcsin(x) + C$
$D\arccos(x)$	$= \frac{1}{-\sqrt{1-x^2}}$	$\int \frac{1}{-\sqrt{1-x^2}}$	$= \arccos(x) + C$
$D\arctan(x)$	$= \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}$	$= \arctan(x) + C$
$D\sinh(x)$	$= \cosh(x)$		
$D\cosh(x)$	$= \sinh(x)$		
$D\tanh(x)$	$= \frac{1}{\cosh^2(x)}$		

Differentiation		Integration	
$Df(g(x))$	$= f'(g(x))g'(x)$	$\int f(g(x))g'(x) dx$	$= f(g(x)) + C$
Special cases			
$D\ln(g(x))$	$= \frac{g'(x)}{g(x)}$	$\int \frac{g'(x)}{g(x)} dx$	$= \ln(g(x)) + C$
$De^{g(x)}$	$= e^{g(x)}g'(x)$	$\int g'(x)e^{g(x)} dx$	$= e^{g(x)} + C$
Dfg	$= f'g + fg'$	$\int f'g dx$	$= fg - \int fg' dx$
$D(f/g)$	$= (gf' - fg')/g^2$		

Physics

Newtonian mechanics

Some forces

- $G = mg$ (gravitation)
- $F = kx$ (spring)
- $F = av^2$ (air resistance)

Differential equation is given by Newton's second law

$$F_{\text{total}} = mx''.$$

Lagrangian mechanics

Lagrange function is the difference of kinetic energy and potential energy. As a formula, $L = T - U$ or $L = K - P$

- Object in free fall $L = \frac{1}{2}my'' - mgy$
- Pendulum $L = \frac{1}{2}mL^2\theta'(t) - mgL(1 - \cos(\theta(t)))$
- Mass and spring $L = \frac{1}{2}mx'(t)^2 - \frac{1}{2}kx(t)^2$

Differential equation is given by Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial x'}\right) - \frac{\partial L}{\partial x} = 0.$$

Other differential equations

- Radioactive decay $N'(t) = -aN(t)$
- Newton's law of cooling $T'(t) = -k(T(t) - T_a)$

Differential equations

Second order linear ODE with constant coefficients

- ODE $y'' + by' + cy = 0$
- Characteristic equation $r^2 + br + c = 0$

Cases

- $r_1, r_2 \in \mathbb{R}$ solution
$$y(x) = A\exp(r_1 x) + B\exp(r_2 x)$$
- $r_1 = r_2 = r$ solution
$$y(x) = A\exp(rx) + Bx\exp(rx)$$
- $r_1 = a + bi$ solution
$$y(x) = \exp(ax)(A\cos(bx) + B\sin(bx))$$

Integrable ODE

The solution of
$$y' = q(x)$$
is $y(x) = \int q(x)dx$

Separable ODE

If you can arrange the equation as
$$a(y)dy = b(x)dx,$$
then you can integrate to obtain
$$\int a(y)dy = \int b(x)dx.$$

First order linear ODE

The solution of
$$y' + p(x)y = q(x)$$
is
$$y(x) = \frac{C}{\mu(x)} + \frac{1}{\mu(x)}\int \mu(x)q(x)dx, \quad \text{where} \quad \mu(x) = e^{\int p(x)dx}.$$

Fourier series

If f is periodic with period 2π and f, f' and f'' are piece-wise continuous, then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx),$$

where

$$\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx \end{aligned}$$

Moreover, if f is odd, that is, $f(-x) = -f(x)$, then

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(nx),$$

and if f is even, that is, $f(-x) = f(x)$, then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

Discrete Fourier transform / FFT

Transform and inverse transform

$$\begin{cases} y_0 &= x_0 + x_1 \\ y_1 &= x_0 - x_1 \end{cases}, \quad \begin{cases} y_0 &= \frac{1}{2}(x_0 + x_1) \\ y_1 &= \frac{1}{2}(x_0 - x_1) \end{cases}$$

Transform and inverse transform

$$\begin{cases} y_0 &= x_0 + x_1 + x_2 + x_3 \\ y_1 &= x_0 - ix_1 - x_2 + ix_3 \\ y_2 &= x_0 - x_1 + x_2 - x_3 \\ y_3 &= x_0 + ix_1 - x_2 - ix_3 \end{cases}, \quad \begin{cases} y_0 &= \frac{1}{4}(x_0 + x_1 + x_2 + x_3) \\ y_1 &= \frac{1}{4}(x_0 + ix_1 - x_2 - ix_3) \\ y_2 &= \frac{1}{4}(x_0 - x_1 + x_2 - x_3) \\ y_3 &= \frac{1}{4}(x_0 - ix_1 - x_2 + ix_3) \end{cases}$$